Impact of profile-averaged soil ice fraction on passive microwave brightness temperature Diurnal Amplitude Variations (DAV) at L-band

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\section*{ARTICLE INFO}

\textbf{Keywords:}
Frozen soil fraction
Soil optical depth
ELBARA-III
Maqu Network
Freeze-thaw state transition

\section*{ABSTRACT}

The dynamic change of frozen soil is crucial to land-surface modeling, carbon feedback studies, ground engineering (e.g., constructions), and microwave remote sensing. L-Band satellite missions Soil Moisture and Ocean Salinity (SMOS) and Soil Moisture Active Passive (SMAP) are currently exploited to characterize soil into freeze-thaw (FT) states. However, brightness temperatures ($T_B$) at L-band contain more information besides the FT state, particularly over permafrost or seasonally frozen soil, which has not been explored via current retrieval algorithms. To examine the potential for L-band $T_B$ observations, we define an index called Profile-Averaged Frozen Soil Fraction ($F_i$) related to Diurnal Amplitude Variation (DAV) of $T_B$ (i.e., $\Delta T_B$) based on the optical depth of the frozen soil column. We evaluated $F_i$ inferred from the 0th-order microwave transfer model with the SMAP L1c product, the ground-based European Space Agency L-Band Radiometer III (ELBARA-III) $T_B$ observations, and temperature profiles collected at the Maqu station in northeastern Tibet. While there is a clear relationship between $F_i$ and $\Delta T_B$, no apparent link exists with the ice content fraction ($F_i$) within a fixed-depth soil column. The proposed model certifies that the profile-averaged soil ice content $F_i$ relates to the dynamic microwave penetration depth by math and field measurement. The model reproduces well $\Delta T_B$ in Period Freezing but has problems in Period Thawing when melted surface water obstruct the microwave signals. Our findings can be used to exploit $\Delta T_B$ between 6 am and 6 pm, as a typically overpassing time by the SMOS and SMAP satellites, for estimating $F_i$, which can be further applied in weather/climate forecasting and for improving land-surface modeling.

\section{1. Introduction}

In cold and high-latitude regions (like Tibetan Plateau), the stability/safety of constructions (e.g., railways, roads, and other engineering projects) is highly susceptible to the dynamic change of frozen soil (Li et al., 2009; Cheng et al., 2008; Qingbai et al., 2002; Niu et al., 2017). Ice has a profound effect on the mechanical properties of frozen soil, significantly increasing its mechanical strength and bringing other unique properties to it. The dynamics of frozen soil also impact regional/global climate patterns (Zhang et al., 2018; Zhang et al., 2007; Hayashi, 2013), carbon feedback (Schuur et al., 2015a), and seasonal weather forecasting (Klepp and McPherson, 2017; Koren et al., 1999). Therefore, it is vital to study the dynamics of frozen soil, especially the surface freeze-thaw (FT) state transition that happens over the top few meters, which interacts with the atmosphere above from synoptic to seasonal scales. Since the ice content fraction ($F_i$) is a key property of frozen soil that can affect albedo, sensible heat flux, and latent heat flux regarding land-atmosphere interactions and change the soil hydraulic parameters in land-surface modeling (Decker and Zeng, 2006), acquiring $F_i$ from modeling and observations is a hotspot in both meteorologies (Schuur et al., 2015b) and engineering communities (Li et al., 2019).

Instead of $F_i$, many datasets offer in-site volumetric ice content ($\theta_i$) datasets. For instance, the dataset from Alaska was provided by the National Snow and Ice Data Center (NSIDC) (at http://nsidc.org/data/ggd903.html), and the data was taken from the Global Energy and Water Experiment (GEWEX) Asian Monsoon Experiment (GAME).
over a Tibet grassland site (at http://monsoon.t.u-tokyo.ac.jp/tibet/). It is to note that \( \theta_i \) in these datasets is not directly measured. \( \theta_i \) needs to be inferred from the observed volumetric liquid water content \( (\theta) \) time series along with the soil temperature \( (T) \) data. The inference assumes that the total mass of water \( (\theta) \) changes little during the subsequent rapid freezing period of approximately one month and then define

\[
F_e = \frac{\theta}{\theta_i} \quad \theta = \theta_i + \theta_f
\]

Eq. (1) depends on the \( T \) profile and the freezing point. Since the existence of super-cold water, the freezing point is usually not just 0 °C but with a soil temperature range. For instance, the Community Microwave Emission Modeling Platform (CMEM) (Rosnay et al., 2008) considers −0.5 °C to −5 °C as a freeze-thaw transition state and \( F_e = 0.5 \). Another example is Community Land Model (CLM), where the freezing point is not a constant but depends on the sand/clay fraction and the bound/unbound water content (Hoffman et al., 2004). The uncertainty of the freezing point affects \( F_e \) estimation. Note that the freezing and thawing processes evolve vertically, so Eq. (1) needs to contain depth information. In other words, \( \theta_i \) is associated with a depth interval in the field measurement or models, which is ignored in the formula. It is also to note that \( F_e \) can be very different for the permafrost, where the sub-surface soil is completely frozen, or for the seasonally frozen ground, where it experiences seasonal FT state transition in the topsoil layers. The former refers to complete frozen soil through the years, while the latter is wet soil below a certain depth.

Besides modeling and field measurement, passive microwave remote sensing is a possible way to explore \( F_e \). The dielectric constant of icewater mixing soil has a considerable difference from frozen or wet soil (Zhang et al., 2003). This difference can be used to estimate the FT state from the passive microwave remote sensing observations (Rautiainen et al., 2016; Rautiainen et al., 2014; Chen et al., 2019). Nevertheless, \( F_e \) is an essential input for microwave transfer forward modeling, and the misinterpretation of \( F_e \) will lead to tens of Kelvin’s difference in simulated brightness temperature \( (T_b) \) (Schwank et al., 2004). Without precise \( F_e \), the weather/climate forecast models cannot take full advantage of microwave observations from satellites via data assimilation, particularly when the soil is experiencing freeze-thaw events (Hirahara et al., 2020). Therefore, \( \theta_i \) as well as \( F_e \) shall be defined clearly with a depth interval. The depth of \( \theta_i \) is critical to this study because we need to clarify \( \theta_i \) at which depth shall be used to compute \( F_e \) and then compare it to the one inferred from \( T_b \). Thus, \( F_e \) in this study is the profile averaged with a precise cut-off depth like 10 cm (i.e., 0–10 cm) or 20 cm (i.e., 0–20 cm).

Monitoring \( F_e \) over large areas is very challenging, and there is no \( F_e \) remote sensing retrieval algorithm yet, which renders the abundant microwave satellite data under-exploited. This study develops a method to infer the profile averaged frozen soil fraction index \( (F_f) \), which corresponds to \( F_e \) with a fixed cut-off depth, related to Diurnal Amplitude Variation (DAV) of passive \( T_b \) signals at the L-band. Section 2.1 introduces the data, including the European Space Agency L Band Radiometer III (ELBARA-III) and SMAP, and \( T_b \) profile data collected at Maqu. Section 2.2 analyses these data and explains the impact of the FT state transitions on soil optical depth \( (\tau) \). Section 2.3 derives the relation between diurnal amplitude variation \( (DAV) \) of \( T_b \) (i.e., \( \Delta T_b \)) observed over a daily FT cycle and \( F_f \) combining \( T_b \)'s \( T_{shf} \) scheme (Lv et al., 2014) with a 9th-order microwave transfer model. Then, Section 2.4 introduces the statistical indicators used for evaluating \( F_f \) results. An evaluation of the relationship between \( F_f \) and \( \Delta T_b \) with the in-situ and SMAP data is presented in Section 3. We discuss the results and conclude in Section 4.

2. Observations and methodology

2.1. Observations

The Maqu network established in 2008 monitors near-surface vertical profiles of soil moisture \( (\theta) \) and \( T \) at several locations spread over an area in the eastern part of the Tibetan plateau. The network covers approximately one SMOS/SMAP footprint (Su et al., 2011; Zeng et al., 2016; Su et al., 2020) and thus contributes to the SMAP Cal/Val project (Collander et al., 2018). The area bounded by the Yellow River at its eastern and northern brinks is covered by meadows interspersed by a few trees or bushes. Due to its elevation of about 3300 m, the rainy summer period caused by the East Asia Summer Monsoon is relatively short (July–August). Thus, soil freeze/thaw state change (i.e., \( T \) minimum in a day is −0 °C) can last more than half a year (October to March the following year) (Su et al., 2011; Zhao et al., 2019). Since December 2015, the L-band (1.41 GHz) radiometer ELBARA-III was installed at Maqu’s central station. It observes the typical meadow vegetation with a 40 m by 25 m footprint at incidence angles from 40° to 70°, varying at 5° steps. In this study, we analyze the observations at 40° to stay consistent with the SMAP and focuses on the about one-year period from July 26th, 2017, to August 15th, 2018, local time at 102°E (GMT + 7, Fig. 1), which contains a complete frozen-soil season together with the leading and trailing freezing and thawing periods, respectively. Since 2016, soil texture (Table 1) and \( T \) (Fig. 3) profiles have been observed at the Maqu center station at 19 depths, with two sensors installed at 2.5 cm followed by one sensor at each step of 0.025 m down to 0.2 m (8 sensors) and then one at each step of 0.05 m down to 0.5 m (6 sensors in total), and one sensor each step of 0.1 m down to 1 m (5 sensors in total). From these profile observations, the local freeze/thaw state can be inferred at the Maqu site. To better distinguish between the annual and daily freezing/thawing cycles, we define the seasonal periods as Period Wet Soil/Freeze/Frozen Soil/Thawing as in vertical dashed lines Figs. 1–5 and \( T \) for the diurnal scale. The periods are defined primarily based on the soil temperature at 2.5 cm by \( T_{2.5cm} \), where Wet Soil is when the daily minimum is \( T_{2.5cm} > −0.5 °C \) and Frozen Soil is the daily maximum \( T_{2.5cm} < −0.5 °C \). Period Freezing is the days from the Wet phase to the Frozen phase, and in reverse for Period Thawing.

SMAP provides \( T_b \) observations over the Maqu network area at least once every three days and sometimes at 6 am and 6 pm in one day, depending on the latitude. We collected the SMAP L1c Radiometer Half-Orbit 36 km EASE-Grid \( T_b \) V4 product observed over the site from July 26th, 2017, to August 15th, 2018, from NSIDC (https://nsidc.org/data/smap/smap-data.html). The vastly different sizes of the SMAP and ELBARA-III footprints cause the differences between both observations in Fig. 1. For instance, the SMAP \( T_b \) is systematically lower by 20 K than the ELBARA-III \( T_b \) during the Wet Soil period and higher by the same amount during other periods due to the landscape heterogeneity in SMAP’s footprint. SMAP \( \Delta T_b \) is computed by the \( T_b \) difference between 6 am and 6 pm if SMAP scans the site twice a day; for ELBARA-III, \( T_b \) at 6 am and 6 pm are selected to compute \( \Delta T_b \) during the Wet Soil period is dominated by rainfall and evaporation. \( T_b \) from SMAP and ELBARA-III and \( T \) are positively correlated during Period Wet Soil. Both are higher in the early afternoon and lowest in the morning,\( T_b \) varying by about 10 °C and 15 °C, respectively. \( \Delta T_b \) is minimal during the Frozen Soil period because the large microwave penetration depth (Wang et al., 2019; Zheng et al., 2020; Zheng et al., 2019; Lv et al., 2019; Roy et al., 2017; Rowlandson et al., 2018) returns \( T_b \) characteristics for a relatively large soil column.

During Period Freezing and Period Thawing (Fig. 2), \( \Delta T_b \) signals are more complex. The large \( \Delta T_b \) cannot be fully explained by either daily soil temperature or penetration depth variations. As shown in Fig. 2, most of \( \Delta T_b \) can reach 20 K in a day. In the Freezing and Thawing periods, this phenomenon can be explained as the radiative heating over the day, which thaws the upper frozen soil leading to wet soil or even
ponded water at the surface. Besides, the ponded water drastically reduces the soil surface emissivity during the Thawing period. SMAP’s $|\Delta T_B|$ during the Freezing and Thawing periods are about 20 K which is less than ELBARA-III’s $|\Delta T_B|$ sometimes. The bias and the daily amplitudes difference between SMAP and ELBARA-III are generally due to the footprint mismatch; SMAP’s footprint is $10^6$ times larger which includes more complex terrain and land-cover than ELBARA-III’s. Fig. 3 shows $T$ profile measurements at Maqu. $|\Delta T_B|$ changes in Fig. 2 visually correlate well with the daily freezing and thawing suggested by $T$ variations between 0 and 0.2 m in Fig. 3. When the topsoil freezes at night, the deeper soil layers — frozen or thawed — will contribute to $T_B$, but with an attenuating effect from the frozen soil above. In this case, the soil profile shows a frozen-moist structure from up to down. Thus, $T_B$ does not change much in the nighttime. In contrast, daytime thawing of the surface soil significantly reduces the penetration depth, and a moist-frozen-moist structure appears in the soil profile, i.e., Fig. 3, in the Freezing/Thawing periods with the white splines between 0 and 20 cm. Especially during the Freezing and Thawing periods, this moist-frozen-moist structure appears shortly during the daytime and completely

### Table 1

Soil Properties measured by Field and Laboratory Experiments (cited from Zhao et al. 2018).

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand(%)</td>
<td>26.95</td>
<td>29.03</td>
<td>29.2</td>
<td>31.6</td>
<td>34.83</td>
</tr>
<tr>
<td>Clay(%)</td>
<td>9.86</td>
<td>9.95</td>
<td>10.15</td>
<td>10.43</td>
<td>9.35</td>
</tr>
<tr>
<td>Bulk density (g/cm$^3$)</td>
<td>0.76</td>
<td>0.95</td>
<td>1.23</td>
<td>1.4</td>
<td>1.49</td>
</tr>
</tbody>
</table>
disappears at night. Besides the different vertical structures, another mechanism that may cause $|\Delta T_B|$ is open water due to melted snow or ice. During the Thawing period, the frozen soil layer below the surface impedes infiltration/drainage and increases the ponding water at the surface.

2.2. Estimating freeze/thaw state and microwave soil optical depth

As stated in Introduction, $F_i$ is usually estimated by considering the $T$ profile, and we follow the same strategy in FT state inference here. First of all, the microwave soil optical depth, $\tau$, depends on the profile of the soil dielectric constant (Lv et al., 2019, 2018) and, thus, on the FT state of the soil at a particular depth. The general expression of $\tau$ is
modules to simulate the soil dielectric constant according to strategy as in CMEM. CMEM (Lange and de Rosnay, 2019) uses different profile regarding frozen-wet soil vertical structure, we adopt the same \( \tau \)

\[
\tau = \frac{2\pi \varepsilon'}{\lambda \sqrt{\varepsilon''}} z
\]

where \( \varepsilon' / \varepsilon'' \) are the real/image part of the uniform soil medium, \( \lambda \) is the wavelength (i.e., 21 cm for ELBARA-III and SMAP). For where the soil medium is not uniform vertically, such as a frozen/wet soil layer mixture, \( \tau \) is the integration from the surface (0 cm) to depth \( z \) as

\[
\tau = \int_0^z \frac{2\pi \varepsilon'}{\lambda \sqrt{\varepsilon''}} \, dz
\]

Eqs. (2) and (3) shall apply to frozen and wet soil. To calculate the \( \tau \) profile regarding frozen-wet soil vertical structure, we adopt the same strategy as in CMEM. CMEM (Lange and de Rosnay, 2019) uses different modules to simulate the soil dielectric constant according to \( \tau \) at a depth layer \( k \), and we follow the same strategy too as:

1) Case I: For \( T_k \leq -5 \) °C, Grimm’s model (Grimm et al., 2015) is used to calculate the relative dielectric constant of pure ice in the microwave region \( \varepsilon_{\text{ice}} \) which is then combined with other components for the determination of the soil dielectric constant.

2) Case II: For \( T_k \geq -0.5 \) °C, different models are available to compute the dielectric constant for moist, unfrozen soil \( \varepsilon_{\text{soil}} \) including Wang and Schmugge’s model (Wang and Schmugge, 1980), Mironov’s model (Mironov et al., 2009), Dobson’s model (Peplinski et al., 1995), and Calvet’s model (Calvet et al., 1995). Mironov’s model proved to be more suitable for the L-band is used in this study (Mironov et al., 2009; Wigneron et al., 2017) (Fig. 5).

3) Case III: For \( -5 \) °C < \( T_k \) < -0.5 °C, the dielectric constant is assumed as the linear average of Case I and II, i.e., \( \varepsilon_{\text{mix}} = (\varepsilon_{\text{mix}} + \varepsilon_{\text{ice}})/2 \).

Fig. 4 shows the temporal evolution of the resulting three soil states. Accordingly, soil freezing started at the end of November 2017 and reached almost 1 m in the end of February 2018, when thawing again began at the surface and persisted until April 2018. The mixed soil layers (grey zones in Fig. 4) indicate the freezing front. By combining the dielectric constant models with the soil characterization in CMEM, we derive the \( \tau \) profile (Fig. 5). The penetration depth (depth at \( \tau = 1 \), dotted line in Fig. 5) ranges from about 0.1 m down to 1 m over the year. Penetration depth is a vivid index to indicate the shape dielectric constant change during FT transition. The penetration depth we used in this study is calculated by its definition from the soil optical depth, i.e., one time of the soil optical depth. In this study, \( F_B \) and \( F_f \) will be estimated from Figs. 3 & 4 as ground truth for validation.

2.3. Methodology

The proposed \( F_f \) simulation model the daily thawing/freezing front (\( \varepsilon_f / \varepsilon_g \)) and the daily FT state cycles that give rise to the observed \( \Delta T_B \) between morning and evening (Fig. 6). The model is based on the 0th-order microwave transfer model and relates \( \Delta T_B \) between two observation times within 24 h to the respective change of the depth \( z_f \) of the near-surface thawing front and its optical depth \( \tau_f \) here, we show in particular that \( \Delta T_B \) can be related to the frozen soil fraction \( F_f \) regarding \( \tau \).

For clearness, we note here that \( z_f \) and \( \tau_f \) refer to daily freezing and thawing dynamics, while \( z_g \) and \( \tau_g \) refer to annual freezing dynamics, e.g., during the Freezing period (Fig. 6).

The 0th-order incoherent model extensively used in passive microwave remote sensing of soil moisture is formulated as

\[
T_B = E \cdot T_{\text{eff}}
\]

where \( T_B \) is the observed brightness temperature in K, \( E \) is the soil slab’s emissivity contributing to \( T_B \) and \( T_{\text{eff}} \) is the so-called effective temperature of the soil slab. \( E \) depends on soil moisture \( \theta \) and other factors that affect the dielectric constant of a soil column; \( T_{\text{eff}} \) depends on the \( T \) profile and dielectric constant. The penetration (e-folding) depth – as a measure of the soil slab depth mainly contributing to \( T_B \) - depends on the dielectric constant profile and can reach at the L-band meters down in extremely dry but also frozen soil.

Seeing from Figs. 4 & 5 that in the first few days of freezing, the soil is frozen down to about 0.2–0.4 m that the soil below the frozen layer does not contribute significantly to the observed \( T_B \) (e.g., a situation shortly before the daily thawing front develops in Fig. 6). We assume that both temperature and dielectric constant are vertically constant in the frozen layer; then Eq. (4) can be rewritten as

\[
T_B = E_f \cdot T
\]

with \( T \) the temperature of the frozen layer, and \( E_f \) the emissivity of the frozen layer. According to the white stripes in Fig. 3, a thin upper soil layer (<0.2 m) has thawed at 6 pm due to the positive surface radiation balance over the day, giving rise to \( z_f \), and correspondingly the \( \tau_f \) (Fig. 5). Thus, Eq. (5) can be formulated as

\[
T_B = E_f \cdot T (1 - e^{-\tau_f}) + E_t \cdot e^{-\tau_f}
\]

where \( E_t \) is the emissivity of the upper thawed soil layer; we further
can be calculated from $\tau$, assume that the thawed and frozen soil is predominantly ice-liquid mixture in the soil. Then $\Delta T_d$ between morning and late afternoon is

$$\Delta T_d = E_t(T(1-e^{-\tau}) + E_t e^{-\tau} - E_t T)$$

$$= (E_t - E_i) T (1-e^{-\tau})$$

(7)

With $\varepsilon_r$ and $\varepsilon_i$ the real and imaginary part of the dielectric constant of the thawed soil and $\tau_d$, the corresponding $\tau_f$ (Lv et al., 2018; Wilheit, 1978) is given by

$$\tau_f = \frac{2\pi\varepsilon_i^\prime}{\sqrt{\varepsilon_f^\prime}}$$

(8)

where $\tau_f$ is related to soil liquid water content $\theta \cdot \theta_1 = \theta (1 - F_i)$.

We now define the frozen soil fraction $F_i$ in terms of the soil optical depth as

$$F_i = \frac{\tau_f}{\tau_c}$$

(9)

where $\tau_c$ is the soil optical depth of the frozen soil in a certain soil optical depth $\tau_c$ (e.g., the penetration depth, $\tau_c = 1$). Thus $\tau_f + \tau_c = \tau_c$, and further, $\tau_f = (1 - F_i) \tau_c$. By adopting this $\tau_c = 1$, we can reformulate Eq. (7) as

$$\Delta T_B = (E_t - E_i) T (1 - e^{-\varepsilon_i^\prime})$$

(10)

The applicable depth of Eq. (10) depends on the penetration depth, as in Fig. 5. For Period Freezing, the depth ranges from <10 cm to 50 cm and goes deep to >1 m.

As we extend $F_i$ to the optical frozen soil fraction $F_i$ in this study, $F_i$ can be calculated from $\varepsilon_i$ and $\varepsilon_i^\prime$, with a cut-off $\tau$ (e.g., $\tau = 1, 2, \ldots$) for various soil layers. In this study, the default value of $\tau_c$ is the penetration depth, i.e., $\tau_c = 1$. On the other hand, $F_f$ is defined as $F_f = \theta_f / \theta_f$, which can only indicate the FT state at a certain depth. Fig. 7 compares $F_f$ and $F_i$ while setting up a cut-off depth at various depths. Fig. 7 compares $F_f$ and $F_i$ while setting up a cut-off depth at various depths. The bottom panel shows the increase of $F_f$ depending on the cut-off depth and soil optical depth along with the time.

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2.4. Statistical methods

We evaluate \( F_f \) with \( T_B \)-observations from ELBARA-III, SMAP \( T_B \) and \( F_f \) in geometric frameworks with a specified depth of 0.1 m and 0.2 m. While \( F_f \) refers to the soil optical depth frame, \( F_{f_zc} \) in a geometric frame is defined similarly to Eq. (1), for instance, when \( z_c = 0.1 \) m (or 0.2 m) as the cutting-off depth. 0.1 m and 0.2 m are selected because \( F_f \) and \( F_{f_zc} \) have the closest values at these depths, as in Fig. 7. Together with \( T_B \) observations at Time \( t_a \) and \( t_b \) where \( 0 < b-a < 24 \) h, we then create \( \Delta T_B = T_B(t_b) - T_B(t_a) \). These two time-series \( F_f \) and \( \Delta T_B \), are taken as input to compute the correlation coefficient as

\[
r = \frac{\sum_{j=1}^{n} (F_f_j - \bar{F}_f)(\Delta T_B_j - \bar{\Delta T}_B)}{\sqrt{\sum_{j=1}^{n} (F_f_j - \bar{F}_f)^2 \sum_{j=1}^{n} (\Delta T_B_j - \bar{\Delta T}_B)^2}}
\]

where the subscript \( j \) means \( j \)th day. \( r \) indicates the linear correlation between \( F_f \) at Time \( t_a \) and \( \Delta T_B \) between Time \( t_a \) and Time \( t_b \).

3. Results

We now compare the observed \( \Delta T_B \) at both polarizations from ELBARA-III and SMAP between 6 am and 6 pm over the Maqu site with both \( F_{f_zc} \) (Fig. 8a, b) with ELBARA-III’s \( T_B \) and \( F_f \) (Fig. 8c, d) with SMAP’s \( T_B \) according to Eq. (10). For \( F_{f_zc} \), comparisons are made for the cut-off depths \( z_c = 0.1 \) m and \( z_c = 0.2 \) m. \( \Delta T_B \) and \( F_{f_zc} \) are uncorrelated, and data points are randomly distributed or clustered. This is mainly because the top 0.1 m or 0.2 m of soil is frozen in the Freezing and Frozen periods (meaning the cut-off depth assumed here is not adequate). However, a clear correlation exists between \( \Delta T_B \) and \( F_f \) (Fig. 8b, d). The linear relation becomes even more apparent when the samples are separated into the Freezing and Thawing periods. For most points, H-polarization

![Fig. 8.](image-url)
coincides with V-polarization in Fig. 8b&d.

ELBARA-III measures once every half an hour and it is twice a day for SMAP. Thus, with the ELBARA-III observations, we are not restricted to using 6 am and 6 pm, i.e., we can look at different time intervals in a day. Fig. 9 shows the correlation coefficient between the resulting $\Delta T_B$ and the $F_f$ by selecting all possible $t_b$ and $b$. According to Fig. 9, the correlation between $\Delta T_B$ and $F_f$ is maximum for $b = 7$ am independent of $a$; here, $r$ can reach 0.4-0.7 depending on Time $a$. Since the daily air temperature minimum happens mostly before sunrise, it makes sense that $r$ is positive because $T_{B,b} - T_{B,a} > 0 ^\circ$ C. $r$ always reaches negative for $b$ from 2 pm towards the middle of the night with a minimum ($-0.89$) at $a = 6$ am and $b = 10$ pm. In general, we can conclude that 6 am and 6 pm is a pair of suitable observation times for inferring $F_f$ from $\Delta T_B$.

In Period Freezing, the theoretical estimates of $\Delta T_B$ from $F_f = 1 - \exp(-\tau_f)$, and ELBARA-III’s $\Delta T_B$-$F_f$ relations with $r = 1$ in V-polarization (dots in Fig. 8b&d) are established as the black curve in Fig. 10. While $F_f$ ranges from 1 to 0, $\Delta T_B$ changes from 60 K to 0 K. The curve is modeled by Eq. (10), and it is not a linear line but an exponent curve in shape. When $F_f$ is small, $\Delta T_B$ is relatively large because most ice content in a detectable depth of the L-band will melt during the daytime; otherwise, $\Delta T_B$ will not change if no ice content melts in a day. The former case is common in the Freezing and Thawing periods, and the latter only happens in the Frozen periods. Most of the $\Delta T_B$ estimated from ELBARA-III fit the curve, and RMSE is 12 K while $r = 0.83$.

4. Discussion and conclusions

When the lower front of the frozen soil layer progresses to deeper soil layers (Period Freezing), we lack a theory that relates the observed $T_B$ to the observed state of the soil profile, which hampers the usability of derived products during this period, e.g., from SMAP or SMOS. Here, we propose a model that can simulate $T_B$ DAV signals from $F_f$ in Period Freezing. To this goal, the model uses a parameterization of the daily daytime thawing of the upper soil layers. We could conclude that $\Delta T_B$ between 7 am and 7 pm, which is close to the overpassing time for SMOS/SMAP, is optimal for the retrieval of $F_f$ and $\Delta T_B$ between 6 am and 6 pm is also suitable for identifying a clear $\Delta T_B$-$F_f$ relationship. $T_B$ observations from SMAP and ELBARA-III are used to validate the results.

For the retrieval of $F_f$ from $\Delta T_B$ by Eq. (11), profile observations of $T$ are not needed because the change of dielectric constant of frozen soil with temperature is much smaller than its change due to the FT state transition, while $T$ in deeper layers does not vary much during this period. We only used the $T$ profiles observed at the Maqu center station to validate the result, which needs further validation in other sites.

However, we proved that $F_f$ can be retrieved from a $\Delta T_B$ observation regarding $r$ (not geometric depth), which can be applied to any land surface type. The $r$ of fixed geometric layers may change due to freezing and thawing, making it impossible to interpolate neighboring samples (at annexed layers). In addition, we do need soil parameters like the clay fraction to obtain the geometrical depth $F_f$ regarding $F_f$.

The seasonal Freezing and Thawing periods have different dynamics impacting $\Delta T_B$. For wet soil, $T$ profiles change $T_B$ and, furtherly, $\Delta T_B$. Thus $\Delta T_B$ is also related to $T$ changes, especially in the top 10 cm. However, for frozen soil, $\Delta T_B$ is dominated by the surface FT state transitions, while the topsoil temperatures can be ignored for $T_B$ under frozen conditions. These conditions simplify the microwave transfer model, limiting the method presented here to the Freezing period. The concept of $r$ is seldom used to analyze $T_B$ at L-band with ground measurement. This concept has advantages over the geometric depth frame.
in analyzing $T_B$ signals in certain circumstances, such as freezing/thawing. The method presented in this study is expected to improve our understanding of the freezing/thawing process.

Besides, regarding the application of the method, it is challenging to determine $F_I$ in land-surface models. For instance, SMOS needs the Hydrology Tiled ECMWF Scheme for Surface Exchanges over Land (HTESSEL) to provide land surface inputs for retrieval or forward simulations. At the same time, SMAP uses the Modern-Era Retrospective Analysis for Research and Applications, Version 2 (MERRA2). Both models have too few layers for determining $F_I$ which makes the $F_I$ retrievals (from satellites) possible for calibration/validation of the land surface model’s outputs.

CRediT authorship contribution statement

Shaoning Lv: Conceptualization, Methodology, Software, Validation, Visualization, Investigation, Writing – original draft. Clemens Simmer: Writing – review & editing. Yijian Zeng: Writing – review & editing. Zhongbo Su: Supervision. Jun Wen: Writing – review & editing.

Appendix A. The variable definitions in Section 2

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>Soil bulk emissivity</td>
</tr>
<tr>
<td>$E_t$</td>
<td>Thawed soil bulk emissivity</td>
</tr>
<tr>
<td>$\varepsilon_t$</td>
<td>The real part of the dielectric constant of thawed soil</td>
</tr>
<tr>
<td>$\varepsilon_t^*$</td>
<td>Imagery part of the dielectric constant of thawed soil</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength $\lambda = 21$ cm</td>
</tr>
<tr>
<td>$\theta$</td>
<td>the volumetric soil moisture content</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>volumetric ice content</td>
</tr>
<tr>
<td>$\theta_l$</td>
<td>volumetric liquid water content</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Soil optical depth, $\tau = \frac{2\pi}{\lambda \sqrt{\varepsilon}}$ for uniform soil medium, or $\tau = \frac{2\pi}{\lambda \sqrt{\varepsilon}}$ for non-uniform soil medium</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>Cutting-off soil optical depth</td>
</tr>
<tr>
<td>$\tau_ff$</td>
<td>Soil optical depth of frozen soil, $\tau_ff = \tau_c - \tau$</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>Soil optical depth of thawed front, $\tau_f = \frac{2\pi}{\lambda \sqrt{\varepsilon}}$</td>
</tr>
<tr>
<td>$\Delta T_B$</td>
<td>temporal $T_B$ changes between two random times within 24 cycles at Climate Scale in the Third Pole Environment</td>
</tr>
<tr>
<td>$F_{SW}$</td>
<td>Alias as $f = (E_t - E_t')T$</td>
</tr>
<tr>
<td>$F_t$</td>
<td>Frozen soil fraction in terms of soil optical depth</td>
</tr>
<tr>
<td>$T$</td>
<td>Soil temperature, $T = T_{sw}$ for uniform soil column</td>
</tr>
<tr>
<td>$T_B$</td>
<td>1-band brightness temperature</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Soil effective temperature</td>
</tr>
<tr>
<td>$z$</td>
<td>the ground surface temperature</td>
</tr>
<tr>
<td>$\eta_f$</td>
<td>Soil geometric depth of the frozen front at the annual scale</td>
</tr>
<tr>
<td>$\eta_d$</td>
<td>Soil geometric depth of the thawing front at the daily scale</td>
</tr>
</tbody>
</table>

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This research was funded by the National Natural Science Foundation of China (Grant 42075150) and by the Natural Science Foundation of Shanghai (No. 21ZR1405S00). Also, by the Deutsche Forschungsgemeinschaft (DFG) via the research group FOR2131 on “Data Assimilation for Improved Characterization of Fluxes across Compartmental Interfaces”, subproject P2. This work was also partly supported by the ESA MOST Dragon IV Program (Monitoring Water and Energy Cycles at Climate Scale in the Third Pole Environment) and partly by the Netherlands Organization for Scientific Research under Project ALW-GO/14-29.

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