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CONDENSED MATTER

Abrikosov Vortices in SF Bilayers[1]

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We study the spatial distribution of supercurrent circulated around an Abrikosov vortex in an SF bilayer in perpendicular magnetic field. Within the dirty limit regime and circular cell approximation for the vortex lattice, we derive the conditions when the Usadel equations the F-layer can be solved analytically. Using the obtained solutions, we demonstrate the possibility of reversal of direction of proximity induced supercurrents around the vortex in the F-layer compared to that in the S-layer. The direction of currents can be controlled either by varying transparency of the SF interface or by changing an exchange field in a ferromagnet. We argue that the origin of this effect is due the phase shift between singlet and triplet order parameter components induced in the F-layer. Possible ways of experimental detection of the predicted effect are discussed.

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It is well known that the critical temperature, T_C, of superconductor–ferromagnet (SF) sandwiches and critical current, I_C, of SFS Josephson junctions are nonmonotonic functions of thickness, d_F, of the ferromagnetic layer [1–3]. This nonmonotonic behavior can be used for developing superconducting spin valves. Adding another ferromagnetic layer allows one to control T_C or I_C by changing mutual orientation of magnetic moments of the F films in SFF or SFFS spin valve devices (see recent reviews [4–6] and references therein). It should be noted that these theoretical predictions [7–13], as well as their experimental confirmations [4–6] were obtained in structures, which are homogeneous along SF interfaces.

However, it was recently demonstrated [14] that in-plane inhomogeneity of the S-layers in S/F/F spin valves causing the suppression of the superconducting spin valve effect (SSVE). Such inhomogeneity significantly increases when morphology of the S-layer changes from the form of overlapping islands to a smooth case. Another type of inhomogeneity of superconducting state in superconductor can be provided by Abrikosov vortices. Superconducting correlations in an SF bilayer in a vortex state monotonously increase with increasing distance from the vortex core in both S and F films.

Despite large number of studies devoted to flux pinning and flux dynamics in superconductor/ferromagnet bilayers and multilayers, it is so far an open question, should one (by analogy with the oscillations of T_C and I_C) expect non-monotonic alterations in the structure of Abrikosov vortex in SF sandwich. The purpose of this paper is to show that it is indeed possible. We demonstrate that by varying the exchange field in an F-layer or by varying S/F interface transparency one can achieve vortex current reversal in the F-layer.

We consider an SF bilayer in an external magnetic field H perpendicular to the plane of the bilayer. We assume that the conditions of dirty limit are valid for both films and pair potential Δ is zero in the F film. The F-layer is supposed to be a single domain ferromagnet with out-of-plane direction of its easy axis. We direct the z axis along the magnetic field and place the coordinate origin at the interface between S and F metals located at $-d_S \le z \le 0$ and $0 \le z \le d_F$, respectively. To define the coordinate dependence of the Green's function it is convenient to use the Wigner–Seitz approximation [15, 16] for elementary vortex cell. This approximation has been previously used in study of Abrikosov vortex lattice and flux flow regimes in superconducting films, as well as in theoretical analysis of influence of trapped Abrikosov vortices on properties of tunnel Josephson junctions [17–25].

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According to the Wigner–Seitz approximation, the hexagonal unit cell of the vortex lattice is replaced by a circular one with the radius

$$r_S = r_C\sqrt{\frac{H_{C2}}{H}}, \quad r_C = \sqrt{\frac{\Phi_0}{\pi H_{C2}}}. \tag{1}$$

For a single S film, the second critical field, H_{C2} and, hence, r_S are determined by the well-known relation [15]

$$\ln t + \psi\left(\frac{1}{2} + \frac{t}{r_S^2}\right) - \psi\left(\frac{1}{2}\right) = 0. \tag{2}$$

Here, $\psi(x)$ is the psi-function, Φ_0 is the magnetic flux quantum, $\xi_S = (D_S/2\pi T_C)^{1/2}$ is the superconductor decay length, D_S is the diffusion coefficient, $t = T/T_C$, T is the temperature of the bilayer, and r_S is normalized on ξ_S. Below, we will define the radius of the circular cell, r_S, using Eqs. (1) and (2), thus neglecting the magnetic field generated by the ferromagnetic film as compared to the external magnetic field H.

Under the above assumptions the system of Usadel equations [26] describing the behavior of SF sandwich in magnetic field in the polar coordinates has the form [1–3, 22, 23]

$$\frac{d^2\theta_S}{dz^2} + \frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_S}{dr}\right) - (\Omega + Q^2\cos\theta_S)\sin\theta_S = -\Delta\cos\theta_S, \tag{3}$$

$$\frac{d^2\theta_F}{dz^2} + \frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_F}{dr}\right) - \frac{\widetilde{\Omega} + k^2Q^2\cos\theta_F}{k^2}\sin\theta_F = 0, \tag{4}$$

$$Q = \frac{1}{r}\left(1 - \frac{r^2}{r_S^2}\right), \tag{5}$$

$$\Delta\ln t + 2t\,\mathrm{Re}\sum_{\Omega\geq 0}^{\infty}\left(\frac{\Delta}{\Omega} - \sin\theta_S\right) = 0. \tag{6}$$

Here, $\widetilde{\Omega} = \Omega + iE$, $\Omega = (2n+1)t$ are the Matsubara frequencies, E is the exchange energy, $\xi_F = (D_F/2\pi T_C)^{1/2}$, D_F is the diffusion coefficient in the F film, Q is the component of the vector potential $\mathbf{Q} = (0, Q, 0)$ normalized on $\Phi_0/2\pi\xi_S$, d_S is the thickness of the S film, the order parameter, Δ, and exchange energy in (3)–(6) are normalized on πT_C, coordinates r and z are normalized on ξ_S, $k = \xi_F/\xi_S$, and $\mathrm{Re}(z)$ is the real part of the function z.

To write the solution of the Maxwell equation, $\mathrm{curl\,curl}\,\mathbf{Q} = \kappa^{-2}\mathbf{J}$ for the vector potential $\mathbf{Q}$ in the form of Eq. (5), we have supposed that the Ginzburg–Landau parameter $\kappa = \lambda_{S\perp}/\xi_S \gg 1$. This condition allows neglecting the magnetic field produced by current in comparison with the applied external field H. The external field is constant inside a circular vortex cell provided that the cell radius r_S is less than $\lambda_{S\perp} = \max(\lambda_S, \lambda_S^2/d_S)$, where λ_S is the London penetration depth.

Equations (3)–(6) should be supplemented by the boundary conditions [27] at SF interface ($z = 0$)

$$\gamma_B k\frac{d\theta_F}{dz} = \sin\theta_F\cos\theta_S - \sin\theta_S\cos\theta_F, \tag{7}$$

$$\frac{d\theta_S}{dz} = \gamma k\frac{d\theta_F}{dz}, \tag{8}$$

where γ_B and γ are the suppression parameters

$$\gamma_B = \frac{R_{BF}\mathcal{A}_B}{\rho_F\xi_F}, \quad \gamma = \frac{\rho_S\xi_S}{\rho_F\xi_F}. \tag{9}$$

Here, R_{BF} and $\mathcal{A}_B$ are the resistance and area of the FS interface, respectively, and $\rho_{S,F}$ are the normal state resistivities of the metals. At free interfaces the boundary conditions has the form

$$\frac{d\theta_S}{dz} = 0, \quad z = -d_S, \tag{10}$$

$$\frac{d\theta_F}{dz} = 0, \quad z = d_{N,} \tag{11}$$

and at $r = r_S$ we have

$$\frac{d\theta_F}{dr} = 0, \quad \frac{d\theta_S}{dr} = 0. \tag{12}$$

The boundary-value problem given by Eqs. (3)–(12) can be simplified in the limit of small F-layer thickness.

If $d_F \ll \xi_F/\mathrm{Re}(\sqrt{\widetilde{\Omega}})$, then in the first approximation $\theta_F = \theta_F(r)$ is independent on z, and in the next approximation we have

$$\frac{d\theta_F(d_F)}{dz} = \frac{d_F}{k\xi_F}\left[(\widetilde{\Omega} + \kappa^2Q^2\cos\theta_F)\sin\theta_F - \frac{k^2}{r}\frac{d}{dr}\left(r\frac{d}{dr}\theta_F\right)\right]. \tag{13}$$

Substitution of (13) into the boundary conditions (7), (8) gives

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\theta_F\right) - \left(\frac{\widetilde{\Omega}}{k^2} + Q^2\cos\theta_F + \frac{\cos\theta_S}{k^2\gamma_{BM}}\right)\sin\theta_F + \frac{\sin\theta_S\cos\theta_F}{k^2\gamma_{BM}} = 0, \tag{14}$$

$$\frac{d\theta_S}{dz} = \gamma_M \times\left[(\widetilde{\Omega} + k^2Q^2\cos\theta_F)\sin\theta_F - \frac{k^2}{r}\frac{d}{dr}\left(r\frac{d\theta_F}{dr}\right)\right], \tag{15}$$

where $\gamma_M = \gamma d_F/\xi_F$, $\gamma_{BM} = \gamma_B d_F/\xi_F$. This equation reduces to the derived earlier in [28] at $E = 0$. At $H \ll H_{C2}$ in a vicinity of $r \lesssim r_S$ both Q and spatial derivatives on r are small and from (14) it follows that at $r \to r_S$ Usadel functions in the F film asymptotically approach to the solution obtained earlier for SF sandwich in the limit of the small F-layer thickness [29]:

$$\tan\theta_F = \frac{\sin\theta_S}{(\tilde{\Omega}\gamma_{BM} + \cos\theta_S)}. \quad (16)$$

Substitution of this solution into (15) gives that at $r \lesssim r_S$:

$$\frac{d\theta_S}{dz} = \frac{\gamma_M\tilde{\Omega}\sin\theta_S}{\sqrt{\tilde{\Omega}^2\gamma_{BM}^2 + 2\tilde{\Omega}\gamma_{BM}\cos\theta_S + 1}}. \quad (17)$$

It follows from (17) that for sufficiently small $\gamma_M \ll \mathrm{Re}\left(\tilde{\Omega}/(\gamma_{BM}\tilde{\Omega} + 1)\right)$ one can neglect the suppression of superconductivity in the S-layer and consider $\theta_S(r)$ as known function describing Abrikosov vortex in the individual S film. Thus the boundary value problem is reduced to solution of Eq. (14), in which $\cos\theta_S$ and $\sin\theta_S$ are the solutions for the vortex state in a superconducting film as functions of coordinate r. There are two characteristic lengths in Eq. (14). The first one is ξ_S, at which the variation of $\theta_S(r)$ takes place. The second one, $\xi_{ef} = \xi_F\sqrt{\gamma_{BM}/(\gamma_{BM}\tilde{\Omega} + 1)}$, is characteristic scale in Eq. (14).

As a rule, in ferromagnetic materials $\mathrm{Re}(\xi_{ef})$ is much smaller than ξ_S. In the limit $\xi_S \gg \mathrm{Re}(\xi_{ef})$ at $r \lesssim \xi_S$ functions $\sin\theta_S \approx \alpha r$ and Eq. (14) for the case of a single vortex ($r_S \gg \xi_S$) transforms to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_F}{dr}\right) - \frac{\theta_F}{r^2} - \frac{\tilde{\Omega}\gamma_{BM} + 1}{\kappa^2\gamma_{BM}}\theta_F + \frac{\alpha r}{\kappa^2\gamma_{BM}} = 0. \quad (18)$$

Substitution $\theta_F = \beta r$ into (18) gives

$$\theta_F = \beta r, \quad \beta = \frac{\alpha}{(\tilde{\Omega}\gamma_{BM} + 1)}. \quad (19)$$

For $r \gtrsim \xi_S$ all derivatives and the item proportional to Q^2 in (14) are of order of $(\mathrm{Re}(\xi_{ef})/\xi_S)^2$ and $(\mathrm{Re}(\xi_{ef})/r_S)^2$, respectively, and can be dropped leading to

$$\tan\theta_F(r) = \frac{\sin\theta_S(r)}{\gamma_{BM}\tilde{\Omega} + \cos\theta_S(r)}. \quad (20)$$

In the limit of small r values, the solution of Eq. (14) asymptotically transforms into Eq. (19). This allows one to use it for all values of r.

The substitution of this solution into the expression for the supercurrent density in the F-film

$$J_S(r) = \frac{2\pi T\sigma_F}{e}\mathrm{Re}\sum_{\Omega\geq 0}^{\infty}\sin^2\theta_F(r)Q \quad (21)$$

results in

$$\frac{e\rho_F\xi_F J_S(r)}{2\pi T_C} = t\sum_{\Omega\geq 0}^{\infty}\frac{p}{\sqrt{p^2 + q^2}}\sin^2\theta_S(r)Q, \quad (22)$$

where

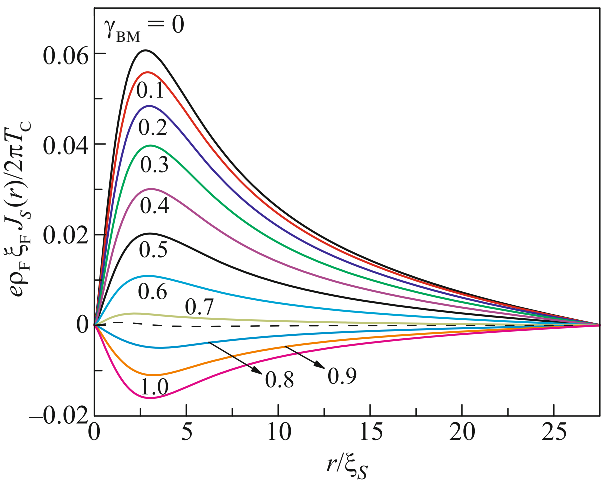
$$p = \left(1 + 2\gamma_{BM}\Omega\cos\theta_S(r) + \gamma_{BM}^2\Omega^2\right) - E^2\gamma_{BM}^2, \quad (23)$$

$$q = 2\gamma_{BM}E\left(\cos\theta_S(r) + \Omega\gamma_{BM}\right). \quad (24)$$

It follows from the above expression that with an increase in E or γ_{BM} the transformation takes place when proximity induced vortex supercurrent around the core in the F-layer changes its direction compared to the current in the S-layer.

To illustrate this effect, we performed numerical calculations of the supercurrent within the vortex unit cell in the F-layer. The results for the spatial dependence of the supercurrent are shown in figure for different values of γ_{BM} and fixed exchange energy $E/\pi T_C = 2$. We have chosen $H/H_{C2} = 0.01$, which corresponds to $r_S = 27.5\xi_S$ (the regime of single vortex). It is seen that for $\gamma_{BM} = 0$, the proximity-induced circulating supercurrent flows in the F-layer. The current density achieves its maximum value at $r \approx 2.5\xi_S$ and then goes to zero at $r \to r_S$. Increase in γ_{BM} results first in gradual suppression of this current and at $\gamma_{BM} \approx 0.73$ (the dashed line in figure) two regions are formed inside the cell with currents flow in opposite directions. Further increase in γ_{BM} leads to reversal of the supercurrent direction in the F-layer compared to that in the S-layer.

The physical mechanism of this transformation is the same as discussed previously for the formation of so-called π-junctions in SFS Josephson devices, for the non-monotonic dependence of the effective magnetic field penetration depth on the thickness of F-layer [30–33] as well as for the peculiarities of surface impedance in SF bilayers [34, 35] and the paramagnetic Meissner effect [36]. The superconducting correlations nucleated at SF interface consist of two parts. They are singlet and triplet pairings. There is $\pi/2$ phase shift between anomalous Green's function describing the pairings. As a result, the superconducting current in the SF structures has two contributions. The first one, defined by the singlet superconducting correlations, is always positive. The second, negative, contribution to the current is due to the triplet order parameter component. Such separation of the current into two components is realized in the present case, as follows from Eqs. (22) and (23). In a certain range of



(Color online) Spatial distribution of the supercurrent within the vortex unit cell in the F part of SF bilayer for $T = 0.2T_C$, $E = 2\pi T_C$, and for various values of the suppression parameter γ_{BM}. The dashed line corresponds to $\gamma_{BM} \approx 0.73$.

parameters of the studied SF structures, the negative contribution to the current may prevail over the positive one thus resulting to the vortex current reversal discussed above. Similar effect is responsible for the change of sign of the critical current in SFS junctions (π-junctions), as well as for the change of the direction of shielding supercurrents in the problem of penetration of the magnetic field in the SF bilayers.

It is necessary to mention again (see discussion after Eq. (6)) that in our model we neglect the magnetic field produced by supercurrent in comparison with the applied external field $H = \Phi_0/\pi r_S$. In this approximation, the magnetic field is independent of r and its integration over circular unit cell results in magnetic flux inside the cell exactly equal to Φ_0, independently on a direction of supercurrent circulating around the vortex core. In the next approximation with respect to the Ginzburg–Landau parameter $\kappa \gg 1$ there should be corrections to spatial distribution of the magnetic field inside the unit cell proportional to κ^{-2}. In the case of an SN bilayer ($E = 0$) or for $\gamma_{BM} \lesssim 0.7$ and $E = 2\pi T_C$ (see figure) the correction has maximum in the center of the vortex core and decreases monotonically with an increase in r. Therefore, the net magnetic field should exhibit small spatial modulation typical for an Abrikosov vortex: there is maximum of H in the core region ($r \lesssim \xi_S$) and monotonous decay to a constant value at $r = r_S$.

Contrary to that, for $\gamma_{BM} \gtrsim 0.75$ and $E = 2\pi T_C$ the correction to magnetic field generated by circulating supercurrent in the area $r \lesssim \xi_S$ should have direction opposite to that of external field H. As a result, the net magnetic field should have a minimum in the core region and should increase with r.

Note that magnetic flux per unit cell exactly equals to Φ_0 in both cases considered above. At the same time, the difference between magnetic field distributions can be detected by means of magnetic force microscopy [37], by muon scattering experiments [38] or by means of nano-SQUID [39–41].

It is worth to note that near the SF interface there are also triplet components induced into the S part of the bilayer. It means that transformations of spatial distribution of circulating supercurrent should also occur on the superconducting side of the SF bilayer. This problem will be analyzed elsewhere.

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