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In Memoriam Walter Kern

Walter Kern died on January 29, 2021, at the age of 63, after fighting around half a year against cancer. Walter graduated in mathematics at the University of Erlangen and received his Ph.D. from the University of Cologne in 1985 and his habilitation in 1989. He joined the University of Twente in 1988, first as an assistant professor and later as an associate professor in the Department of Applied Mathematics.

Walter's research contributions

Walter had an incredibly broad spectrum of scientific interests and made significant contributions not only in combinatorial optimization – ranging from design and analysis of algorithms for many different problems to discrete mathematics and graph theory – but also in algorithmic game theory and traffic engineering. His papers highlight the strong ties between the theory of combinatorial optimization and algorithmic game theory, linking mathematics, computer science and economics.

In the following, we describe some of his research, but we are fully aware that this can only be a glimpse at Walter's contribution of the last decades.

The "greedy algorithm" represents one of the most naive approaches to attack an optimization problem algorithmically: construct a solution iteratively by always taking the locally best step. In terms of combinatorial packing problems of type $\max\{c^T x | Ax \leq b, x \geq 0\}$, where the constraint matrix A has values in $\{0, 1\}$, the greedy approach orders the components of the objective function according to non-increasing values and then constructs a feasible solution for the linear program greedily by iteratively raising the next augmentable variable as far as possible until one of the constraints becomes tight. Certainly, such greedy approaches can easily fail. In his various papers, Walter studied the question which conditions of the underlying problem structure guarantee the optimality of the greedy algorithm.

Walter's contributions in this research area are, without any doubt, significant. Let us mention a few examples: he characterized a class of non-negative real constraint matrices in terms of forbidden submatrices that guarantees the optimality of the greedy algorithm relative to any objective function, right-hand side and arbitrary box constraints [3]. Walter introduced and analyzed a model of a primal–dual pair of combinatorial packing and covering problems for which it can be guaranteed that a certain, natural, primal–dual greedy approach terminates with an optimal solution [6]. This model can be viewed as a common generalization of various existing extensions of matroids and generalizes the idea of the Lovász extension of set functions and the discrete Choquet integral. Thirdly, he investigated a natural greedy approach for packing problems of type $\max\{1^T x | Ax \leq b, 0 \leq x \geq u\}$, where the underlying matrix A has entries in $\{-1, 0, 1\}$. The greedy approach may be viewed as an extension of the Ford–Fulkerson algorithm for computing a maximum flow in a network [7]. Walter identified conditions for the underlying system (A, b) that guarantee that the greedy algorithm terminates after a quadratic number of augmentations and/or terminates with an optimal solution.

Walter's investigations on the performance of algorithms for combinatorial packing and covering problems also play an important role in cooperative game theory that models and analyzes situations where several decision takers ("players") are, in principle, willing to cooperate and negotiate on how to share the overall benefit or cost in a fair and stable manner. The most prominent solution concept in this area is the core, which is the set of all feasible outcomes of an n-player game that cannot be improved upon by any coalition of players. As it turns out [4,5] combinatorial models for which the greedy algorithm "works" are in strong connection to game theoretic models, e.g., Shapley's "convex games" [9], for which core solutions exist and can be computed efficiently.

Walter was a recognized expert in the area of cooperative game theory and also had a deep knowledge of other classical solution concepts, such as the nucleolus and the Shapley value. His papers include general results that set conditions for solution concepts to be tractable as well as many complexity results for specific cooperative games, such as matching games, flow games, min-cost spanning tree games and so on. Many cooperative games have an underlying discrete

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structure that can be modelled as a graph. Walter also had a thorough knowledge of graph theory. His publications display a large variety of algorithmic techniques and include the design of exact algorithms, approximation algorithms, and online algorithms for solving well-known graph problems. In recent years, Walter also worked in the more applied area of international kidney exchange. In this application, kidney patients who have a willing but incompatible donor are placed in an international pool of patient–donor pairs. The goal is to swap donors to obtain solutions that are seen as fair by each of the participating countries. By applying his knowledge on cooperative game theory and graph theory, Walter was able to make novel contributions in this area of research as well.

Since 2010, Walter became interested in models from traffic engineering, in particular in so-called traffic assignment problems. In these models, a transportation network with links and routes is given and the assignment is described by a vector x of link flows and a vector f of route flows. The aim is to predict and to improve the traffic flow on the network. Typically, these assignment models lead to convex or non-convex problems from continuous optimization.

A classical model is the Wardrop equilibrium traffic assignment that is characterized by the property that all travelers with the same departure and arrival place only use routes with the same travel time. It is well-known that, unlike the link flow x of the Wardrop equilibrium, the route flow f of the Wardrop equilibrium might not be unique. With several co-authors [1], Walter studied this non-uniqueness problem and established a special set of additional so-called proportionality conditions on f that leads to a unique equilibrium route flow. Walter's other publications deal with the boundedly rational user equilibrium model [2]. Here, the drivers also use routes where the travel time is within a certain "indifference band". This model leads to so-called bilevel programs. Walter also investigated the effect of link and route tolls on the flow of traffic networks. For instance, he considered the situation where different stakeholders compete for the toll settings [8]. In such a model it is natural to apply techniques from game theory and to look for Nash equilibria or for stable coalitions.

Walter's relation to the Cologne Twente Workshop on Graphs and Combinatorial Optimization (CTW)

For the Cologne Twente Workshops, Walter was one of the constant factors over the whole period of their existence. He was involved in these workshops from the start in 1989, back then under the name Twente Workshops on Graphs and Combinatorial Optimization. He was not the person in the spotlight – something he really was not looking for – but he liked more to be in the background, asking questions, starting discussions, giving valuable feedback, putting things into different perspectives. He was known as a "real problem solver". This attitude and his broad interest in the topics of the workshop made Walter for many of the participants of the 24 workshops a valuable researcher and as a friendly and open-mined colleague, who will be greatly missed, not only at future editions of CTWs.

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