Vortex nozzle interaction in solid rocket motors: A scaling law for upstream acoustic response

L. Hirschberg,[1,a)] S. J. Hulshoff,[2] J. Collinet,[1] C. Schram,[3,b)] and T. Schuller[4,c)]

[1] *ArianeGroup, 51-61 Route de Verneuil, BP 3002, 78133 Les Mureaux Cedex, France*
[2] *Faculty of Aerospace Engineering, Delft University of Technology, Delft, The Netherlands*
[3] *Environmental and Applied Fluid Dynamics Department, von Karman Institute for Fluid Dynamics, 72 Chaussée de Waterloo,1640 Rhode-St-Genèse, Belgium*
[4] *EM2C Laboratory, CNRS, CentraleSupélec, Université Paris-Saclay, 3, rue Joliot Curie, Gif-sur-Yvette Cedex 91192, France*
lionel.hirschberg@centralesupelec.fr, S.J.Hulshoff@tudelft.nl, jean.collinet@ariane.group, christophe.schram@vki.ac.be, thierry.schuller@imft.fr

Abstract: In solid rocket motors, vortex nozzle interactions can be a source of large-amplitude pressure pulsations. Using a two-dimensional frictionless flow model, a scaling law is deduced, which describes the magnitude of a pressure pulsation as being proportional to the product of the dynamic pressure of the upstream main flow and of vortex circulation. The scaling law was found to be valid for both an integrated nozzle with surrounding cavity and a nozzle geometry without surrounding cavity that forms a right angle with the combustion chamber side wall. Deviations from the scaling law only occur when unrealistically strong circulations are considered.

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1. Introduction

In solid rocket motors (SRMs), time-dependent hydrodynamic force reactions due to the interaction of vortices with the nozzle are a significant source of sound. These result in acoustic standing waves, which in turn trigger periodic vortex shedding at frequency f close to the frequency f_1 of the first longitudinal acoustic mode of the combustion chamber. The result is a self-sustained oscillation of considerable magnitude. In cold-gas scale model (1/30) experiments of Ariane 5 SRMs, in which the combustion of the solid propellant is mimicked by injection of air through porous walls, relative pressure pulsation amplitudes $p'/p = \mathcal{O}(10^{-3})$ (with p the static pressure) have been observed. These are of the same order as in real flight with combustion.[1,2]

The nozzles used in SRMs are integrated, viz., they contain a cavity with an opening near the nozzle inlet [see Fig. 1(a)]. The presence of such a cavity has been shown to be important for the establishment of self-sustained pressure pulsations in SRM systems.[1,3] In these publications it was observed that the limit-cycle amplitude p'/p increases linearly with V_c, where V_c is the cavity volume.

An incompressible and frictionless two-dimensional planar analytical flow model for the sound generation of vortex nozzle interaction was proposed by Hirschberg *et al.*[4] This model, which uses a point vortex representation, suggests that the acoustic response due to vortex nozzle interaction is generated on approach to the contraction and is proportional to the dynamic pressure $\rho U^2/2$ times the square of the dimensionless circulation $\tilde{\Gamma}^2 = \Gamma^2/(S_1 U)^2$ carried by the point vortex, where ρ is the density, U is the upstream (from the nozzle) flow velocity, and S_1 is the upstream channel height. However, this model does not consider integrated nozzles with cavities. A later model[3] takes this into account by placing an acoustic volume point source in the right angle corner of the nozzle inlet.

In this letter, the results of a systematic numerical parameter study of vortex nozzle interaction is reported. Rather than considering the limit-cycle behavior of the combustion chamber the sound pulse generated by vortex-nozzle interaction is studied

[a)] Author to whom correspondence should be addressed. Also at: EM2C Laboratory, CNRS, CentraleSupélec, Université Paris-Saclay, 3, rue Joliot Curie, Gif-sur-Yvette Cedex 91192, France.
[b)] Also at: Aeronautics and Aerospace Department, von Karman Institute for Fluid Dynamics, 72 Chaussée de Waterloo, 1640 Rhode-St-Genèse, Belgium.
[c)] Also at: Institut de Mécanique des Fluides de Toulouse (IMFT), Université de Toulouse, CNRS, INPT, UPS, Toulouse, France.

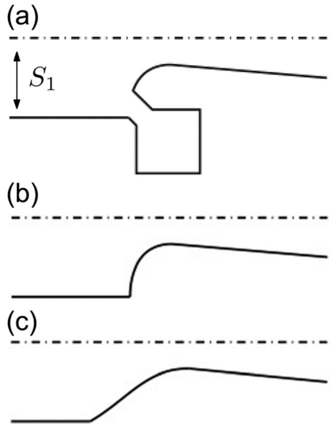

Fig. 1. Three nozzle inlet geometries considered: integrated nozzle with cavity volume (a), nozzle inlet with a right angle corner (b), and a gradual ramp nozzle inlet (c). For all inlet geometries the flow direction is from left to right. The ratio of nozzle inlet to nozzle throat cross-sectional area is $A_1/A^* = 3$ corresponding to $M = 0.197$ at the inlet and $M = 1$ in the throat.

in a semi-infinite duct. This allows for a systematic variation of parameters such as the vortex circulation, the viscous core-radius, and the position at which the vortex is released. This makes the systematic study of scaling behavior possible. This type of approach has been used experimentally for entropy inhomogeneity-nozzle interaction[5] and for an axially swirling vortex-nozzle interaction.[6]

A more detailed compressible two-dimensional planar model is considered, in which the vortices have a finite core size. The study was carried out using the Euler Internal Aeroacoustics (EIA) code, which has previously been used by Hulshoff *et al.*[7] for the study of vortex nozzle interactions. The present study focuses on choked nozzles, whereas previous results were obtained for subcritical flow conditions.[7] Furthermore, different nozzle geometries are considered, including an integrated nozzle geometry with surrounding cavity. The results described here lead to the following scaling law: the sound produced due to the interaction of vortices with typical nozzle inlet geometries encountered in SRMs is proportional to the dynamical pressure $\rho U^2/2$ times the dimensionless circulation $\tilde{\Gamma} \equiv \Gamma/(S_1 U)$ carried by the vortex when realistic circulations are considered, viz., $p' \propto \rho U^2 \tilde{\Gamma}/2$.

An explanation for this new scaling law and the deviation from the $\tilde{\Gamma}^2$ proportionality predicted for an incomprehensible flow by Hirschberg *et al.*[4] is provided using Howe's Vortex Sound Theory.[8]

2. Estimation of the dimensionless circulation range carried by vortices in SRMs

The maximum circulation carried by vortices in SRMs Γ_{max} can be estimated as the product of the upstream mean flow velocity U at the nozzle inlet with a hydrodynamic wavelength of order U/f. The flow velocity U is the product cM of the local sound speed c and the upstream Mach number M for a choked nozzle. The Mach number of the upstream flow can be estimated using a quasi-one-dimensional flow approximation[9] from the ratio A_1/A^* of the nozzle inlet cross-sectional area A_1 to the cross-sectional area of the nozzle throat A^* (where locally $M = 1$). The oscillation frequency f is on the order of $c/2L$, where L is the length of the combustion chamber.

Considering a two-dimensional flow in a channel of square cross-section ($A_1 = 4S_1^2$) with upstream half channel height S_1 the maximum dimensionless circulation is $\tilde{\Gamma}_{max} = 2LM/S_1$ where $\tilde{\Gamma} \equiv \Gamma/(S_1 U)$, for typical SRMs $M = \mathcal{O}(10^{-1})$ and $L/S_1 = \mathcal{O}(10)$, hence $\tilde{\Gamma}_{max} = \mathcal{O}(1)$. The relationship between the actual axi-symmetrical geometry and the two-dimensional planar flow model is discussed in more detail by Hirschberg *et al.*[3] Using $\tilde{\Gamma} = \tilde{\Gamma}_{max}/3$, an analytical model[3] was able to reproduce the order of magnitude of pressure pulsation amplitudes observed in cold-gas scale experiments.[1] Laminar axi-symmetrical numerical simulations by Anthoine[1] show vortex circulation $\tilde{\Gamma} = \mathcal{O}(10^{-1})$. In summary, one expects the range of circulations carried by vortices in SRMs to be $0.1 \leq \tilde{\Gamma} \leq 1$. All the following results are presented for a vortex core radius $0.3S_1$. A variation of the core radius in the range $0.1 \leq R_\Gamma/S_1 \leq 0.4$ leads to a variation of 30% in the pulse amplitude generated by vortex nozzle interaction. This effect is neglected in the further discussion.

3. Nozzle inlet geometries considered

For the present study three nozzle inlet geometries are considered: an integrated nozzle with cavity volume [Fig. 1(a)], a nozzle inlet with a right angle corner [Fig. 1(b)], and a gradual ramp nozzle inlet [Fig. 1(c)]. The calculations are limited to $A_1/A^* = 3$ or $M = 0.197$, which is an upper bound for SRMs.[1]

The surface used to model the presence of the integrated cavity volume V_c was chosen on the order of a square segment $0.7S_1^2$. This corresponds to the cavity volume $V_c = 2.8S_1^3$ of nozzle 2 used in the cold-gas scale experiments reported by Anthoine.[1] In this case, the cavity volume V_c is equivalent to that of a square upstream duct segment ($A_1 = 4S_1^2$) of length equal to seven-tenths its half height S_1.

The right angle inlet geometry [Fig. 1(b)] was drawn using Henrici's[10] conformal mapping as used in previous analytical models.[3,4] This does not precisely correspond to a geometry used in scale-model experiments. However, similar right angle inlet geometries have been used in both hot-gas (with combustion) and cold-gas scale model experiments.[1,11,12]

The gradual ramp inlet geometry has, to our knowledge, not been used in experimental studies previously reported in the literature. It has been included in the present study to investigate its impact on the scaling behavior of the vortex nozzle interaction sound source and investigate the possible reduction of pulsation amplitude.

4. Numerical simulation results

Two-dimensional planar numerical simulations of vortex nozzle interaction are carried out with EIA.[7] In view of the symmetry of the problem only half of the duct is considered in the calculations. Starting at a distance of approximately $6S_1$ upstream from the nozzle inlet and at height h from the bottom wall, circular vortices of core radius $R_\Gamma = 0.3S_1$ are generated, by imposing a force field moving with the vortex. This corresponds to approximately 50 cells per vortex core diameter. Spatial integration is performed with a second order total variation diminishing Roe method.[13] The meshes used for the present vortex nozzle interaction simulations produce results in the asymptotic region of the discretization. An observed order of accuracy of 2.06 (second order convergence) was found, with an estimated relative discretization error of 1%.

Calculations are carried out for an anechoic upstream pipe termination (infinite pipe). In Fig. 2, one observes that the results for an integrated nozzle are quite insensitive to the initial vortex release height h in the range $0.3 \le h/S_1 \le 0.5$. Furthermore, one sees that the dimensionless pressure pulse $2p'/\rho U^2\tilde{\Gamma}$ is almost independent of the sign and magnitude of $\tilde{\Gamma}$. In other calculations $\tilde{\Gamma}$ was varied over the full range $-1.7 \le \tilde{\Gamma} \le 1.7$ confirming this result.

To explain this one should consider $-\rho(\boldsymbol{\omega} \times \mathbf{v}) \cdot \nabla G$ integrated over time and space, which according to the theory of Howe[8] is the pressure pulse generation term due to vortex dynamics.[3,14] In this relation ρ is the fluid density, $\boldsymbol{\omega} \equiv \nabla \times \mathbf{v}$ is the vorticity vector, $\mathbf{v}$ is the local flow velocity, and G is the Green's function. At the low frequencies considered here, G can be calculated using the reciprocity principle,[15] by

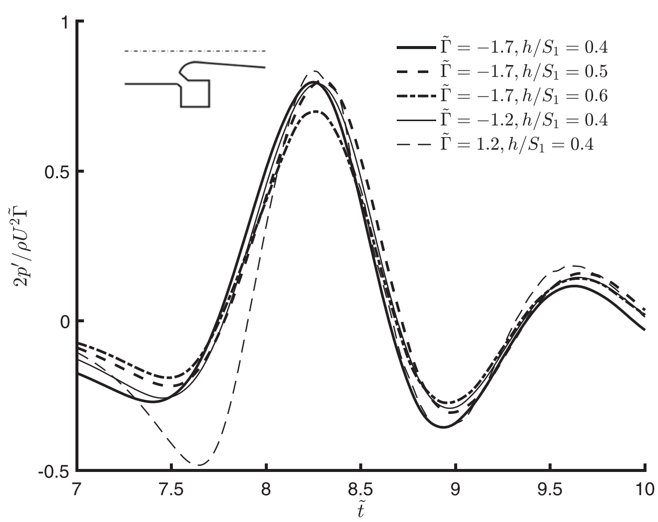

Fig. 2. Dimensionless acoustic pulse generated due to nozzle vortex interaction, as a function of dimensionless time $\tilde{t} \equiv tU/S_1$, for the integrated nozzle geometry.

considering the reflection of a plane wave generated far upstream by a Dirac pulse point sound source. The upstream pipe is assumed to be infinitely long.

Considering the integrated nozzle inlet geometry [Fig. 1(a)], one distinguishes two contributions to G. First the acoustic field $G_{\rm rad}$ due to radiation through the nozzle, and second the acoustic field due to the compressibility of the fluid in the cavity $G_{\rm cav}$ around the nozzle. For values of $|\tilde{\Gamma}| = \mathcal{O}(1)$, the vortex path is close to that of a steady potential flow through the nozzle, along the streamlines of $\nabla G_{\rm rad}$. This implies that $-\rho(\boldsymbol{\omega} \times \mathbf{v}) \cdot \nabla G_{\rm rad}$ is negligibly small, as the vector $\mathbf{v}$ and $\nabla G_{\rm rad}$ are almost parallel. The angle between $\mathbf{v}$ and $\nabla G_{\rm rad}$ is assumed to be proportional to $\tilde{\Gamma}$, which explains a $\tilde{\Gamma}^2$ proportionality because $\boldsymbol{\omega}$ is proportional to $\tilde{\Gamma}$. By contrast, the acoustic flow due to the compressibility of the cavity $\nabla G_{\rm cav}$ is almost normal to $\mathbf{v}$ for release heights in the expected range $0.3 \le h/S_1 \le 0.5$. Thus, the sound pulse scales linearly with $\boldsymbol{\omega}$ and therefore with $\tilde{\Gamma}$ and also with V_c, which determines the magnitude of $G_{\rm cav}$. This is exactly what can be seen in Fig. 2, where dimensionless results for $\tilde{\Gamma} = -1.7$ are very close to those for $\tilde{\Gamma} = \pm 1.2$. The linear scaling with V_c was already observed in the experiments performed by Anthoine.[1]

This raises the following question: which scaling law should be used when at first glance there appears to be no nozzle cavity volume? To answer it one considers the right angle inlet geometry in Fig. 1(b). From Fig. 3, one concludes that in the case for $|\tilde{\Gamma}| \le 0.43$ the pulse remains globally proportional to $\tilde{\Gamma}$. Indeed scaled responses for $\tilde{\Gamma} = \pm 0.11$ and $\tilde{\Gamma} = \pm 0.43$ are globally similar in shape and magnitude. One could argue that the right angle corner acts as an effective nozzle cavity volume V_0. From simulations with various cavity volumes V_c, by extrapolation it was deduced that $0.4 S_1^3 \le V_0 \le S_1^3$, the value of V_0 depends on the upstream release height h and the dimensionless circulation $\tilde{\Gamma}$.

However, for $\tilde{\Gamma} = \pm 1.7$, one observes a large effect of the sign of $\tilde{\Gamma}$. This effect is caused by the deviation between the path of the vortex $\mathbf{v}$ and the acoustic radiation flow $\nabla G_{\rm rad}$. It is due to the influence of an "image" vortex on the path of the vortex[4] (influence of the walls). One can deduce that for very large $\tilde{\Gamma}$ this contribution to the pressure pulse scales with $\tilde{\Gamma}^2$ rather than with $\tilde{\Gamma}$. Indeed around $\tilde{t} = 8.8$ in the response for $\tilde{\Gamma} = 1.7$ in Fig. 4, one notices an upwards pulse which can be attributed to the emerging $\tilde{\Gamma}^2$ effect.

To check this hypothesis a third nozzle inlet geometry is considered. The right angle inlet is replaced by a gradual ramp in Fig. 1(c). This should further reduce any volume compressibility effect. The response of the gradual ramp nozzle inlet geometry due to an interaction with vortices carrying very large $\tilde{\Gamma} = \pm 1.7$ and unrealistically large $\tilde{\Gamma} = \pm 6.8$ circulations is shown in Fig. 4. One clearly sees the $\tilde{\Gamma}^2$ scaling for $\tilde{\Gamma} = \pm 6.8$. For $\tilde{\Gamma} = \pm 1.7$ one deduces that this scaling behavior is being approached when compared to results for the right angle inlet geometry (Fig. 3). However, additional simulations for $\tilde{\Gamma} = \pm 0.1$ show a linear scaling behavior with $\tilde{\Gamma}$ even for this gradual inlet

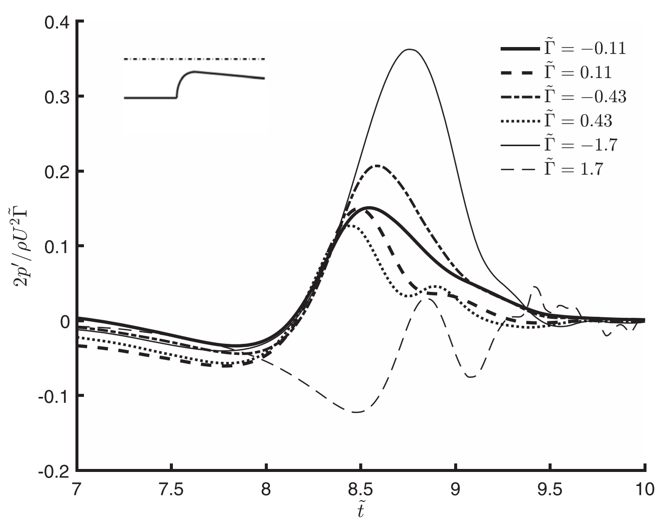

Fig. 3. Scaled acoustic response generated due to vortex nozzle interaction, as a function of dimensionless time $\tilde{t} \equiv tU/S_1$, for the right angle corner inlet nozzle geometry.

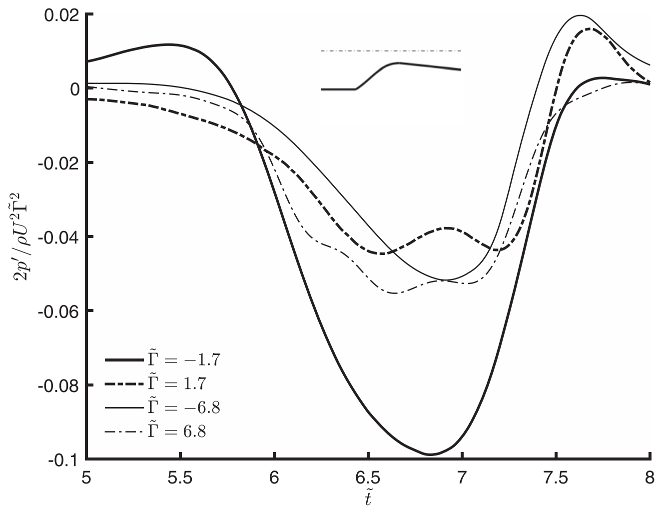

Fig. 4. Scaled acoustic pulse generated due to nozzle vortex interaction, as a function of dimensionless time $\tilde{t} \equiv tU/S_1$, for the gradual slope inlet nozzle geometry.

nozzle geometry. This indicates that even for this gradual ramp nozzle inlet geometry there is a significant effect of compressibility associated to an effective nozzle "cavity" volume.

It appears from these simulations that the use of a gradual ramp nozzle inlet reduces the pulse amplitude by a factor of 2 compared to the results for the right angle corner found in Fig. 3 and a factor of 10 compared to those shown in Fig. 2 for the integrated nozzle. The gradual ramp pulse, however, is longer this could affect the limit-cycle behavior.

This leads to the conclusion that the linear scaling law in $\tilde{\Gamma}$ should be used, as it covers behavior expected for the range $0.1 \leq \tilde{\Gamma} \leq 1$ for reportedly used and most pertinent nozzle inlet geometries[1,11,12] [Figs. 1(a) and 1(b)].

5. Conclusion

When considering realistic dimensionless circulations in the range $0.1 \leq \tilde{\Gamma} \leq 1$ and typical nozzle inlet geometries of SRMs reported in the literature [integrated nozzle Fig. 1(a) and right angle corner geometry Fig. 1(b)], one finds that the acoustic response due to vortex nozzle interaction scales with $\rho U^2 \tilde{\Gamma}/2$ where $\tilde{\Gamma} = \Gamma/S_1 U$. Deviations from this scaling law only occur when unrealistically strong circulations are considered.

When a cavity of volume V_c is present around the nozzle the upstream pressure pulse amplitude scales almost linearly with this volume. Even in the absence of a nozzle cavity, for the case of the inlet with a right angle corner similar behavior is observed. This indicates that a right angle corner behaves as an effective nozzle cavity volume and the acoustic pressure pulse still scales with $\rho U^2 \tilde{\Gamma}/2$. Thus, the upstream acoustic response is largely generated by the flow compressibility on approach to the contraction.

The generated acoustic pulse is quite insensitive to the upstream vortex release height h, which here is measured with respect to the outer wall, especially for cases where $h/S_1 \leq 1/2$, which is the range where one expects vortices to be formed in SRMs.

The scaling law reported in this letter opens the door to the development of simplified semi-analytical lumped element models for the hydrodynamic sound source in SRMs.

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