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Is interacting vacuum viable?

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ABSTRACT

We study the asymptotic dynamics of dark energy as a mixture of presssureless matter and an interacting vacuum component. We find that the only dynamics compatible with current observational data favours an asymptotically vanishing matter-vacuum energy interaction in a model where dark energy is simulated by a generalised Chaplygin gas cosmology.

1. Introduction

In recent years most of the cosmological studies have been focused on variations of General Relativity and modifications of the Standard Cosmological model [1]. This is done in order to provide a more reliable framework to explain the present physical evidence of the universe. It is observed today that only 5% of the matter content is of baryonic form and additional evidence coming from the high redshift surveys of type I supernovae [2,3] indicate that we currently live in a universe that undergoes an accelerated expansion.

In the present literature many cosmological models involve the presence of an exotic type of matter component that lies beyond the framework of standard cosmology, cf. [4–7], in an attempt to explain the present acceleration of the observed universe. Additionally, the case of coupling and energy transfer between dark energy and dark matter leads to research efforts that try to alleviate the so-called coincidence problem [8,9]. One unified dark energy model that has attracted the interest for research is the Generalised Chaplygin Gas model (GCG in short). This model has a dual character since at early times it satisfies the properties of a matter-dominated universe whereas at late times it approaches the limiting behaviour of dark-energy dominated universe [10].

In previous works [11–13], we have studied the asymptotic dynamics near finite-time singularities of flat and curved universes filled with two interacting fluids using an interaction term that was first introduced by Barrow and Clifton, cf. [14,15]. In the present work, we consider the case of energy exchange between dark matter (as pressureless dust) and dark energy (as vacuum) with the local energy transfer being associated with the energy density of the vacuum (that is ρ_V) so that $Q_\mu = -\nabla_\mu \rho_V$ [16].

In the limit of zero energy exchange ($Q_\mu = 0$), or equivalently if the vacuum energy is covariantly conserved ($\nabla_\mu \rho_V = 0$), then the vacuum energy must be homogeneous in spacetime and equal to a cosmological constant [17]. Under these conditions, we address the question of the viability and stability of the interacting vacuum model on approach to the finite-time singularity by studying the asymptotic properties of solutions of the scale factor, the total energy density and total pressure of the universe.

The cosmological model is expected to be stable, and therefore acceptable, if asymptotically it reproduces the dominant features of dark matter and dark energy at both early and late times respectively. We show that asymptotically at early times the energy exchange is vanishing and the energy density of the vacuum is approximately zero, in contrast to what occurs in the standard cosmological model. Hence, at early times and in the absence interaction, our model is indistinguishable from the CDM model [18].

The asymptotic analysis of the solutions is carried out using the method of asymptotic splittings, cf. [19,20]. The analysis provides a complete description of all possible dominant features that the solution possesses as it is driven to a blow-up.

The plan of this paper is as follows. In the next section, we write down all possible asymptotic decompositions of the basic differential equations of our problem describing the GCG model. Sections 3–5 present a detailed study of the various asymptotic

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solutions. In the last section we discuss our results and point out some interesting open problems in this field.

2. Decomposed dark energy models

We study the case of the generalised Chaplygin gas model in flat FRW universe as a mean to explain the accelerated expansion of the universe [21]. In the GCG approach the exotic cosmological fluid is defined by the barotropic equation of state

$$P_{cgc} = A\rho_{cgc}^{-\alpha}, \tag{1}$$

where A is a positive constant and $0 < \alpha \leq 1$. This leads to a cosmological solution for the density

$$\rho_{cgc} = \left(A + Ba^{-3(\alpha+1)}\right)^{1/(\alpha+1)}, \tag{2}$$

where a is the scale factor of the universe and B is a positive integration of constant for a well defined ρ_{cgc} at all times. From Eq. (2), one can conclude that at early times the asymptotic solution for the energy density reproduces the CDM model as described by

$$\rho_{cgc} \sim a^{-3} \quad a \to 0, \tag{3}$$

in the limit of vanishing constant $\alpha \to 0$ [22,25]. At late times the solution (2) implies that the fluid behaves as a cosmological constant

$$\rho_{cgc} \sim A^{1/(\alpha+1)} \quad a \to \infty. \tag{4}$$

This interpolation of the model between two different fluids at different stages of the evolution of the universe suggests that the GCG model can be interpreted as a mixture of two cosmological fluids with energy exchange.

Now, any unphysical oscillations or exponential blow-up in the matter spectrum produced by such a unified model [22] can be avoided, if one excludes coupling with phantom fields [18]. Therefore, the unique coupling between dark matter (pressureless dust) and dark energy (cosmological constant) makes the GCG model a well-behaved model both at early times (approach the successful CDM model) and at late times (approach de Sitter Universe).

It is interesting to mention here that the interaction between the fluid components allows energy to be transferred from dark matter to dark energy, since α is a positive constant. As we will show below, this energy transfer is vanishingly small at early times making the model indistinguishable from a CDM model in the past. Whereas when interaction starts off the contribution of the cosmological constant is significant and the model approaches de Sitter universe [18].

The Einstein equations[1] for a flat Friedman universe filled with pressureless dust (ρ_m) and vacuum (ρ_V), scale factor $a(t)$ and Hubble expansion rate $H = \dot{a}/a$ reduce to the Friedman equation

$$3H^2 = \rho_m + \rho_V = \rho_{gcg}. \tag{5}$$

The total energy momentum tensor of the pair is the algebraic some of the individual energy–momentum tensor given by

$$T^{\mu\nu}_{total} = T^{\mu\nu}_m + T^{\mu\nu}_V. \tag{6}$$

[1] Here we consider the case where the baryons have a similar behaviour to that of a pressureless dust, i.e. dark matter and we exclude the possibility of an energy exchange between baryons and dark energy (see [18] for more information).

Since the two fluids are not separately conserved it occurs that

$$\nabla_\nu T^{\mu\nu}_m = -u^\mu \quad \nabla_\nu T^{\mu\nu}_V = u^\mu, \tag{7}$$

where u^μ is the total 4-velocity so that $\nabla_\nu T^{\mu\nu}_{total} = 0$.

It is shown in [13] that by taking the covariant derivative of each energy density momentum separately one obtains

$$-u^0 = \nabla_\nu T^{0\nu}_m = -a^3\dot{p_m} + \frac{d}{dt}[a^3(p_m + \rho_m)] \tag{8}$$

and similarly for the second one

$$u^0 = \nabla_\nu T^{0\nu}_V = -a^3\dot{p_V} + \frac{d}{dt}[a^3(p_V + \rho_V)]. \tag{9}$$

The forms of the continuity equations then read

$$\dot{\rho_m} + 3H\rho_m = -\frac{u^0}{a^3}$$
$$\dot{\rho_V} = \frac{u^0}{a^3}.$$

If we set $Q = u^0/a^3$ as the interaction term, one can show after some calculations that the interaction term reads

$$Q = \frac{u^0 H}{\dot{a}a^2}. \tag{10}$$

Therefore, the interaction function (10) is generally dependent on the expansion rate H, the scale factor $a(t)$, its time derivative $\dot{a(t)}$, as well as on the energy densities and pressures of the fluid components. We note here that if the expansion of the universe ceases, that is for $H = 0$, the interaction between the fluid components will also vanish due to the fact that interaction is coupled to the 3-geometry of the slice with a mean curvature described from the Hubble parameter H.

We assume a fluid interaction of the form [16]

$$Q = 3\alpha H\left(\frac{\rho_m\rho_V}{\rho_m + \rho_V}\right), \tag{11}$$

and the final forms of the continuity equations for matter and vacuum are given by

$$\dot{\rho_m} + 3H\rho_m = -Q \tag{12}$$
$$\dot{\rho_V} = Q, \tag{13}$$

respectively. Equations (5), (12) and (13) describe a 3-dimensional system with unknowns (a, ρ_m, ρ_V) satisfying the constraint given by Eq. (5). After some manipulations it is proved that the above set of equations leads to the following master differential equation

$$\ddot{H} + 3(\alpha+1)H\dot{H} + 2\alpha\frac{\dot{H}^2}{H} = 0. \tag{14}$$

Equation (14) is a nonlinear differential equation of second order for the Hubble parameter H. If we assume a power-law type solution for the scale factor $a = t^p$ that is $H = p/t$, where $p \in \mathcal{Q}$, then one can provide all possible exact solutions for the Hubble parameter as well for the scale factor based on the GCG parameter α at both early and late times. For the interaction term described in Eq. (11), exact solutions have been found in [23,24] under a more general framework of two interacting fluids.

Indeed if we substitute the form $H = p/t$ in Eq. (14) we get the following equation for the exponent p

$$p^2[2 - 3p(\alpha+1) + 2\alpha] = 0. \tag{15}$$

We find two possible solutions to the equation above; The first one describes the case of no interaction with $p=0$ while the non-trivial solution satisfies the form

$$p=\frac{2+2\alpha}{3(\alpha+1)}, \tag{16}$$

and the exact solution for the scale factor reads

$$a(t)=t^{(2+2\alpha)/[3(\alpha+1)]}. \tag{17}$$

However, in this work we are interested in an asymptotic analysis of solutions of (14) near finite-time singularities. To do so, it will be very useful for our calculations to rewrite the master equation (14) in a suitable dynamical system form. In this respect, we rename $H=x$ and find the 2-dimensional system

$$\dot{x}=y$$
$$\dot{y}=-3(\alpha+1)xy-2\alpha\frac{y^2}{x}. \tag{18}$$

Equivalently, we have the vector field

$$f(x,y)=[y,-3(\alpha+1)xy-2\alpha\frac{y^2}{x}]. \tag{19}$$

The vector field can split [19] in three different ways namely

$$f_I(x,y)=[y,-3(\alpha+1)xy]+(0,-2\alpha\frac{y^2}{x}), \tag{20}$$

$$f_{II}(x,y)=(y,-2\alpha\frac{y^2}{x})+[0,-3(\alpha+1)xy], \tag{21}$$

$$f_{III}(x,y)=[y,3(\alpha+1)-2\alpha\frac{y^2}{x}]. \tag{22}$$

In the following sections we apply the method of asymptotic splittings, analytically expounded in [19,20], to describe the asymptotic properties of the solutions of the dynamical system (18) in the vicinity of its finite-time singularities.

3. Early times asymptotics

In this section, we give necessary conditions in terms of the parameter α for the existence of generalised Fuchsian series type solutions [20] towards the finite-time singularity of the first decomposition $f_I(x,y)=[y,-3(\alpha+1)xy]+(0,-2\alpha\frac{y^2}{x})$.

To do so, we look for possible dominant balances by substituting the forms $x(t)=\theta t^p,\quad y(t)=\xi t^q$ in the dominant part of the decomposition described by the system below

$$\dot{x}=y$$
$$\dot{y}=-3(\alpha+1)xy. \tag{23}$$

We assume here that $\Xi=(\theta,\xi)\in\mathbb{C}$ and $\mathbf{p}=(p,q)\in\mathbb{Q}$. This leads to the unique balance

$$\mathcal{B}_I=[\Xi,\mathbf{p}]=\left[\left(\frac{2}{3(\alpha+1)},-\frac{2}{3(\alpha+1)}\right),(-1,-2)\right], \tag{24}$$

for $0<\alpha\leq 1$. The subdominant part of the splitting (20) satisfies

$$\frac{f_I^{(sub)}(\Xi,t^{\mathbf{p}})}{t^{\mathbf{p}-1}}=\left(0,-\frac{4\alpha}{3(\alpha+1)}\right), \tag{25}$$

and is asymptotically subdominant [11] in the sense that

$$\lim_{t\to 0}\frac{f_I^{(sub)}(\Xi,t^{\mathbf{p}})}{t^{\mathbf{p}-1}}\to 0, \tag{26}$$

only if $\alpha\to 0$. We therefore conclude that in the neighbourhood of the finite-time singularity the asymptotic solution is meaningful only in the limit of vanishing α, that is in the absence of interaction between the two fluids.

Next we calculate the Kovalevskaya matrix given by,

$$\mathcal{K}_I=\mathcal{D}f_I(\Xi)-diag(\mathbf{p}), \tag{27}$$

where $\mathcal{D}f$ is the Jacobian matrix of the decomposition. For this case the Kovalevskaya matrix reads

$$\mathcal{K}_I=\begin{bmatrix}1 & 1\\ 2 & 0\end{bmatrix}. \tag{28}$$

As discussed in [19] the number of non-negative $\mathcal{K}$-exponents equals the number of arbitrary constants expected to appear in the series solution, while the -1 exponent corresponds to the arbitrary constant relevant to the position of the singularity (for notational convenience taken to be $t=0$). Therefore, if the balance is to correspond to a general solution, two arbitrary constants are expected to appear in the series expansion (since the original system (18) is two dimensional). Here we find

$$spec(\mathcal{K}_I)=(-1,2), \tag{29}$$

with a corresponding eigenvector

$$v_2^T=(1,-1). \tag{30}$$

Hence, it is expected that the balance $\mathcal{B}_I$ will correspond to a general solution. Substituting the series expansions

$$x=\sum_{j=0}^{\infty}c_{j1}t^{j-1},\quad y=\sum_{j=0}^{\infty}c_{j2}t^{j-2}, \tag{31}$$

in the system (18) we arrive after manipulations at the following asymptotic solution around the singularity

$$x(t)=\frac{2}{3}t^{-1}+c_{21}t+\cdots,\quad \text{as}\quad t\to 0,\quad \alpha\to 0. \tag{32}$$

The y-expansion is derived from the above by differentiation. As a final test for the validity of this solution, a compatibility condition has to be satisfied for every positive $\mathcal{K}$-exponent [11]. For the positive eigenvalue 2 and an associated vector $v_2^T=(1,-1)$ it reads

$$c_{21}=c_{22}, \tag{33}$$

and this is indeed true based on previous recursive calculations.

It follows from Eq. (32) that all solutions are dominated by the $x=H\sim\frac{2}{3}t^{-1}$ solution which in terms of the scale factor reads

$$a(t)\sim t^{2/3}\quad \text{as}\quad t\to 0,\quad \alpha\to 0. \tag{34}$$

The dominant term of the series expansion (34) is the same as the exact solution for the scale factor described by Eq. (17) in the limit $\alpha\to 0$. It also follows from Eq. (34) that in the vicinity of the finite-time singularity and in the absence of interaction the total energy density of the model satisfies the form

$$\rho_{tot}\sim a^{-3}\quad t\to 0,\quad \alpha\to 0. \tag{35}$$

Now, the geometric character of the singularity is completely described in terms of the asymptotic behaviour of the total energy density and pressure of the model, the asymptotic behaviour of the scale factor and the Hubble parameter [12]. For the solutions above it occurs that

$$\rho_{tot} \to \infty \quad P_{tot} \to 0, \quad a \to 0, \quad H \to \infty, \tag{36}$$

as $t \to 0$ and $\alpha \to 0$. The asymptotic conditions above describe the case of a Big Bang type of singularity. Consequently, the singularity is necessarily placed at early times. It is discussed in cf. [18] that the contribution from the cosmological constant is negligibly small at early times, hence we conclude that our decomposition describes a model that is indistinguishable from a CDM dominated universe in the past.

It is discussed in [18] that the energy density perturbations at early times regarding the dark matter component (and the baryon perturbations) are linear and small in scale ($\delta_m << 1$) and in the absence of interaction one can easily recover the standard energy perturbations in the CDM model.

4. Quasi de Sitter Universe

Let us now move on to the asymptotic analysis of the decomposition with dominant part given by the vector field

$$f_{II}(x, y) = (y, -2\alpha y^2/x). \tag{37}$$

Now by substituting in the asymptotic system $(\dot{x}, \dot{y}) = [y, -2\alpha(y^2/x)]$ the forms $x(t) = \theta t^p$ and $y(t) = \xi t^q$ we find the following dominant balance

$$\mathcal{B}_{II} = \left[\left(\theta, \frac{\theta}{2\alpha+1}\right), \left(\frac{1}{2\alpha+1}, \frac{1}{2\alpha+1}\right)\right], \tag{38}$$

where θ is an arbitrary constant. The candidate subdominant part of the vector field, namely $f_{II}^{(sub)}(x, y) = [0, -3(\alpha+1)xy]$ is vanishing asymptotically without any restrictions on the values of the parameter α nor the constant θ. Hence the decomposition is acceptable. To continue with, the Kovalevskaya matrix is given by

$$\mathcal{K}_{II} = \begin{bmatrix} -1/(2\alpha+1) & 1 \\ (2\alpha)/(2\alpha+1)^2 & -(2\alpha)/(2\alpha+1) \end{bmatrix}, \tag{39}$$

with corresponding eigenvalues

$$spec(\mathcal{K}_{II}) = (-1, 0), \tag{40}$$

and an eigenvector

$$v_2^T = \left(1, \frac{1}{2\alpha+1}\right). \tag{41}$$

We note here that the second $\mathcal{K}$-exponent is zero. Hence the arbitrary constants at the $j = 0$ level of expansion (cf. [11,12,20,13] for this terminology) are the coefficients given by the dominant balance (38), that is $(c_{01}, c_{02}) = \left(\theta, \frac{\theta}{2\alpha+1}\right)$. Therefore, the asymptotic solution is general since two arbitrary constants appear in the asymptotic solution as described below

$$x(t) = \theta t^{1/(2\alpha+1)}, \quad t \to 0, \tag{42}$$

for $0 < \alpha \leq 1$. Since we are interested in expanding universes ($H > 0$), it follows that the arbitrary constant θ attains only positive values. By integrating the solution above one obtains asymptotically the general solution for the scale factor described by the expression

$$a(t) = a_0 \exp\left(\theta C t^{1/C}\right) \quad \text{as} \quad t \to 0, \tag{43}$$

where $C = (2\alpha+1)/(2\alpha+2)$.

A comment about the asymptotic behaviour of the scale factor is in order. The specific form of Eq. (43) describes an exponential evolution of the universe, with slower rate of expansion than the de Sitter universe, valid for a time interval. Clearly, as interaction kicks off the transfer of energy from dark matter to dark energy (described by Eqs. (12)–(13)) results in an important growth of the energy density of the vacuum. However the presence of dark matter decelerates the rate of expansion.

As shown in the asymptotic solution (43) the α parameter determines the asymptotic states of the universe. For $0 < \alpha \leq 1$ the universe enters (for a time interval) a quasi de Sitter space where the total energy density, total pressure, scale factor and Hubble parameter are asymptotically equal to

$$\rho_{tot} \to 0, \quad |P_{tot}| \to \infty, \quad a \to a_0, \quad H \to 0, \tag{44}$$

respectively as $t \to 0$, while higher derivatives of H diverge. This is a new type of singularity, a combination of Type IV [26–29] and Type II (sudden) singularity placed at late times.

It is interesting to note here that the present decomposition describes an intermediate phase in the evolution of our interacting model. In the limiting case $\alpha \to 0$ (limit of no interaction) it is expected that the universe asymptotically (as $t \to 0$) will approach the CDM model. This is indeed true since for $\alpha \to 0$ the asymptotic analysis is identical to the one performed for the first decomposition in section 3. Consequently, the particular decomposition successfully reproduces the CDM model (as $t \to 0$ and $\alpha \to 0$).

In addition, it is also expected at late times that the dominance of dark energy will drive the evolution of the universe towards de Sitter space. This is indeed feasible in the limit $\alpha \to \infty$, that is the case where energy is being transferred from dark matter to dark energy without bound. This results in the following asymptotic forms for the total energy density, total pressure, scale factor and the Hubble parameter[2]

$$\rho_{tot} \to \rho_0, \quad |P_{tot}| \to \infty, \quad a \sim a_0 \exp(\theta t), \quad x = H \sim \theta, \tag{45}$$

as $t \to 0$, $\alpha \to \infty$. The forms above describe a dark energy dominated universe with a sudden type singularity placed at late times. It is discussed in [18] that at late times the energy density perturbations of dark matter and baryons deviate from the linear behaviour explaining the large energy transfer from dark matter to dark energy.

We note here that the exact solution described by Eq. (17) fails to reproduce this specific behaviour of the scale factor at late times since it describes only possible power-law type solutions.

5. Interacting vacuum

We now focus on the asymptotic analysis of all-terms-dominant case, that is the decomposition (13), or equivalently described by the asymptotic system

$$\dot{x} = y$$
$$\dot{y} = -3(\alpha+1)xy - 2\alpha\frac{y^2}{x}. \tag{46}$$

The subdominant vector field is the zero field in this case and there is one distinct balance given by

$$\mathcal{B}_{III} = \left[\left(\frac{2}{3}, -\frac{2}{3}\right), (-1, -2)\right]. \tag{47}$$

[2] If we assume here for purposes of notation that the constant θ is positive and plays the role of the cosmological constant it can be proved that our solution (43) can also describe a de-Sitter Universe at a finite time at late epoch $t_f \neq 0$.

The Kovalevskaya matrix is given by

$$\mathcal{K}_{III} = \begin{bmatrix} 1 & 1 \\ 4\alpha+2 & 2\alpha \end{bmatrix}, \tag{48}$$

with corresponding eigenvalues

$$spec(\mathcal{K}_{III}) = [-1, 2(\alpha+1)]. \tag{49}$$

Even though the parameter α is present in the second $\mathcal{K}$-exponent, the form of the dominant the balance (47) indicates that on approach to the finite-time singularity the dominant part of asymptotic solution $x \sim (2/3)t^{-1}$ is independent from the choice of the parameter α.

For the whole series expansion though, the choice of the parameter α will determine the level of expansion at which the second arbitrary constant is expected to appear. For purposes of illustration we choose $\alpha = 1/2$ so that $spec(\mathcal{K}_{III}) = (-1, 3)$. Then the associated eigenvector reads

$$v_2^T = (1, -1). \tag{50}$$

The candidate asymptotic solution is expected to be general if two arbitrary constants (the position of the singularity and one constant at the $j = 3$ level of expansion) appear in the series expansion. After substituting the forms (44) into the asymptotic system (46), and for $\alpha = 1/2$, we find the following asymptotic solution

$$x(t) = \frac{2}{3}t^{-1} + c_{31}t^{-2} + \cdots, \quad t \to 0. \tag{51}$$

The compatibility condition at the $j = 3$ level reads

$$2c_{31} = c_{32} \tag{52}$$

and it is indeed satisfied after recursive calculations. Hence the asymptotic solution found above is general. In particular, the dominant behaviour of the solution (51) on approach to the finite-time singularity is identical to the one of the decomposition $f_I(x, y)$.

We conclude here that the decomposition describes asymptotically the model in the very early universe before interaction becomes significantly large. Having said that, the model described here has the same asymptotic features as in the case where the interaction is switched off asymptotically and the universe is matter dominated. Hence, at early times the contribution of dark energy is negligible.

6. Discussion

In this paper we analysed the stability of the singular flat space solutions that arise in the content of a unified dark energy model (GCG model) on approach to the finite-time singularity. We have shown that spacetime evolves from a phase that is initially dominated by dark matter to a phase that is asymptotically de-Sitter under some restrictions. The transition period in our model, between dark matter and dark energy domination corresponds to a quasi-inflationary regime that posses a new type of singularity asymptotically.

We conclude that the current observational data are supportive towards an asymptotically vanishing interaction in a model where dark energy is simulated by a generalised Chaplygin gas cosmology. In particular, it is shown that for such unified model the interaction is asymptotically vanishing at early times and the contribution of dark energy (as cosmological constant) is negligible. Hence, the model is indistinguishable from CDM universe. Such a model attains a pole-like [13] type of singularity and it is proved in previous works [11,12] that such a dominant behaviour is an attractor of all possible asymptotic solutions on approach to the finite-time singularity.

An interesting era of expansion arises in an intermediate phase of expansion when the vector field decomposition admits a quasi de-Sitter solution on approach to the finite-time singularity for $0 < \alpha \leq 1$. In particular, the decomposition reproduces the successful CDM model at early times (in the limit as $\alpha \to 0$) and approaches de-Sitter Universe at late times respectively. This intermediate phase of evolution is in alignment with the predictions of the GCG model for both early and late times.

To conclude with, it would be interesting to apply the central projection technique of Poincarè to the dominant part of each of the asymptotic solutions on approach to the finite-time singularity to discuss the asymptotic stability of the model at infinity. This is examined in [30].

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