

41st European Transport Conference 2013, ETC 2013, 30 September – 2 October 2013, Frankfurt, Germany

## Modelling Public Transport Route Choice, with Multiple Access and Egress modes

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### Abstract

Public transport (PT) is important, because the current traffic system faces well known problems like congestion, environmental impact and use of public space. To be able to assess the effects of policy measures properly, it is necessary to model the behavior of the (PT) traveler in a realistic way. An aspect that lacks realism in a lot of current models is the rigid separation between modes: within the model a traveler cannot choose to switch between modes, so multimodal trips that combine a public transport trip with the car or with the bicycle are not (or at least not explicitly) taken into account, while the use of the bicycle as an access mode is very popular in the Netherlands, and getting more popular in other countries. Easy bike rental systems enable use as an egress mode as well. The use of the car as an access mode is very popular in the US. Furthermore, multiple routing is important, because different users have different preferences (i.e. a fast route or a route without a transfer). These two aspects are addressed in this paper, to achieve more realistic transit modeling.

Multiple routing is included by further developing the method of optimal strategies, where the departure time of vehicles is taken into account in order to determine whether a choice option is the shortest route for some moment in time. This results in a static route choice algorithm that is capable to assess large scale networks. By defining a search radius for different access and egress modes and by defining a sensible set of transit lines, the calculation time of the algorithm is kept limited. Logit choice models are used for stop choice and line choice, to calculate the fractions of travelers that take each route alternative.

The route choice model calculates cost matrices for several mode chains. These mode chains include single mode travel options (like the car or PT combined with walking), but also multimodal travel options (that always include a PT leg). These cost matrices are incorporated in the mode choice process with a nested logit model to determine mode choice. This results in an OD matrix for all modes and mode chains. Finally, these OD matrices are assigned to the network, again using the route choice model.

Applying this modeling framework to a real world case study in the Amsterdam metropolitan area shows that the computation times are reasonable, the results are plausible and conceptually sound. This enables modelers for example to assess infrastructural network developments in large scale networks, taking into account realistic behavior of travelers, namely the combination of multiple modes to reach their destination.

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Peer-review under responsibility of the Association for European Transport

*Keywords:* PT assignment, multimodal passenger network modelling, multiple routing

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## 1. Introduction

The current traffic system faces well known problems like congestion, environmental impact and use of public space. Public transport (PT) is an important mode to alleviate these problems. To be able to assess the effects of policy measures properly, it is important to model the behaviour of the (public transport) traveller in a realistic way.

One aspect that lacks realism in a lot of current multi-modal models is the rigid separation between modes in the traditional 4-step model (Ortúzar and Willumsen, 2001). Within many models a traveller cannot choose to switch between modes, so multimodal trips that combine a public transport trip with car or bicycle are not (or at least not explicitly) taken into account. The use of the bicycle as an access mode is very popular in the Netherlands (40% of all home based train trips has bicycle as access mode, see van Nes et al., 2014), and becoming more and more popular in other countries too. With bike rental systems popping up in cities over the world, the bicycle becomes interesting as an egress mode as well. The use of the car as an access mode is very popular in the US and Australia, where at suburban stations, it is not uncommon for over half of all passengers to arrive by car.

Another important issue is the treatment of heterogeneous preferences among travellers. These preferences can relate to the utility provided by different modes of transit, different transit stops, or even different services (i.e. a fast route, or a route without transfer). To model the variation in preferences for transit users, multiple routing was developed. Multiple routing is generally achieved in two ways: stochastic assignment (draw preferences or attributes such as link time from a distribution and search for shortest paths in the modified network multiple times) or probabilistic approaches which calculate the likelihood that any alternative is the shortest path. The well-known method of “optimal strategies” is an example of a probabilistic approach which assumes random the arrival of passengers and services. Furthermore, it is possible to introduce crowding, which results in an iterative equilibrium assignment. Finally, it is possible to do a dynamic assignment, taking the complete schedule into account. With scheduling a spread over different routes is achieved in time. However, this method is data intensive and computationally expensive, and estimating dynamic OD matrices (that are needed as input) is complicated.

The notion that any physical transit itinerary depends on the arrival of the first carrier of the set of attractive lines at any boarding or transfer stop has led to the introduction of the strategy concept. This concept has been implemented in several software packages and used extensively. A strategy specifies the set of transit lines that are feasible to board at a stop that bring you closer to your destination. A hyperpath is the unique acyclic support graph of this strategy. Specifically for transit assignment the hyperpath framework has been described by Nguyen and Pallattino (1989) and Spiess and Florian (1989). Several modifications and improvements of this theory have been published. For example in Nguyen, Pallottino and Gendreau (1998) and more recently by Florian and Constantin (2012). Where Nguyen introduced a logit model to calculate the distribution over the different paths in the hypergraph, Spiess initially used an equation based on the different headways of the transit lines. In 2012 Florian also introduced a logit model for splitting passengers among options, specifically to get better distributions and also to overcome problems with walking links.

Another well-known method simplifies the transit network by constructing direct links between all boarding and alighting stops (De Cea and Fernández, 1993). This results in a large increase of the number of links in the network, but has the advantage of shorter routes to be found in the network.

The method described in this paper was originally developed by Veitch Lister Consulting in Australia and part of the Zenith software system (Veitch and Cook, 2011). In 2007 the method was adopted in the software package OmniTRANS (DAT.Mobility, 2014) and improved over the years. It has been applied in many studies. The method has a lot of similarities with the strategies approach but introduces a few extra constraints to make the method more

efficient and flexible. Furthermore the concept of trip-chains and the rigid separation between line choice and stop choice is an addition and makes to method particularly useful for multi-modal networks. In this paper this algorithm is described.

In the next section the algorithm is described followed by a real world case for the Amsterdam area. The paper finishes with a conclusion.

## 2. Model description

A public transport trip is typically a multimodal trip. Every home-based trip starts with an access leg to the public transport system. This leg could be travelled on foot, but also by bicycle. In The Netherlands, access by bicycle is responsible for a large percentage of the trips. Car access is typically used in large wide-spread urban areas, such as Australian or North American cities, whereas in European cities, car access is mainly used to prevent parking and congestion problems.

The second leg of the trip is travelled in public transport. This part itself could contain multiple legs when the traveller is changing between services or public transport systems. Sometimes a small walking interchange leg is required.

At the destination stop the last leg to the destination starts. Again the traveller has a choice for walking, cycling (hire a bike) or car (taxi). A work based trip traverses these legs in the reverse order.

The algorithm that is proposed in this paper, referred to as the Zenith method, models these multi-modal public transport trips in several steps, which are all integrated in a single algorithm.

### 2.1. Integrated multimodal network

The methodology is based on the assumption that a multi-modal network is present as a graph built up with nodes and links. In this paper a link is assumed to support one or more modes. So a link can be open for walking only, with a given walking speed or the link could be open for walking and cycling, with a different speed per mode. Links that are open for transit can carry transit lines. A transit line traverses several consecutive links in the graph. At some nodes, stops are placed allowing boarding and alighting of passengers. At these stops exchange to other transit lines or other modes is possible. Each transit line has its own travel time per section of the transit line. A section contains the links between two consecutive stops (see Fig. 1).

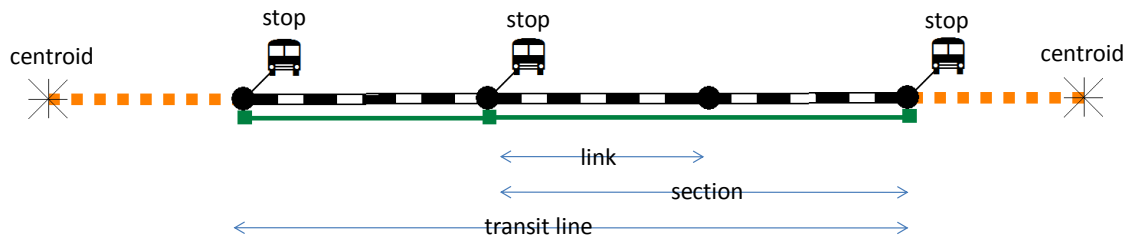


Fig. 1: Network with one transit line and three stops

In fig. 1 a small sample network is displayed with three links, three stops, two centroids and one transit line. The centroids are connected with walking links (dashed) to the network, i.e. links that allow only walking. The transit line traverses three links, i.e. links that allow transit. The transit line passes three stops. This means the transit line has two sections. The first section contains one link, the second section contains two links.

Mathematically the multimodal transportation network is defined as a directed graph  $G$ , consisting of node set  $N$  and link set  $A$ . For each link  $a \in A$  one or more modes  $m \in M$  are defined that can traverse the link. Each link has its own characteristics per mode. Transportation zones  $z \in Z$  are a subset of  $N$  and act as origins and destinations. Total fixed transportation demand  $D$  is stored in a matrix with size  $|Z| \times |Z|$ .

Furthermore a set  $S$  is defined with transit stations or stops. A stop  $s$  is related to a node  $n$  with a one to one relation. Consequently the set  $S$  could be defined as a subset of  $N$ . Transit lines  $l \in L$  are defined as ordered subsets  $A_l \subseteq A$  and can be stop services or express services. Consequently, a line  $l$  passes several stops. The travel time between two stops is determined by the travel time of the consecutive links between two stops, denoted as a subset of links  $\hat{A}_q \subseteq A$  that defines section  $q$ . Each link  $a$  has a travel time  $T_{al}$  for transit line  $l$ . Therefore each section  $q$  has its own travel time  $T$  per transit line:

$$\hat{T}_{ql} = \sum_{a \in \hat{A}_q} T_{al} \quad (1)$$

Each transit line  $l$  has a frequency  $F_l$ . Transfers between transit lines or between modes can only take place at nodes where a stop exists. Whether a line calls at a stop  $s$  or not, is indicated by stop characteristics.

All together, the transportation network is defined by  $G(N,A,L,S)$ .

## 2.2. Mode chains

In the transportation network all possible paths between origins and destinations are sought. These are paths with at least one leg traversed with a transit line. The access mode and the egress mode however, are different modes. For example the access mode could be bicycle and the egress mode could be walk. This gives a trip with bicycle – transit – walk as modes. A mode chain is defined as a combination of access mode, PT and egress mode. In a typical transport study several of these mode chains can exist, for example walk-transit-walk, bicycle-transit-walk and car-transit-walk. Depending on the time of day you can have these chains in reversed order. The Zenith algorithm can handle several mode chains at once.

## 2.3. Generalized costs

Before the algorithm is explained, a short notion of generalized cost is given. Throughout the explanation we will refer to generalized cost as cost. For the non-transit leg of a trip (access, egress and interchange) this function depends on the distance and travel time on a link and has a linear form:

$$C_{ma} = \alpha_m K_a + \beta_m T_{am} \quad (2)$$

where:

$C_{ma}$  the cost to traverse link  $a$  with mode  $m$ .

$K_{am}$  the length of link  $a$  using mode  $m$ .

$T_{am}$  the travel time on link  $a$  using mode  $m$ .

The factors  $\alpha$  and  $\beta$  are different per mode, i.e. walk, bicycle, car.

For the transit leg of the trip, the generalized cost function is a linear function:

$$\hat{C}_{sz} = \sum_{m \in M} [\alpha_m K_{sz} + \beta_m T_{szm} + \gamma_m W_{szm} + \delta_m P_{szm}] \quad (3)$$

where:

$\hat{C}_{sz}$  Transit costs to travel from stop  $s$  to destination  $z$ .

$K_{szm}$  Distance in transit mode  $m$  when travelling from stop  $s$  to destination  $z$ .

$T_{szm}$  Travel time in transit mode  $m$  when travelling from stop  $s$  to destination  $z$ .

$W_{szm}$  Waiting time for mode  $m$  when travelling from stop  $s$  to destination  $z$ .

$P_{szm}$  Penalty for mode  $m$  when travelling from stop  $s$  to destination  $z$ .

This function is valid for the transit part of the trip, i.e. from the boarding stop to the alighting stop. The factors  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  could be different per transit sub mode  $m$ , such as train, bus, metro, etc.

The waiting time depends on the transit line and the stop where the transit line is boarded. Typically the waiting time is a function of the headway, which is the inverse of the frequency of the transit line. In many applications the headway is divided by two to calculate the waiting time. In this algorithm (and many others) a distinction is made between the first boarding of a line and any consecutive boarding of a line. The explanation for this is that for the first boarding a waiting time of 0.5 x headway is too long for low frequency lines. The passenger typically anticipates for this when leaving home. For any consecutive waits this is no longer true.

For coordinated transit lines the waiting time does not depend on the frequency, but on the timetable. For these lines a constant waiting time is applicable. In the Zenith algorithm either waiting factors or constant waiting times are possible. These waiting factors and constants depend on the mode or sub mode. In reality the constants depend on the stop and the pair of transit lines:

$$W_{sl} = f_m F_l \quad (\text{when waiting time is defined as function of the frequency}) \quad (4a)$$

$$W_{sl} = wc_{l'l} \quad (\text{when the transit line is coordinated}) \quad (4b)$$

where:

$W_{sl}$  waiting time to board transit line  $l$  at stop  $s$ .

$f_m$  wait factor when boarding a line of mode  $m$ .

$F_l$  frequency of transit line  $l$ .

$wc_{l'l}$  waiting time as a constant when changing from line  $l'$  to line  $l$ .

A penalty is typically used to put any extra cost on a transit line which is not addressable as travel time or waiting time. This could be called an inconvenience component. Interchanging between two transit lines is in general perceived as inconvenience: given the same travel time and total waiting time, such an option will be perceived as less attractive than an alternative with no transfer component. In order to distinguish between the two an extra cost factor called penalty is added, this factor is imposed when boarding a transit line or changing between two lines. The penalty is either a function of the waiting time or a constant:

$$P_{sl} = p_m W_{sl} \quad (\text{when penalty is defined as function of the waiting time}) \quad (5a)$$

$$P_{sl} = pc_{l'l} \quad (\text{when the transit line is coordinated}) \quad (5b)$$

where:

$P_{sl}$  penalty to board transit line  $l$  at stop  $s$ .

$p_m$  penalty factor for mode  $m$ .

$W_{sl}$  waiting time at stop  $s$  for transit line  $l$ .

$pc_{l'l}$  penalty as a constant when changing from line  $l'$  to line  $l$ .

#### 2.4. Access to the transit system

In many models the zones are directly connected to stops by means of special connector links. These links typically get a distance and travel time which is an average representation for accessing the transit system. For zones which are further away a higher speed is specified in order to represent access by bicycle.

In this paper we assume a multimodal network to be present. This means that origins and destinations are linked to stops utilizing some underlying network, where links may be accessible by only a subset of modes, or speeds to traverse a link could be different for walk, bicycle and car. For most networks this implies that from one origin or destination almost all the stops in the network are reachable.

The first step in the model is to identify a set of stops that are relevant for a given zone. These stops serve as starting point of a transit trip. Because this set of stops is only a small subset of the total number of stops in the network, the path finding becomes a lot more efficient.

Criteria for accessing the transit network can be different for different access modes. In the Zenith algorithm the following criteria are used:

1. Distance radius. With this criterion the modeller can specify that all stops within a certain radius from the zone are included in the candidate set. This is particularly relevant for walk and bicycle access.
2. Type of system reached. With this criterion the modeller can specify the zone should have access to at least a minimal number of certain, typically higher order, transit systems. For example, that each zone is connected with at least one train station.
3. Type of station. With this criterion the modeller can specify that the zone should have access to certain type of stations. This is particularly relevant for car access and stations with Park & Ride facilities.
4. Minimal number of stops. With this criterion the modeler can specify the minimal number of stops in a candidate set. In case of remote zones this could be important to prevent zone disconnection when none of the other criteria can be satisfied.

The candidate set is complete when every criterion is satisfied. The resulting set of stops that is found to be relevant for an origin zone  $i$  is referred to as the candidate set  $S_i^d$ .

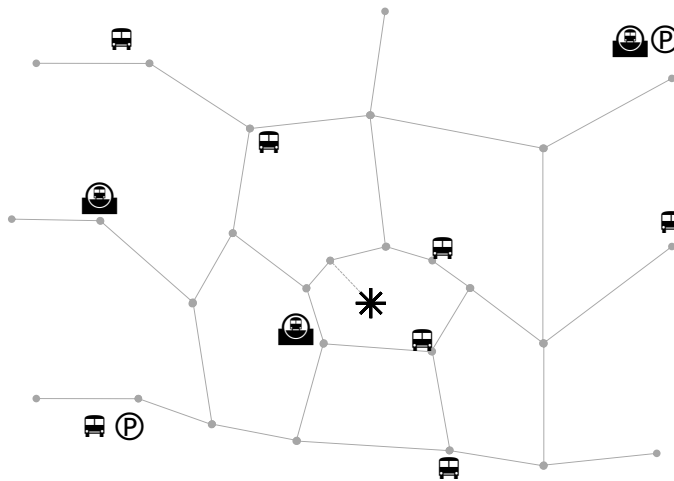


Fig. 2: Example access to the transit system

In the example in fig. 2 access to the transit system is explained for the centroid in the middle of the network for different access modes. In the example, 10 different stops are accessible for this centroid. Three of these stops give access to the train system, displayed with a different icon (🚆). Two stops have park & ride facilities denoted as an additional (P).

First consider the access mode to be walking. In that case it might suffice to only use the first criterion: radius. When a small search radius is applied only the three stops just around the centroid are considered as access points to the transit system for this origin.

When the access mode is bicycle, the access radius could be increased, resulting in two more bus stops to be included in the set of stops. Furthermore it becomes more likely that a cyclist prefers to access a higher order transit system, i.e. a train station. For this access mode the second criterion could be added, for example at least two train stations should be reached. In that case one more station is considered.

Finally when access mode is car, the search radius becomes less relevant but the facilities at the station become more important. For this access mode the search radius criterion could be dropped and only stations with park & ride facilities are considered. The criterion could be set such that at least two park & ride stations should be found, in this case the two stations with the additional (P). In fig. 3 the access stops per access mode are displayed.

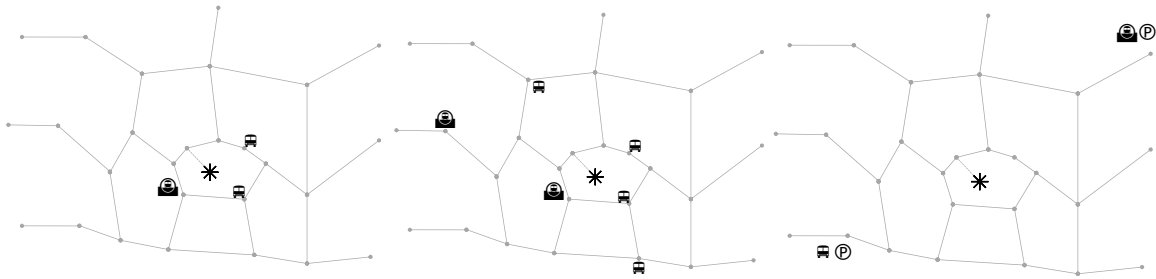


Fig. 3: Candidate set per access mode: walk (left), bicycle (middle) and car (right).

The path finding from the centroid to the set of access stops is done with a traditional Dijkstra algorithm based on generalized cost (see equation 2).

### 2.5. Egress from the transit system

When alighting the transit system the exact same criteria and algorithms are used as for accessing the transit system. In practice this does not necessarily mean that the access stop set is equal to the egress stop set, even when the access mode and egress mode is equal, due to different characteristics of the links in opposite directions, such as one-way streets. When multiple mode-chains are evaluated these stop sets are determined for each relevant mode. The resulting set of stops that is found to be relevant for a certain destination zone  $j$  is referred to as the candidate set:  $S_j^E$ .

### 2.6. Walk interchanges

When interchanging between lines the only mode allowed is walking. For every stop a set of possible interchange stops is identified based on a distance criterion only. So all stops within a certain radius are considered as stops where an interchange could take place. These stops need to be connected by a walking network. So for every possible stop  $s$  in the network a set of transfer stops is calculated, referred to as the interchange candidate set  $S_s^I$ .

### 2.7. Line choice model

As could be read in section 2.4 and 2.5, for every origin  $i$  and destination  $j$  in the network the set of candidate stops for a transit trip to begin (the first boarding stops)  $S_i^A$  and the set of candidate stops for a transit trip to end (the last alighting stops)  $S_j^E$  are determined a priori, containing only relevant stops per zone. This does not necessarily mean that these stops are actually used. This depends on the attractiveness of the connection by transit between these two stops.

From the set of stops at the destination  $S_j^E$  all transit lines that serve these stops are followed backwards towards the beginning of these transit lines. At every stop upstream along the transit line the generalized cost is calculated. These costs are stored at the stop and could be different per transit line.

When these generalized cost are set for all stops reachable from the collection of stops around the destination, the probabilities for boarding each transit line at each stop are calculated using :

$$P_{lsij} = \frac{F_l e^{-\lambda C_{lsij}}}{\sum_{x \in L_s} F_x e^{-\lambda C_{xsij}}} \quad (6)$$

where:

$P_{lsij}$  fraction for line  $l$  at stop  $s$  to reach  $j$  from  $i$ .

$F_l$  frequency of line  $l$ .

$C_{slj}$  generalized costs when using line  $l$  at stop  $s$  to reach  $j$  from  $i$ .

- $L_s$  set of candidate lines at stop  $s$ .  
 $\lambda$  service choice parameter.

From the set of stops reached the process is repeated until a given maximum number of interchanges (typically four). All trips that require more interchanges are not considered.

At any given stop a passenger can either stay in the same line or change to another line. From all the transit lines passing through a stop, only some will provide an effective means of reaching the chosen destination. Many will travel away from the destination, or at least not towards it. Others may travel towards the destination, but require too many interchanges, or travel too slowly to be classed as effective. In order to reduce the number of options, the following criterion is introduced:

Given a transit line  $l$  which is imminently departing, if there is another transit line which has lower expected generalized cost even if it involves waiting its full average headway, then transit line  $l$  is deemed to be illogical, and is ruled out of the choice set.

## 2.8. Stop choice model

Once the access candidate set is known for each origin zone and the line choice is known at every stop travelling to a given destination, the stop choice can be determined with a standard logit formulation:

$$P_{sij} = \frac{e^{-\theta C_{sij}}}{\sum_{x \in S_i^A} e^{-\theta C_{xij}}} \quad (7)$$

where:

- $P_{sij}$  fraction of travellers that choose stop  $s$  to reach  $j$  from  $i$ .  
 $S_i^A$  set of candidate stops for a given origin  $i$ .  
 $C_{sij}$  total generalized cost for travelling from stop  $s$  to reach  $j$  from  $i$ .  
 $\theta$  logit scale factor for stop choice

The total generalized cost to reach a stop (backwards from the destination) is calculated using the generalized cost per option multiplied with the probability of that option.

## 2.9. Summary

The Zenith algorithm tackles the public transport assignment problem in several steps:

1. For every origin and every destination the set of relevant stops are calculated. This set of stops could be different per access/egress mode. The set is bounded by using various constraints.
2. For every stop in the network a set of relevant interchange stops are calculated. This set of stops is bounded by a walking distance constraint.
3. For every destination zone in the network the shortest path tree is build backwards, starting at the relevant egress stops for the given destination. Paths are built backwards while following the entire line with a label setting algorithm. The various options to reach a stop are limited by constraints. The utility is calculated using a logit equation.
4. From the set of stops reached in the previous step and based on the blended utility, the process is repeated until a given maximum number of interchanges.
5. Based on the paths between the stops and the predetermined access and egress legs, the total chain for different access and egress modes is calculated.



### 3. Amsterdam Case

In this section an application of the described algorithm in Amsterdam (in The Netherlands), is described, that includes the use of multiple access and egress modes. Apart from PT, car is considered as a separate mode  $m$ . Bicycle is not considered, because the focus here is on interregional trips. Bicycle can only play a minor role on its own, so the bicycle trips are not included in the demand data. However, the bicycle is considered to be an important mode for access and egress, especially in the Dutch situation. In the following section, each component is described. After that, results are shown for this specific case study.

#### 3.1. Assignment

For PT assignment the following mode chains  $m$  are considered as separate modes with their specific costs:

- Walk – PT – Walk
- Bicycle – PT – Walk
- Car – PT – Walk
- Walk – PT – Bicycle
- Walk – PT – Car

Besides PT modes, car is considered as an additional mode. For car the generalized costs consist of travel time and of distance to represent fuel costs and other variable costs, for example maintenance costs (see equation 8). Parking charges at inner-city locations are included in the network on the connector links. Parking charges at stations are included in the transfer penalties, so these are included in the PT model when car is involved as access or egress mode:

$$C_{ma} = \alpha_m K_a + \beta_m T_{am} \tag{8}$$

The car-only trips are assigned to the network using the standard capacity dependent user equilibrium assignment of Frank-Wolfe. The car travel times depend on the flow following a standard BPR curve.

#### 3.2. Modal split

Using the costs of the mode chains and the costs of car, a nested logit model is used as a mode choice model. The OD matrices per mode chain are iteratively assigned to the multimodal network. For each iteration the costs are updated (see fig. 4). A fixed number of  $n_{max}$  iterations is executed, where a trade-off value for  $n_{max} = 8$  is chosen between calculation time and convergence to user equilibrium.

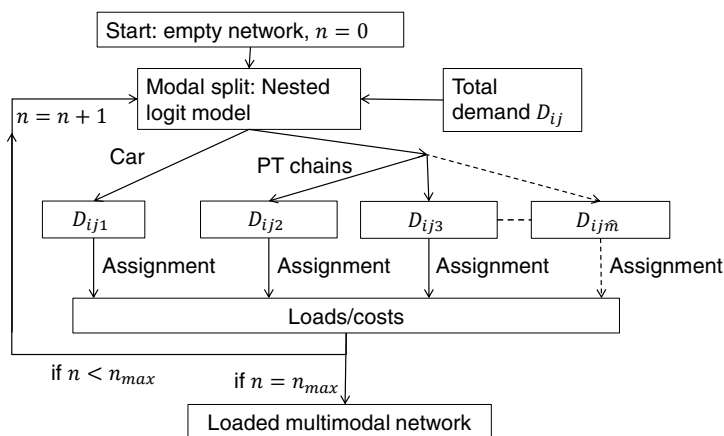


Fig. 4: Multimodal traffic assignment model used in the lower level

Depending on the costs per mode, a distribution over the modes is calculated using a nested logit model (Ben-Akiva and Bierlaire, 1999). This step splits the total OD matrix  $D_{ij}$  into several OD matrices  $D_{ij\hat{m}}$ , one for every mode chain  $\hat{m}$ . Within the nested logit model, we use two nests: one for the mode car and one for all mode chains (that include PT). The (generalized) costs  $C_{ij\hat{m}}$  of a mode follow from the route choice models as defined in equations 6 and 7. The use of a logit model as a choice model implies variation in preferences and generalized costs perception among travelers. Equation 9 calculates the composite costs (logsum) of the mode chains containing a PT leg. Using these costs, equation 10 calculates the share of car. The remaining mode share is distributed among mode chains that include PT by equation 11.

$$C_{ijM_{PT}} = -\frac{1}{\omega} \ln \sum_{\hat{m} \in M_{PT}} e^{-\omega C_{ij\hat{m}}} \quad (9)$$

$$D_{ij\hat{m}} = D_{ij} \frac{e^{-\varphi C_{ij\hat{m}}}}{e^{-\varphi C_{ij\hat{m}}} + e^{-\varphi C_{ijM_{PT}}}} \quad \text{for } \hat{m} \text{ is car} \quad (10)$$

$$D_{ij\hat{m}} = D_{ij} \frac{e^{-\varphi C_{ijM_{PT}}}}{e^{-\varphi C_{ijcar}} + e^{-\varphi C_{ijM_{PT}}}} \frac{e^{-\omega C_{ij\hat{m}}}}{\sum_{m \in M_{PT}} e^{-\omega C_{ijm}}} \quad \text{for } \hat{m} \in M_{PT} \quad (11)$$

$\varphi$  Logit scale parameter for the choice between car and PT

$\omega$  Logit scale parameter for the choice between mode chains that contain a PT leg

$C_{\hat{m}}$  Generalized costs for using mode chain  $\hat{m}$ .

$M_{PT}$  Set of mode chains that contain a PT leg

### 3.3. Study area

The case study area covers the Amsterdam Metropolitan Area in The Netherlands (Fig. 5). This area has an extensive multimodal network with pedestrian, bicycle, car and transit infrastructure. Transit consists of 586 bus lines, 42 tram and metro lines and 128 train lines, that include local trains, regional trains and intercity trains. Bicycles can be parked at most stops and stations. A selection of transit stops facilitate park-and-ride transfers. Origins and destinations are aggregated into 102 transportation zones. Important commercial areas are the city centres of Amsterdam and Haarlem, the business district in the southern part of Amsterdam, the harbour area and airport Schiphol. Other areas are mainly residential, but still small or medium scale commercial activities can be found.

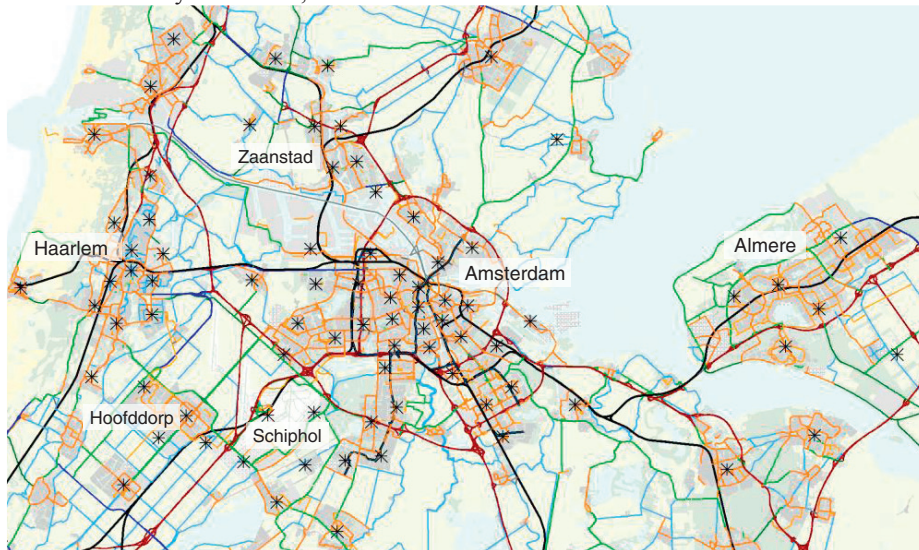


Fig. 5 Map of the study area, showing transportation zones, railways, roads

### 3.4. Route choice examples

The access leg of an example trip from Haarlem (zone 7 on the map) to Amsterdam (West of Haarlem, not on the map) is shown in fig. 6. When travellers use walk as access mode, they mainly take the bus to the main station of Haarlem, to take an express train to Amsterdam: the fraction taking this bus is 0.97, where a majority boards at the closest stop (0.94) and a minority at a stop further away (0.03). A small fraction (0.03, not on the map) takes the bus to Schiphol airport and changes there. When a traveller uses bicycle as access mode, a majority of the travellers cycle to Haarlem main station to take a fast connection to Amsterdam. However, a fraction of 0.22 cycles to a smaller and closer station in Haarlem to take the local train there. In the case study, approximately 40% of all PT traveller uses the bicycle as access mode.

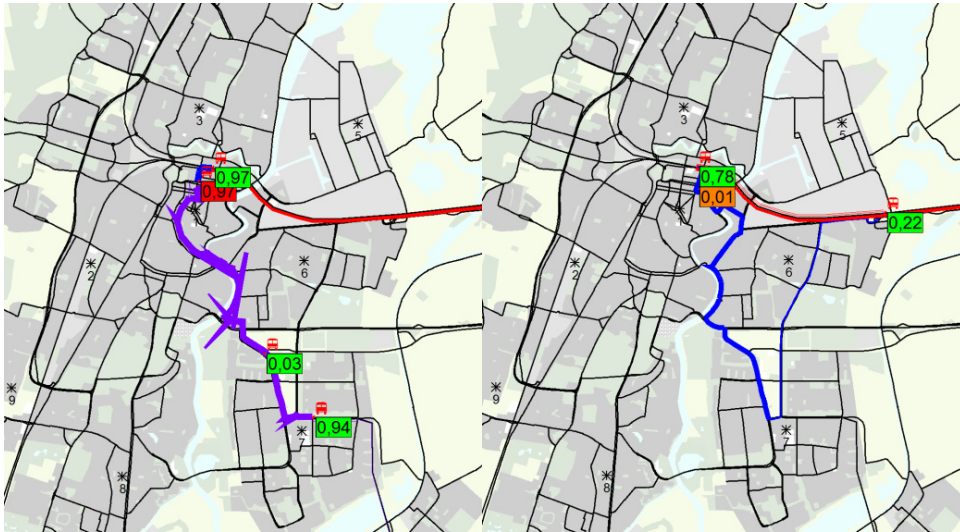


Fig. 6: route choice results for walk as access mode (left) and for bicycle as access mode (right)

### 3.5. Calculation time

Using a fixed number of 8 mode choice iterations (and consequently do 8 car assignments and 8 PT assignments), in the Amsterdam case study, the time used for PT calculations is 187 seconds (using a computer with an Intel® Core™ i7 CPU 860 @ 2.8GHz and a 4 GB RAM). Car calculations took 202 seconds and mode choice calculations 7 seconds. Overall, the calculation time is limited. As a result, the described case study can be repeatedly used to assess a multimodal network design during an optimization process. Moreover, in practice the PT assignment algorithm is suitable for much larger networks: it has for example been used to perform PT assignments for the complete PT network of the Netherlands, using around 7000 transportation zones.

## 4. Conclusions

This paper described a specific PT assignment algorithm in detail, referred to as the Zenith method. This frequency based algorithm includes multiple routing and multiple access and egress modes and uses an integrated multimodal network as input. This allows flexibility by using appropriate parameter settings and mode chains, including the possibility to include bicycle legs to stations and to include park and ride. Furthermore, it accounts for different preferences and perceptions among travellers.

A practical application in the Amsterdam metropolitan area showed that the assignment algorithm can be incorporated in a wider modelling framework that includes mode choice and car assignment. The application resulted in finding realistic routes in a real network. An important aspect in the Netherlands is the high share of the bicycle as

an access mode: taking this into account results in more realistic routes through the PT network, leading to more realistic loads and a better way of modelling. Finally, calculation times are limited, allowing for applications in large networks.

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