Regulated Output Synchronization of Heterogeneous Multi-agent Systems with Desired Convergence Rate via Scale-free Protocol Design

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***Abstract*—This paper outlines scalable collaborative protocols for achieving regulated output synchronization in heterogeneous multi-agent systems (MAS) in presence of arbitrary desired convergence rates. The proposed protocol design is scale-free, i.e., only requires knowledge of the agents' models, without any networks information and the number of agents. With our protocols, only adjusting a single parameter allows output synchronization to be achieved at any convergence rates.**

***Index Terms*—Regulated output synchronization, arbitrary desired convergence rate, scale-free collaborative protocol design, heterogeneous multi-agent systems**

I. Introduction

In recent years, multi-agent systems' (MAS) synchronization or consensus problem has gained increasing recognition due to its practical applications in cooperative control of various MAS including autonomous vehicles, distributed sensor networks, and satellites/robotic systems, the references regarding this topic see [1], [2], [6], [11], [12], [17]. The fundamental goal of achieving synchronization for MAS is to reach an asymptotic agreement on a trajectory for systems' states or output by using local interactions among agents. Note that state synchronization inherently calls for homogeneity within the given system.

Many protocols for synchronization of MAS described in the literature rely on knowledge of the communication network, including spectral bounds and agent count. However, such approaches are prone to scale fragility issues as noted by [13], [15], [16], making them unstable for larger networks or changing communication graphs. Thus, current research efforts prioritize consensus law scalability. Our recent work introduces a class of scale-free protocols to synchronize or almost synchronize the states of MAS under external disturbances and input saturation respectively, see [9], [10]. The developed protocols can be implemented without any prior knowledge about the topology or size of the network used for communications by utilizing localized information exchange among neighbors.

Designing consensus protocols for practical applications heavily relies on performance, with the asymptotic convergence rate serving as a key indicator which is defined in [18] and utilized in this paper. In protocol design, the communication topology between agents plays a crucial role in determining its convergence rate. More specifically, existing continuous-time MAS protocols' convergence rates are influenced by their associated communication graph's second smallest eigenvalue or algebraic connectivity. However, some undirected graphs exhibit decreasing algebraic connectivity concerning network size growth, i.e., an effect studied extensively through recent works like [14]. Directed circulant graphs also demonstrate similar characteristics outlined by another analysis mentioned here in passing from thesis [2, Chapter 16]. Finally, [7], [8] propose scalable regulated state synchronization capable of achieving any desired level of convergent rate within homogeneous MAS frameworks.

This paper's primary objective is to establish a scale-free collaborative protocol that can be regulated by a positive scalar parameter μ to achieve output synchronization for heterogeneous MAS and maintain an arbitrary desired convergence rate. Our proposed protocols possess the following properties.

- The regulated output synchronization of MAS can be attained for any network with arbitrary size and communication topology, as long as the positive parameter μ is used to adjust the convergence rate.
- By appropriately selecting the parameter μ, the proposed protocols can accomplish any desired convergence rate. Specifically, raising μ allows us to reach any predetermined asymptotic convergence rate.
- The protocols' structure is not contingent on the parameter μ, which allows for developing the design at one stage and adjusting μ later to achieve the desired convergence rate. Tuning can even happen in real-time due to its continuity, resulting in a non-iterative one-shot approach..

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Notations and definitions

Firstly, we provide an important concept, i.e., asymptotic convergence rate (or convergence factor) see in [18] and [6]. For a stable linear system,

$$\dot{s}(t) = Hs(t),$$

with $s \in \mathbb{R}^n$ and Hurwitz stable H, we have that the *asymptotic convergence rate* as follows

$$r_{\text{asym}} = \max_{s(0)\neq 0} \lim_{t\to\infty} \left(\frac{\|s(t)\|}{\|s(0)\|}\right)^{\frac{1}{t}}. \tag{1}$$

Meanwhile, we obtain the associated *convergence time* as follows

$$\tau_{\text{asym}} = \frac{-1}{\log(r_{\text{asym}})}. \tag{2}$$

Thus, one have

$$\max_{s(0)\neq 0} \lim_{t\to\infty} \left(\frac{\|s(t)\|}{\|s(0)\|}\right)^{\frac{1}{t}} = \lim_{t\to\infty}(\|e^{Ht}\|)^{\frac{1}{t}} = \rho(e^H)$$

where Gelfand's spectral radius formula [5] is used to connect a matrix's norm and spectral radius $\rho(\cdot)$. Thus

$$\begin{aligned} r_{\text{asym}} &= \rho(e^H) = \max_i(\|e^{\text{Re}(\lambda_i(H))}\|) \\ &= \max_i(e^{\text{Re}(\lambda_i(H))}) = e^{\text{Re}(\lambda_1(H))} \end{aligned}$$

where $\lambda_i(H)$ is H's the i^{th} eigenvalue, and H's eigenvalues ordered by their real parts, i.e.

$$0 > \text{Re}\,\lambda_1(H) \geqslant \text{Re}\,\lambda_2(H) \geqslant \cdots \geqslant \text{Re}\,\lambda_n(H).$$

If the system output is $y(t) = Cx(t)$ with a constant matrix C, we can still use the above definition to measure or evaluate the output's asymptotic convergence rate.

II. Problem formulation and system description

We introduce the following agent model to describe the heterogeneous MAS with N agents

$$\begin{aligned} \dot{\bar{x}}_i &= \bar{A}_i\bar{x}_i + \bar{B}_i\bar{u}_i, \\ y_i &= \bar{C}_i\bar{x}_i, \end{aligned} \tag{3}$$

where $\bar{x}_i \in \mathbb{R}^{n_i}$ is the state of agent i, $\bar{u}_i \in \mathbb{R}^{m_i}$ is the input of agent i, and $y_i \in \mathbb{R}^p$ is the output of agent i with $i = 1, \ldots, N$.

Each agent is introspective as it can access to its own local information, which might also include part of its state

$$\bar{z}_i = \bar{C}_i^m \bar{x}_i, \tag{4}$$

where $\bar{z}_i \in \mathbb{R}^{q_i}$.

We assume that the agent model (3) satisfies the following conditions.

Assumption 1 *For agent model i with $i \in \{1, \ldots, N\}$, we have*

1) *$(\bar{A}_i, \bar{B}_i)$ is controllable.*
2) *$(\bar{C}_i, \bar{A}_i)$ is observable.*
3) *$(\bar{C}_i, \bar{A}_i, \bar{B}_i)$ is right-invertible.*
4) *$(\bar{C}_i^m, \bar{A}_i)$ is observable.*

Each agent receives a linear combination of its own output and that of neighboring agents through the communication network. Each agent i ($i \in \{1, \cdots, N\}$) can access to the following quantity,

$$\sigma_i = \sum_{j=1, j\neq i}^{N} s_{ij}(y_i - y_j) \tag{5}$$

with $s_{ij} \geqslant 0$ and $s_{ii} = 0$. $\mathcal{G}$ is a weighted directed graph and utilized to portray the communication network's topology. In this context, each node in the network represents an agent while the coefficient s_{ij} determines edge weight. When considering the Laplacian matrix $L = [\ell_{ij}]_{N\times N}$ with

$$\ell_{ij} = \begin{cases} \sum_{k=1}^{N} s_{ik}, & i = j, \\ -s_{ij}, & i \neq j, \end{cases}$$

σ_i can be expressed as

$$\sigma_i = \sum_{j=1}^{N} \ell_{ij} y_j. \tag{6}$$

A localized information exchange among agents is introduced. Each agent i with $i \in \{1, \ldots, N\}$ can access this $\hat{\sigma}_i$, which is shown as follows

$$\hat{\sigma}_i = \sum_{j=1, j\neq i}^{N} s_{ij}(\theta_i - \theta_j) \tag{7}$$

where θ_i is an internal variable for agent i.

Our objective is to achieve regulated output synchronization of heterogeneous MAS. Here, regulated output synchronization is that the outputs produced by each agent converge asymptotically towards a trajectory denoted as y_r, i.e.,

$$\lim_{t\to\infty}(y_i(t) - y_r(t)) = 0 \tag{8}$$

where y_r is generated by an exosystem

$$\begin{cases} \dot{\bar{x}}_r = \bar{A}_r\bar{x}_r, & \bar{x}_r(0) = \bar{x}_{r0}, \\ y_r = \bar{C}_r\bar{x}_r, \end{cases} \tag{9}$$

with $\bar{x}_r \in \mathbb{R}^{n_{\bar{r}}}$ and $y_r \in \mathbb{R}^p$.

The following assumption is provided for the above exosystem,

Assumption 2 *For exosystem, we have*

1) *$(\bar{C}_r, \bar{A}_r)$ is observable.*
2) *$\bar{A}_r$'s all eigenvalues are on the imaginary axis.*

To ensure that every agent receives exosystem's information, we establish a nonempty subset $\mathscr{C}$ for agents. This subset allows each agent to access their output in relation to that of the exosystem. So, we have that agent i can access to the following quantity

$$\Delta_i = \kappa_i(y_i - y_r), \qquad \kappa_i = \begin{cases} 1, & i \in \mathscr{C}, \\ 0, & i \notin \mathscr{C}. \end{cases} \tag{10}$$

Therefore, the information exchange among agents linked by the above exosystem is provided by

$$\tilde{\sigma}_i = \sum_{j=1, j\neq i}^{N} s_{ij}(y_i - y_j) + \kappa_i(y_i - y_r). \tag{11}$$

$\tilde{\sigma}_i$ can be rewritten as $L_{EP} = L + \text{diag}\{\kappa_i\} = [\tilde{\ell}_{ij}]_{N\times N}$, i.e.,

$$\tilde{\sigma}_i = \sum_{j=1}^{N} \tilde{\ell}_{ij}(y_j - y_r). \tag{12}$$

It should be noted that the matrix L_{EP} is not a regular one for graphs as its rows can not add up to zero. As stated in [4, Lemma 7], it can be proven that L_{EP}'s all eigenvalues have positive real parts, and this L_{EP} is invertible.

The following definition about the graph is provide to solve our problem.

Definition 1 *There is a node set $\mathscr{C}$, then the set of all graphs with N nodes in the node set $\mathscr{C}$ is defined as $\mathbb{G}^N_{\mathscr{C}}$. The network graph $\mathcal{G} \in \mathbb{G}^N_{\mathscr{C}}$'s every nodes are the members of a directed tree, speciallythe node set $\mathscr{C}$ contains the directed tree's root. Then, the node set $\mathscr{C}$ is defined as root set. On the other hand, it is clear that a graph $\mathcal{G} \in \mathbb{G}^N_{\mathscr{C}}$ does not necessarily contain a directed spanning tree.*

Remark 1 *The matrix L_{EP}'s eigenvalues $\lambda_{EP,i}$ $(i = 1, \cdots, N)$, which is corresponding to graph $\mathcal{G} \in \mathbb{G}^N_{\mathscr{C}}$, are arranged in ascending order based on their real parts, i.e.,*

$$0 < \text{Re}\,\lambda_{EP,1} < \text{Re}\,\lambda_{EP,2} \leqslant \cdots \leqslant \text{Re}\,\lambda_{EP,N}. \tag{13}$$

We now give the problem formulation for **scalable** regulated output synchronization.

Problem 1 *There is a network communication described by* (12).

For a heterogeneous MAS (3) *with local information* (4)*, associated exosystem* (9)*. The* ***scalable regulated output synchronization with arbitrarily desired convergence rate utilizing collaborative protocols*** *is to establish by the following parameterized linear dynamic protocol for each agent by using only knowledge of the agent models, i.e. $(\bar{A}_i, \bar{B}_i, \bar{C}_i)$ and r^*_{asym}, for any $r^*_{\text{asym}} > 0$ and a scalar parameter $\mu > 0$,*

$$\begin{cases} \dot{x}_{i,p} = A^{\mu}_{p,i}x_{i,p} + B^{\mu}_{p,i}\tilde{\sigma}_i + C^{\mu}_{p,i}\hat{\sigma}_i + D^{\mu}_{p,i}\bar{z}_i \\ \bar{u}_i = F^{\mu}_{p,i}x_{i,p} + G^{\mu}_{p,i}\bar{z}_i, \end{cases} \tag{14}$$

where $\hat{\sigma}_i$ is defined in (7) *with $\theta_i = M_p x_{i,p}$, and $x_{i,p} \in \mathbb{R}^{n_p}$, such that*

- *for any N, any $\mu > 0$, and any graph $\mathscr{G} \in \mathbb{G}^N_{\mathscr{C}}$, regulated output synchronization* (8) *can be achieved by the above protocol , where $\mathbb{G}^N_{\mathscr{C}}$ is defined as Definition 1.*
- *for any given graph $\mathscr{G} \in \mathbb{G}^N_{\mathscr{C}}$ and any y_r, a desired convergence rate (for the synchronization error) less than r^*_{asym} is achieved by choosing parameter μ sufficiently large.*

III. Main result

Firstly, we provide a lemma to achieve homogenization for heterogeneous MAS, which can be found in [19, Appedix C].

Lemma 1 *Consider an new exosystem denoted by*

$$\begin{cases} \dot{x}_r = A_r x_r, \quad x_r(0) = x_{r0} \\ y_r = C_r x_r. \end{cases} \tag{15}$$

For all $\bar{x}_{r0} \in \mathbb{R}^r$, there exists $x_{r0} \in \mathbb{R}^r$ such that (15) *generates exactly the same output y_r as the original exosystem* (9)*. By finding a matrix B_r, one have the triple (C_r, A_r, B_r) is of uniform rank n_q, invertible, and has no invariant zeros, where n_q is an integer greater than or equal to $(\bar{C}_i, \bar{A}_i, \bar{B}_i)$'s maximal order of infinite zeros with $i \in \{1, \cdots, N\}$, and $(\bar{C}_r, \bar{A}_r)$'s all the observability indices (see [3] for the definition). Meanwhile, A_r's all eigenvalues consist of $\bar{A}_r$'s all eigenvalues and additional zero eigenvalues.*

For the scalable regulated output synchronization, we develop a four-step protocol design process.

Step 1: Remodeling the exosystem

The original exosystem is remodeled by choosing suitable the target model (C_r, A_r, B_r). The detailed design procedure is provided in Lemma 1 or in [19].

Step 2: Designing pre-compensators

According to the instruction in [19], we can establish the following precompensator for each agent i, such that the agent can homogenize to the target model (C_r, A_r, B_r) which is designed in Step 1.

$$\begin{cases} \dot{\eta}_i = \bar{A}_{i,h}\eta_i + \bar{B}_{i,h}\bar{z}_i + \bar{E}_{i,h}u_i, \\ \bar{u}_i = \bar{C}_{i,h}\eta_i + \bar{D}_{i,hr}u_i, \end{cases} \tag{16}$$

Then we can obtain the following compensated agent model

$$\begin{cases} \dot{x}_i = A_r x_i + B_r(u_i + \phi_i), \\ y_i = C_r x_i, \end{cases} \tag{17}$$

where ϕ_i is given by

$$\begin{cases} \dot{\delta}_i = \bar{A}_{i,s}\delta_i, \\ \phi_i = \bar{C}_{i,s}\delta_i, \end{cases} \tag{18}$$

and $\bar{A}_{i,s}$ is Hurwitz stable with the real part of all eigenvalues less than $\ln(r^*_{\text{asym}})$. As a result, we can homogenize the compensated agents to targel model (C_r, A_r, B_r).

Step 3: Designing collaborative protocols for the homogenized compensated agents

Then, we can design the following the collaborative protocols, which is parameterized with a positive scalar μ and rely on exchanging localized information,

$$\begin{cases} \dot{\hat{x}}_i = A_r\hat{x}_i - \mu B_r K\hat{\sigma}_i + M(\mu\tilde{\sigma}_i - C_r\hat{x}_i) + \mu\kappa_i B_r u_i, \\ \dot{\rho}_i = A_r\rho_i + B_r u_i + \hat{x}_i - \mu\hat{\sigma}_i - \mu\kappa_i\rho_i, \\ u_i = -F\rho_i, \end{cases} \tag{19}$$

where M and F are designed such that we have that $A_r - MC_r$ and $A_r - B_r F$ are both Hurwitz stable with the real part of all eigenvalues less than $\ln(r^*_{\text{asym}})$. The information exchange $\hat{\sigma}_i$ is defined as follows,

$$\hat{\sigma}_i = \sum_{j=1}^{N} a_{ij}(\rho_i - \rho_j),$$

and $\tilde{\sigma}_i$ is defined in (12).

Step 4: Final protocols
The final protocol is provided as follows

$$\begin{cases} \dot{\eta}_i = \bar{A}_{i,h}\eta_i + \bar{B}_{i,h}\bar{z}_i - \bar{E}_{i,h}F\rho_i, \\ \dot{\hat{x}}_i = A_r\hat{x}_i - \mu B_r F\hat{\sigma}_i + M(\mu\tilde{\sigma}_i - C_r\hat{x}_i) - \mu\kappa_i B_r F\rho_i, \\ \dot{\rho}_i = A_r\rho_i - B_r F\rho_i + \hat{x}_i - \mu\hat{\sigma}_i - \mu\kappa_i\rho_i, \\ \bar{u}_i = \bar{C}_{i,h}\eta_i - \bar{D}_{i,h}F\rho_i. \end{cases} \tag{20}$$

Next, the following theorem is presented to establish scale-free regulated output synchronization result.

Theorem 1 *There are the set of nodes $\mathscr{C}$ and the graph set $\mathbb{G}^N_{\mathscr{C}}$. Let a heterogeneous MAS with N agents* (3) *and local information* (4) *satisfy Assumption 1, and let the exosystem* (9) *satisfy Assumption 2.*

The scalable regulated output synchronization with arbitrarily desired convergence rate as defined in Problem 1 can be solvable. For any desired convergence rate r^*_{asym}, the parameterized linear dynamic protocol (20)

- *for any $\mu > 0$, and any graph $\mathcal{G} \in \mathbb{G}^N_{\mathscr{C}}$ with any N, achieves regulated state synchronization.*
- *for any given graph $\mathscr{G} \in \mathbb{G}^N_{\mathscr{C}}$ and any y_r, by choosing parameter μ sufficiently large, achieves a desired convergence rate less than r^*_{asym} (for the synchronization error).*

Proof: We use two steps to prove this theorem.
Step 1: Proving regulated output synchronization

In order to homogenize to the target model represented by (17), a pre-compensator (16) is established to specific each agent i with $i = \{1, \cdots, N\}$, i.e.,

$$\begin{cases} \dot{x}_i = A_r x_i + B_r(u_i + \psi_i), \\ y_i = C_r x_i, \end{cases}$$

where ψ_i is given in (18). Let $\check{x}_i = x_i - x_r$, and then we can define

$$\check{x} = \begin{pmatrix} \check{x}_1 \\ \vdots \\ \check{x}_N \end{pmatrix}, \hat{x} = \begin{pmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_N \end{pmatrix}, \rho = \begin{pmatrix} \rho_1 \\ \vdots \\ \rho_N \end{pmatrix}, \psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}, \delta = \begin{pmatrix} \delta_1 \\ \vdots \\ \delta_N \end{pmatrix}.$$

And let $\bar{A}_s = \text{diag}(\bar{A}_{i,s})$ and $\bar{C}_s = \text{diag}(\bar{C}_{i,s})$, we have

$$\begin{cases} \dot{\check{x}} = (I \otimes A_r)\check{x} - (I \otimes B_r K)\rho + (I \otimes B_r)\psi, \\ \dot{\hat{x}} = (I \otimes (A_r - MC_r))\hat{x} \\ \qquad -\mu(L_{EP} \otimes B_r F)\rho + \mu(L_{EP} \otimes MC_r)\check{x}, \\ \dot{\rho} = (I \otimes (A_r - B_r F) - \mu L_{EP} \otimes I)\rho + \hat{x}, \\ \dot{\delta} = \bar{A}_s\delta. \end{cases} \tag{21}$$

By defining $\omega = \check{x} - \rho$ and $\bar{\omega} = \mu(L_{EP} \otimes I)\check{x} - \hat{x}$, we obtain

$$\begin{cases} \dot{\check{x}} = (I \otimes (A_r - B_r F))\check{x} + (I \otimes B_r F)\omega + (I \otimes B_r)\bar{C}_s\delta, \\ \dot{\omega} = (I \otimes A_r - \mu L_{EP} \otimes I)\omega + \bar{e} + (I \otimes B_r)\bar{C}_s\delta, \\ \dot{\bar{\omega}} = (I \otimes (A_r - MC_r))\bar{\omega} + \mu(L_{EP} \otimes B_r)\bar{C}_s\delta, \\ \dot{\delta} = \bar{A}_s\delta. \end{cases} \tag{22}$$

One have

$$(Q \otimes I)(I \otimes A_r - \mu L_{EP} \otimes I)(Q^{-1} \otimes I) = I \otimes A_r - \tilde{R} \otimes I, \tag{23}$$

when we have a non-singular transformation matrix Q satisfies the equation $T^{-1}\tilde{R}T = L_{EP}$, where $\tilde{R}$ is matrix L_{EP}'s upper triangular Jordan form. Then it follows that (23) can be partitioned into blocks and has matrices $A_r - \mu\lambda_{EP,i}I$ on its diagonal with $i \in \{1, \cdots, N\}$. Here $(\lambda_{EP,1}, \cdots, \lambda_{EP,N})$ are the matrix L_{EP}'s eigenvalues. Since all eigenvalues of A_r are in the closed left-half plane, $A_r - \mu\lambda_{EP,i}I$ is Hurwitz stable. As a result, the matrix $I \otimes A_r - \mu L_{EP} \otimes I$'s eigenvalues have negative real part.

One know that $I \otimes A_r - \mu L_{EP} \otimes I$, $A_r - MC_r$ and $A_r - B_r F$ are Hurwitz stable. We also know that $\bar{A}_s$ is Hurwitz stable since the matrices $\bar{A}_{i,s}$ are Hurwitz stable. This immediately yields that $\check{x}(t), \omega(t), \bar{\omega}(t)$ and $\delta(t)$ converge to zero as $t \to \infty$.

Since $\check{x}_i(t) \to 0$ as $t \to \infty$ we find that $x_i(t) - x_r(t) \to 0$. Hence, one can obtian $y_i(t) - y_r(t) = C_r(x_i(t) - x_r(t)) \to 0$ as $t \to \infty$, i.e., regulated output synchronization is achieved.
Step 2: Proving the desired convergence rate

From (22), one have

$$\dot{s} = A_H s \tag{24}$$

by letting

$$s = \begin{pmatrix} \check{x} \\ \omega \\ \bar{\omega} \\ \delta \end{pmatrix},$$

where

$$A_H = \begin{pmatrix} I \otimes (A_r - B_r F) & I \otimes (B_r F) & 0 & (I \otimes B_r)\bar{C}_s \\ 0 & I \otimes A_r - \mu L_{EP} \otimes I & I & (I \otimes B_r)\bar{C}_s \\ 0 & 0 & (I \otimes (A_r - MC_r)) & \mu(L_{EP} \otimes B_r)\bar{C}_s \\ 0 & 0 & 0 & \bar{A}_s \end{pmatrix}.$$

The A_H's eigenvalues are equal to the union of the eigenvalues of $I \otimes (A_r - B_r F)$, $I \otimes A_r - \mu L_{EP} \otimes I$, $I \otimes (A_r - MC_r)$ and $\bar{A}_s$.

By construction, the real parts of the eigenvalues of $A_r - B_r F$, $A_r - MC_r$ and $\bar{A}_s$ are all less than $\ln(r^*_{\text{asym}})$. Clearly, for μ large enough we also have that the real parts of the eigenvalues of $A_r - \mu L_{EP} \otimes I$ are all less than $\ln(r^*_{\text{asym}})$.

Together, this implies that the real parts of A_H's all eigenvalues are less than $\ln(r^*_{\text{asym}})$. This implies that the dynamics of (24) has a convergence rate of r^*_{asym}.

Finally, since $C_r(x_i - x_r) = y_i - y_r$, the regulated output synchronization with the desired convergence rate is achieved. ■

IV. Numerical example

We are going to demonstrate the effectiveness of the proposed parameterized protocol designs in this section, by using two numerical examples to achieve heterogeneous MAS' regulated output synchronization.

Consider agent models (3) including the following three groups of parameters

$$\text{Group 1: } \bar{A}_i = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}, \bar{B}_i = I_2, \bar{C}_i^{\mathrm{T}} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \bar{C}_i^m = I_2$$

$$\text{Group 2: } \bar{A}_i = 0, \bar{B}_i = 1, \bar{C}_i = 1, \bar{C}_i^m = 1$$

and

$$\text{Group 3: } \bar{A}_i = \begin{pmatrix} 0 & 1 \\ -3 & 4 \end{pmatrix}, \bar{B}_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \bar{C}_i^{\mathrm{T}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \bar{C}_i^m = I_2.$$

Then, consider exosystem model (9) with the following parameters

$$\bar{A}_r = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \bar{C}_r = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}.$$

Since $n_q = 3$, we only remodel exosystem by $A_r = \bar{A}_r$, $B_r = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^{\mathrm{T}}$, and $C_r = \bar{C}_r$.

We design the following pre-compensator for the agents

$$\begin{cases} \bar{u}_i = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \eta_i, \\ \dot{\eta}_i = -\eta_i + \begin{pmatrix} 1 & 0 \end{pmatrix} \bar{z}_i + u_i, \end{cases}$$

for Group 1,

$$\begin{cases} \bar{u}_i = \begin{pmatrix} 1 & -1 \end{pmatrix} \eta_i, \\ \dot{\eta}_i = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} \eta_i + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_i + \begin{pmatrix} -1 \\ 1 \end{pmatrix} \bar{z}_i, \end{cases}$$

for Group 2, and

$$\begin{cases} \bar{u}_i = \begin{pmatrix} 1 & 2 \end{pmatrix} \eta_i \\ \dot{\eta}_i = -\eta_i + \bar{z}_i + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_i \end{cases}$$

for Group 3.

Then, we choose $F = \begin{pmatrix} 1 & 2.4142 & 2.4142 \end{pmatrix}$ and $M = \begin{pmatrix} 2.4142 & 2.4142 & 1 \end{pmatrix}^{\mathrm{T}}$ such that $A_r - MC_r$ and $A_r - B_r F$ are Hurwitz stable.

We consider two examples for heterogeneous MAS consisting of 6 agents ($N = 6$) and 60 agents ($N = 60$) respectively. Meanwhile, let $\kappa_1 = 1$ and $\kappa_i = 0$ for $i \neq 1$.

Case 1 ($N = 6$) : For agent $i = 1, 6 \in$ Group 1, agent $i = 2, 5 \in$ Group 2, and agent $i = 3, 4 \in$ Group 3. The associated adjacency matrix to the communication network is still assumed to be $\mathcal{A}_I$ where $s_{21} = s_{32} = s_{13} = s_{43} = s_{36} = s_{54} = s_{65} = 1$.

Case 2 ($N = 60$): For agent $i = 1 - 30$ and $101 - 110 \in$ Group 1, agent $i = 31 - 60$ and $91 - 100 \in$ Group 2, and agent $i = 61 - 90$ and $111 - 120 \in$ Group 3, and communication network with the matrix $\mathcal{A}_{II}$, where $s_{1,120} = s_{i+1,i} = 1$ and $i = 1, \cdots, 59$.

Figures 1 and 2 show that scalable regulated output synchronization with any desired convergence rate is achieved for a heterogeneous MAS with both $N = 6$ agents and $N = 60$ agents. Meanwhile, when the parameter μ has a larger value, we can find that the convergence rate goes fast (or the result has shorter convergence time).

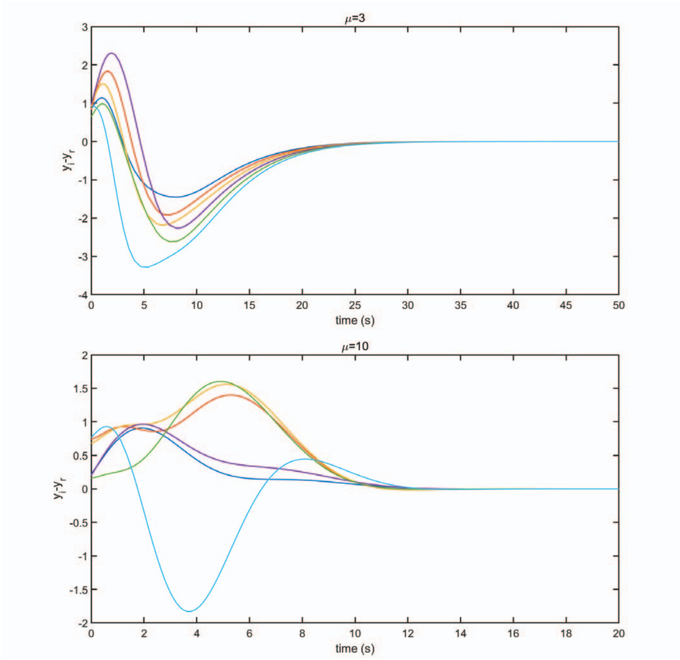

Figure 1. Scalable regulated output synchronization in Case B.I, where $\mu = 3$ and $\mu = 10$.

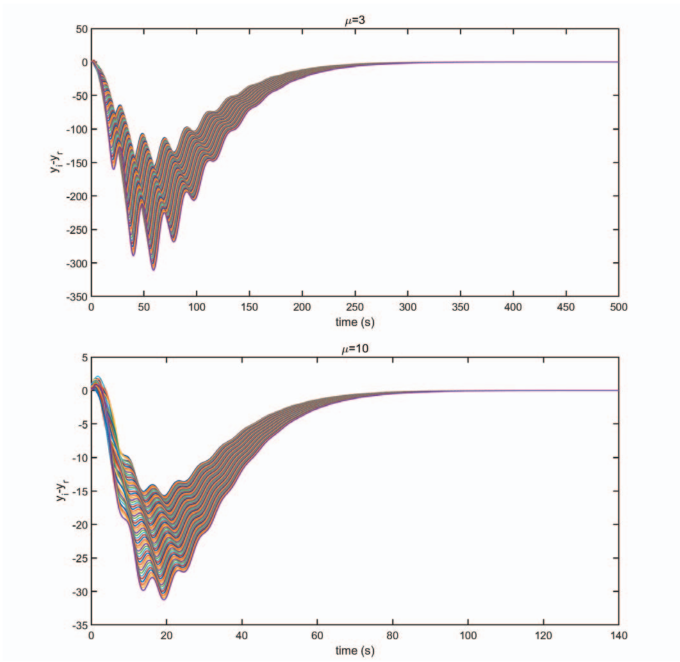

Figure 2. Scalable output synchronization in Case B.II, where $\mu = 3$ and $\mu = 10$.

V. Conclusion

In this paper, regulated output synchronization results with any desired convergence rate for MAS have been studied by introducing a positive parameter μ. The *scale-free* collaborative protocol design has been achieved only based on the agent models' knowledge and without utilizing the network structure information. Scalable regulated output synchronization has been achieved for heterogeneous MAS with introspective agents in presence of any desired asymptotic convergence rate by tuning μ online.

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