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A MARTINGALE APPROACH TO ESTIMATION AND CONTROL OF TRAFFIC FLOW ON MOTORWAYS[1]

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Abstract. Emphasizing the stochastic elements of traffic flow on motorways a stochastic continuous-time aggregate variable model is presented. The model consists of a nonlinear dynamical system with a martingale forcing term. The problem of estimating local traffic conditions from noisy disaggregate data (detector measurements) is discussed. Using results of abstract martingale theory with respect to nonlinear optimal filtering for counting process observations a recursive estimation algorithm is obtained. The algorithm, which is very appropriate for use in automatic motorway control and surveillance systems, performs very well. Finally, the optimal control of traffic flow is discussed with an emphasis on speed control.

Keywords. Road traffic; modelling; stochastic systems; martingales; nonlinear filtering; counting processes; optimal control.

INTRODUCTION

In most of the industrialized countries a stage has been reached in which a perspective of growing congestion on motorways cannot be erased anymore just by expanding the motorway network. Within the broader social context of environmental livability, energy consumption and economic decline other solutions have to be sought.

At present the general policy is to promote the use of public transport and the selective use of private cars. Besides to promote the more efficient and safer use of existing motorways detailed studies are made for the development and design of automatic motorway control and surveillance systems.

In the Netherlands, the Ministry of Roads and Waterways (Rijkswaterstaat, 1982) has been installing such a system in the central and western part of the country, covering some 90 km of road. The system will, at its most complex level, allow ramp-metering, speed control, rerouteing, incident detection, lane reservation and measurement of adverse weather conditions.

The main objective of motorway control and surveillance systems is to improve in some sense the quality of vehicular flow: in particular, better utilization of available road capacity and reduction of traffic unsafety.

These and other goals can be pursued by a large variety of control actions, which can be classified into at least four different categories according to the impacts on the traffic flows:

- control of on- and off-ramp traffic volumes,
- interference with motorway vehicle speeds,
- guidance of flows, i.e. conducting traffic to suitable routes,
- offering effective information, e.g. cautioning against unexpected local traffic conditions upstream with respect to flow, weather, maintenance or road surface.

Usually, loop detectors in special configurations imbedded in the road surface provide the systems with measurements of current local flow conditions. These detectors collect "trigger-time" data that can be transformed into occurrence times and speeds of single vehicles passing the measuring position.

Theoretically, the key problem in the design of motorway control systems can be described as: at each time instant determine the set of local decisions that in view of the motorway network considered is optimal given a continuous stream of noisy, local and disaggregate data. Evidently, optimality refers to the objectives pursued. Two aspects come into prominence, namely estimation and control. Accurate and detailed information about current flow conditions has to be deduced from the available disaggregate data. And secondly, based on this information the optimal set of decisions has to be determined. Solving these problems satisfactorily is hardly

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possible without an adequate mathematical model of motorway traffic flow.

In the paper a stochastic continuous-time aggregate variable model of motorway traffic flow is presented. The model (van Maarseveen, 1979) emphasizes the stochastic elements of traffic flow. It consists of a nonlinear dynamical system with a martingale forcing term. Validation studies (van Maarseveen, 1982) have shown that the model effectively represents both freely moving and congested motorway traffic. Using the concept of stochastic jump processes a related mathematical formulation of detector measurements is given.

The estimation problem forms the main topic of the paper. Based on general results of abstract martingale theory with respect to nonlinear optimal filtering for jump processes (Boel, Varaiya and Wong, 1975) a recursive estimation algorithm is obtained. Because of its recursive nature the algorithm is very appropriate for use in automatic control and surveillance systems. Results of simulation experiments - some are reported in the paper - demonstrate a good performance of the algorithm under various flow conditions.

In formulating a stochastic optimal control problem ramp-metering and speed control are discussed. In particular, attention is paid to the adjustment of local time-varying speed recommendations. The main idea behind this control action is to affect the equilibrium speed-density relationship.

THE TRAFFIC FLOW MODEL

Aggregation

In designing motorway control and surveillance systems aggregate variable models appear to be best suited to describe traffic flow, both in an adequate manner and within an appropriate form. In these models (e.g. Cremer, 1979; Payne, 1971) the motorway is sectioned into lengths of about 0.5 up to 1 km and flow is described in terms of macroscopic, fluid-analog variables named spatial mean speed and density.

The model will be introduced for a simple uncontrolled traffic system. However, it can easily be adapted to more complex systems like an interrelated controlled network of motorways. Consider a portion of a single motorway with at least two lanes and with no on- and off-ramps that is partitioned into a set of n sections. The section boundaries, denoted by the sequence of positions $x_0, x_1, \ldots, x_n$, can be selected at will provided that the number of lanes within each section is a constant. Let L_i be the length of section i (the section between x_{i-1} and x_i), and l_i the number of lanes. Let ξ_j denote a measure of width of the motorway at x_j; it may express a bottleneck situation in cross-section capacity. In general, it is defined by $\xi_j = \min(l_j, l_{j+1})$.

The following variables play a major rôle in the model description:

$\phi_i(t)$, *section density* , is the number of vehicles in section i per lane per unit of length at time t;

$v_i(t)$, *section speed* , is the mean speed of the vehicles in section i at time t.

In addition, let t_0 denote some initial time and define

$\pi_j(t)$, *traffic count* , is the total number of vehicles that passed section boundary x_j in the time interval $(t_0, t]$; $\pi_j(t_0)$ is set equal to zero.

At this stage it should be pointed out that all the above-mentioned traffic variables are considered to be stochastic processes in order to express the random nature of traffic flow. In particular, defining occurrence times of vehicle passages in a suitable way, the random process $\pi_j(t)$ satisfies the conditions for being a counting process: it has sample paths that are right continuous and piecewise constant except for jumps of size +1, almost surely.

Flow Dynamics

The evolution of section density follows from an equation representing the principle of conservation of vehicles. In absence of on- and off-ramp traffic the density in section i satisfies the stochastic differential equation

$$d\phi_i(t) = \frac{1}{l_i L_i} \{d\pi_{i-1}(t) - d\pi_i(t)\} \tag{1}$$

A change in density corresponds to an arrival of a vehicle, $d\pi_{i-1}(t) = 1$, or a departure, $d\pi_i(t) = 1$. Uncertainty in section density, therefore, traces back to the randomness of arrival and departure times.

In elaborating Eq. (1) a representation of counting processes in terms of martingales (Segall and Kailath, 1975) is used. It is based on a general result in abstract martingale theory, the Doob-Meyer decomposition theorem for submartingales. Application of this result to the counting process $\pi_j(t)$ yields that given some information pattern there exists a unique decomposition

$$d\pi_j(t) = \lambda_j(t)dt + dq_j(t) \tag{2}$$

where $\lambda_j(t)$ is a predictable nonnegative random process and $q_j(t)$ a martingale.

Avoiding mathematical details and technicalities (van Maarseveen, 1982) it is emphasized that the uniqueness of the decomposition is essential. The process $\lambda_j(t)$ is called the rate process of the counting process $\pi_j(t)$; it represents the instantaneous rate at which the counting process will jump. Intuitively, the martingale component reflects the completely unpredictable part in $d\pi_j(t)$ with respect to the considered information structure.

Substitution of Eq. (2) into Eq. (1) yields a general stochastic differential equation for the section density $\phi_i(t)$

$$d\phi_i(t) = \frac{1}{l_i L_i} \{\lambda_{i-1}(t) - \lambda_i(t)\}dt + \frac{1}{l_i L_i} d\{q_{i-1}(t) - q_i(t)\} \qquad (3)$$

The martingales $q_{i-1}(t)$ and $q_i(t)$ are completely characterized by uniqueness of the decomposition theorem. It remains to investigate the structure of the rate processes. On the analogy of the theory of a fluid continuum and with the aggregation in mind it is assumed that, given the history of section densities and speeds, the traffic intensity at section boundary x_j satisfies

$$\lambda_j(t) = \qquad (4)$$

$$\xi_j\{a\phi_j(t)+(1-a)\phi_{j+1}(t)\}\{av_j(t)+(1-a)v_{j+1}(t)\}$$

the product of weighted averages of section density and section speed in adjacent sections. The constant weighting factor a is restricted to $0 \leq a \leq 1$.

Considering the evolution of section speed over time two features can be distinguished: continuous behaviour as a result of altering global (or macroscopic) traffic conditions, and (minor) discontinuous changes appearing at a microscopic level due to the definition of section speed and caused by vehicle passages at section boundaries. Using some basic ideas originating in a deterministic model of freeway traffic proposed by Payne (1971) it is assumed that $v_i(t)$ satisfies the stochastic differential equation

$$dv_i(t) = \{ -\frac{1}{T} \{v_i(t) - v^{eq}(\phi_i(t))\} - \frac{\nu}{T(L_i+L_{i+1})} \{ \frac{\phi_{i+1}(t) - \phi_i(t)}{\phi_i(t) + c} \} - \frac{\xi_{i-1}}{l_i L_i} v_{i-1}(t) \{v_i(t) - v_{i-1}(t)\} \}dt + dw_i(t) \qquad (5)$$

In this equation $v^{eq}(\phi)$ stands for a representation of the equilibrium speed-density relationship. The presence of the factor $1/T$ represents the temporal lag in response of a group of vehicles to changing traffic conditions. The term involving the parameter ν represents anticipation of changing density ahead. The parameters ν and T may be considered as a sensitivity coefficient and a reaction time constant, respectively. The positive constant c restricts the effect of anticipation in case of very low densities. Approximately, the term involving ξ_{i-1} compensates for discontinuities in section speed due to the aggregation procedure; it corresponds to a forward shift of moving vehicles with time. Finally, the stochastic process $w_i(t)$ is introduced to stress random effects like, for instance, acceleration noise and to incorporate inaccuracies in the model. It is assumed that $w_i(t)$ can be represented as a Brownian motion.

In principle, any speed-density relationship can be substituted into Eq. (5). In a comparative study Gerlough and Huber (1975) concluded that in the absence of a strong reason to the contrary the simplest appropriate relationship should be considered. In the sequel, therefore, the linear relationship

$$v^{eq}(\phi) = \alpha (1 - \phi / \gamma) \qquad (6)$$

will be used. The parameters α and γ represent the free traffic speed and the jam concentration, respectively.

Observations

Focussing the basic ideas in modelling the observations only measurements of vehicle passage times are considered in the paper. These passage times are transformed into vehicle counts yielding an observation model that consists of counting process formulations. Measurements of other quantities, like velocities of passing vehicles, can be treated in a similar, yet more complicated way using a more general jump process formulation (van Maarseveen, 1982).

In the considered traffic system detectors are placed at a number of (but not necessarily all) section boundaries. For these locations define

$n_j(t)$ is the *observed number of vehicle passages* at section boundary x_j in the time interval $(t_0,t]$; $n_j(t_0)$ is set equal to zero.

Obviously, the random process $n_j(t)$ constitutes a counting process. In absence of measurement errors the process would satisfy the relation $n_j(t) = \pi_j(t)$. However, inductive loop detectors have to contend with measurement errors. Sources of error are deviation in loop-geometry, dissimilar behaviour of passing vehicles, imperfection of detector, and environmental noise. Dealing with non-ideal devices three situations can be distinguished:

- a registered event corresponds to a real event (vehicle passage);
- registration of a non-existing event, false measurement (error of type 1);
- a real event is not registered, a passing vehicle is not measured (error of type 2).

Time measurements of vehicle passages may be wrong as well. However, these errors, which are very small in general, are not relevant to the application in mind, and therefore will be neglected.

The following compound error processes are defined:

$e_j^k(t)$ is the *number of counting errors of type k*, $k = 1, 2$, at measuring position x_j in the time interval $(t_0, t]$; $e_j^k(t_0)$ is set equal to zero.

By definition these processes are counting processes. Taking into account measurement errors the observation process $n_j(t)$ satisfies

$$n_j(t) = \pi_j(t) + e_j^1(t) - e_j^2(t) \qquad (7)$$

Given the history of section densities and speeds the martingale decomposition of the counting process $e_j^k(t)$ is assumed to have the form

$$de_j^k(t) = \varepsilon_j^k \lambda_j(t)\, dt + dr_j^k(t) \qquad (8)$$

where ε_j^k is some nonnegative constant satisfying $\varepsilon_j^k << 1$, the process $\lambda_j(t)$ is given in Eq. (4) and $r_j^k(t)$ is a martingale. Again it is noted that by uniqueness the martingale $r_j^k(t)$ is completely characterized by the decomposition in Eq. (8). The constant ε_j^k represents the counting error fraction of type k at measuring position x_j.

Taking the differentials in Eq. (7) and substituting Eqs. (2) and (8) yields the martingale decomposition of the observation process $n_j(t)$

$$dn_j(t) = (1 + \varepsilon_j^1 - \varepsilon_j^2)\, \lambda_j(t)\, dt + d\,(q_j(t) + r_j^1(t) - r_j^2(t)) \qquad (9)$$

Model Parameters

The parameters in the traffic flow model were identified using a maximum likelihood estimation procedure for counting observations (van Maarseveen, 1982). The data were collected from a stretch of the northern carriageway of the motorway Utrecht - The Hague. In the identification study five model parameters were considered: α, γ, T, ν and c. The weighting factor a was set equal to 0.5.

A comparison of the main results (table 1) with those of other studies (Cremer and Papageorgiou, 1981; Grewal and Payne, 1976) reveals that the values for α, γ and c are rather consistent. The values for ν and T show considerable differences, probably due to differences in traffic flow and in equilibrium speed-density relationship used, and to the strong correlation between these parameters.

TABLE 1 Parameter identification results

parameter	value
α	106 km/h
γ	116 veh/km/lane
T	16 s
ν	138 km^2/h
c	10 veh/km/lane

The overall performance of the traffic flow model and the transferability of the obtained parameter values with time were investigated using different data sets. The results of this validation study are very promising.

ESTIMATION

The Problem

The traffic flow model can be written in the general and more compact form

$$dX(t) = f(X(t))\, dt + dM(t) \quad , t \geqq t_0 \qquad (10)$$

where the 2n-dimensional vector processes X(t) and M(t) are defined by

$$X_{2i}(t) = \phi_i(t)$$
$$X_{2i-1}(t) = v_i(t) \qquad (11)$$

for $i = 1, 2, \ldots, n$, and

$$M_{2i}(t) = \frac{1}{l_i L_i}\{q_{i-1}(t) - q_i(t)\}$$
$$M_{2i-1}(t) = w_i(t) \qquad (12)$$

for $i = 1, 2, \ldots, n$. The random vector $X(t)$ represents the *unobserved* state of the traffic system at time t. The vector function f in Eq. (10) is nonlinear in the state variables.

Let m be the number of measuring positions with $1 \leq m \leq n+1$, and let $N(t)$ denote the m-dimensional vector process of counting processes $n_j(t)$. The *observed* vector counting process $N(t)$ satisfies a stochastic differential equation

$$dN(t) = g(\ X(t)\ )\ dt + dS(t) \quad , t \geq t_0 \qquad (13)$$

where $S(t)$ denotes the associated m-dimensional martingale vector. It follows from Eqs. (4) and (9) that the vector function g is nonlinear in the state variables.

The estimation problem discussed in the paper deals with estimating at the current time t, $t \geq t_0$, the value of $X(t)$ based on the observation of the counting process $N(t)$ up to time t. That is, reconstructing current local traffic conditions in terms of the aggregate variables, section density and section speed, given the noisy and incomplete detector measurements obtained in the past.

A Recursive Estimation Algorithm

Van Maarseveen (1982) extensively studied a recursive formula for the minimum variance estimate for $X(t)$ given the observations $\{N(s),\ t_0 \leq s \leq t\}$. The formula is not an explicit solution to the filtering problem, but rather a representation for the optimal estimate. In general, the exact solution to the estimation problem involves an infinite set of equations for the conditional moments of the estimation error.

The utility of the filtering formula, however, lies in its possible use as a guide to the construction of implementable approximate filters. Using a truncation of the equations as proposed by Bagchi and van Maarseveen (1980) an approximate filter for nonlinear dynamical systems was derived. The filter may be considered as the analogue of the extended Kalman filter for stochastic dynamical systems in additive Gaussian white noise.

Let $F(x)$ and $G(x)$ denote the matrices of partial derivatives of the functions $f(x)$ and $g(x)$, respectively. Furthermore, let $\hat{X}(t)$ denote the estimate for the state vector $X(t)$. Then, for $t \geq t_0$, the approximate optimal filter is given by

$$d\hat{X}(t)=f(\hat{X}(t))dt+K(t)\{dN(t)-g(\hat{X}(t))dt\} \qquad (14)$$

where the matrix process $K(t)$ satisfies

$$K(t)=(\hat{P}_X(t)G(\hat{X}(t))^T+R_{MS}(t))(R_{SS}(t))^{-1} \qquad (15)$$

taking the left hand limit at time t. The approximate conditional estimation error covariance matrix $\hat{P}_X(t)$ satisfies

$$d\hat{P}_X(t) =\{\ \hat{P}_X(t)F(\hat{X}(t))^T + F(\hat{X}(t))\hat{P}_X(t) + R_{MM}(t) - K(t)R_{SS}(t)K(t)^T\ \}dt \qquad (16)$$

for all $t \geq t_0$. In these equations the superscript T stands for transpose. The processes $R_{..}(t)$ represent time derivatives of conditional predictable quadratic (co)variation processes. Subscripts refer to the processes in question, the martingales $M(t)$ and $S(t)$. Using the relations between state and observation martingales expressions for the processes $R_{..}(t)$ can be derived (van Maarseveen, 1979, 1982).

The filter is formulated in terms of stochastic differential equations for $\hat{X}(t)$ and $\hat{P}_X(t)$. In a straightforward manner an implementable recursive estimation algorithm is obtained by using the standard integration method of Euler.

Results

The main incentive for carrying out a simulation study - in addition to real data experiments - was the need for a precise evaluation of the performance of the proposed approximate filter in estimating the aggregate traffic variables.

In the simulation experiment a part of a (fictitious) 3-lane motorway with a length of 4.5 km was considered. The traffic system contained a bottleneck: a stretch with two lanes only (e.g. bridge or tunnel). The bottleneck was introduced to create a wide range of density and speed values in the simulated data. The system was equipped with four measuring positions at a distance of 1.5 km each.

The motorway was partitioned into 7 sections: boundaries coincided with measuring positions and with beginning and end of the bottleneck (Fig. 1). Lengths of sections that were situated upstream but close to the bottleneck were chosen somewhat smaller to enable a detailed evaluation with respect to the anticipated locally varying behaviour of traffic flow.

Traffic flow was simulated using the aggregate variable model. The simulation period was half an hour. Average input flow levels in successive 10 minute periodes were 2250, 3000, 1500 veh/h/lane respectively. Counting measurements were deduced from simulated vehicle passages at measuring positions. Based on these measurements densities and speeds were estimated using the recursive estimation algorithm in Eqs. (14)-(16). The estimates were compared with the actual (simulated) values.

In van Maarseveen (1982) detailed results of the experiment can be found. Some representative results (for the density and speed in section 3) are given in Fig. 2 and 3. It is found that the proposed estimation procedure performs very well. In spite of the limited number of measuring positions the filtered estimates for section density and section speed follow closely the evolution of the aggregate traffic variables.

Moreover, the estimated standard deviations of the estimation errors, i.e. the square root values of the diagonal elements of the conditional error covariance matrix, yield correct indicators of the real estimation errors. It also appears that these quantities indicate local instabilities in traffic flow. Further research is needed to investigate their potential use in motorway control systems.

CONTROL

Traffic flow control for motorways can be characterized as open-loop (in general time-of-day dependent) or closed-loop (traffic responsive). Because of the highly random nature of traffic flow a traffic responsive control scheme is most preferable. In this context, it is essential to have sufficient information about current local traffic conditions (cf. the estimation problem previously discussed). From the numerous control measures that affect traffic flow on motorways two controls are considered in the paper: ramp-metering and speed control.

Ramp-metering

The most obvious way to control the flow is to regulate the input traffic volume (i.e. on-ramp volumes) by means of stop-and-go signals. Most contributions dealing with flow control on motorways are devoted exclusively to ramp-metering.

Consider a traffic system with on- and off-ramps and define the counting processes $\rho_i^{on}(t)$ and $\rho_i^{off}(t)$ describing the total number of vehicles that have entered (left respectively) the main flow within the range of section i after time t_0. Then, Eq. (1) can easily be adapted to

$$d\phi_i(t) = \frac{1}{l_i L_i} \{d\pi_{i-1}(t) - d\pi_i(t) + d\rho_i^{on}(t) - d\rho_i^{off}(t)\} \qquad (17)$$

This stochastic differential equation can be elaborated employing the martingale decompositions of the 'ramp' counting processes.

Ramp-metering primarily affects the evolution of section densities as can be seen from Eq. (17). Controlling on-ramp traffic volumes can be conceived of as controlling the rate process associated with the counting process $\rho_i^{on}(t)$.

Speed Control

Another control measure is the adjustment of local time-varying speed recommendations (or limitations). The measure intends to help prevent accidents, to promote smooth flow and thereby to increase the throughput of the transportation system. Traffic matrix signals have been designed that allow for the visualization of a range of advisory (or imposed) speeds (e.g. Rijkswaterstaat, 1982).

As opposed to ramp-metering it is to be expected that advisory speed signal settings primarily affect the evolution of section speeds. The character and extent of these effects depend strongly on the framework of the instructions, prevalent neighbouring traffic conditions, experiences of drivers with the control system in the past and, of great importance, the reactions thereupon. It is reasonable to assume that in general automobilists are inclined to adapt their velocity to advisory speed values as long as the recommendations appear to be rational.

Let $u_k(t)$ denote the speed control (i.e. advisory speed value) ruling section i at time t. The evolution of section speed $v_i(t)$ can be adapted to the control by adding the term

$$- \frac{1}{\tau} \{ v_i(t) - u_k(t) \} dt$$

to the right-hand side of Eq. (5). This extra term expresses the tendency of drivers to adapt their speed to the instruction $u_k(t)$. The presence of the factor $1/\tau$ represents the temporal delay in response.

It is interesting to study the effects of the above extension of the speed equation upon the stationary speed-density relationship. The shape of the fundamental diagram of controlled traffic flow (Fig. 4 and 5), obtained for $\tau = T$ and with Eq. (6), fits very well with the empirical results obtained by Zackor (1972). It also agrees with the equilibrium relationship proposed by Cremer (1979). For high density values it also demonstrates the paradox of a higher average velocity under a speed limitation.

Posing the Problem

The general form of the adapted traffic flow model becomes

$$dX(t) = h(\ X(t), U(t)\ )\ dt + dM(t) \qquad (18)$$

(cf. Eq. (10)), where U(t) denotes a vector of control functions including ramp-metering and speed control. The optimal control will depend on the objectives of the motorway control system and on the constraints imposed on the control functions. Van Maarseveen (1979, 1982) has shown that a large variety of objective functions can be expressed in the usual form of a quadratic performance criterion. Theoretical results in abstract martingale theory may be used to solve the indicated stochastic optimal control problems.

CONCLUSION

Traffic counts and, more generally, point characteristics play an important rôle in many transportation systems. These characteristics can be modelled with jump process formulations. In this way, theoretical results in abstract martingale theory with respect to jump processes can be applied fruitfully in modelling, estimation and control.

This was illustrated for motorway control and surveillance systems. Using the concept of counting process a traffic flow model was presented. Moreover, a recursive estimation algorithm was derived for estimating local traffic conditions from loop detector measurements. Further research may be directed towards the construction of an implementable estimation algorithm for jump process observations (velocity measurements) and towards the indicated optimal control problem.

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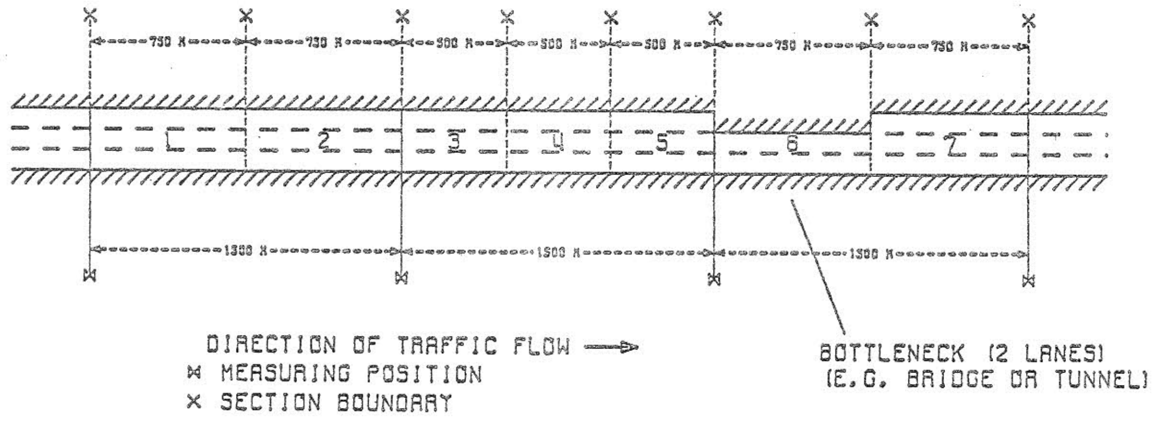

Fig. 1 Design of Simulated Traffic System.

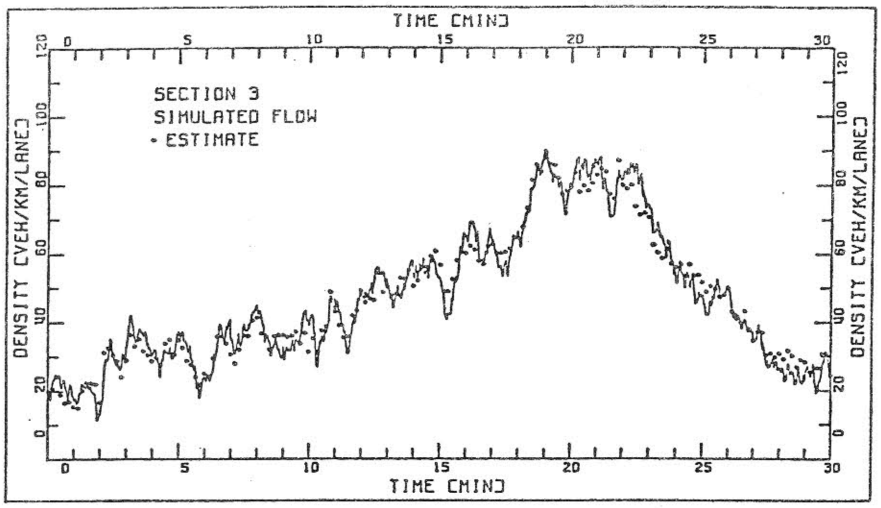

Fig. 2 Filtered Density Estimate versus Simulated Value.

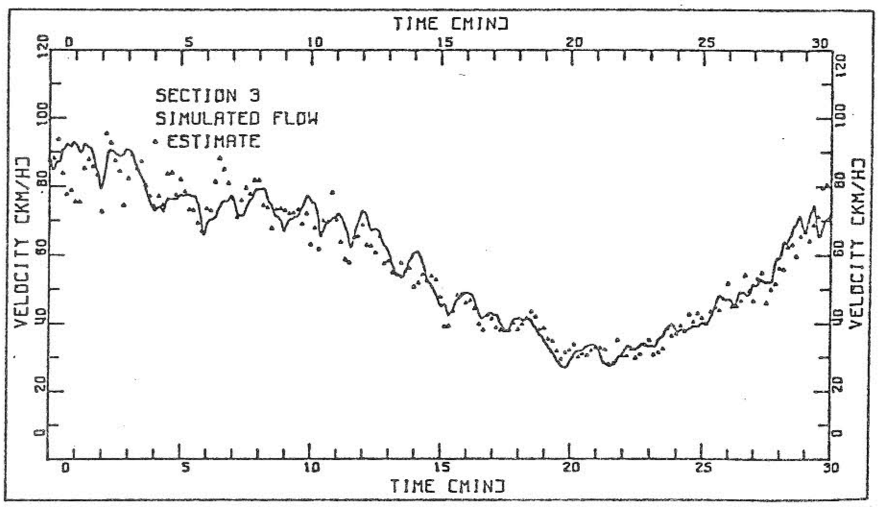

Fig. 3 Filtered Speed Estimate versus Simulated Value.

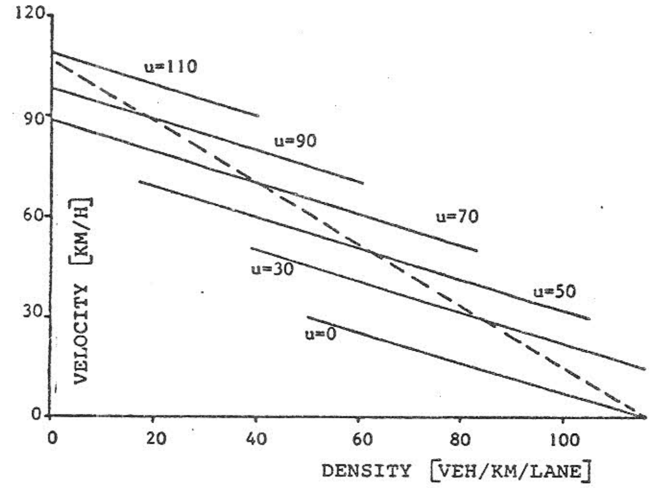

Fig. 4 Speed-Density Relationship for Controlled Flow.

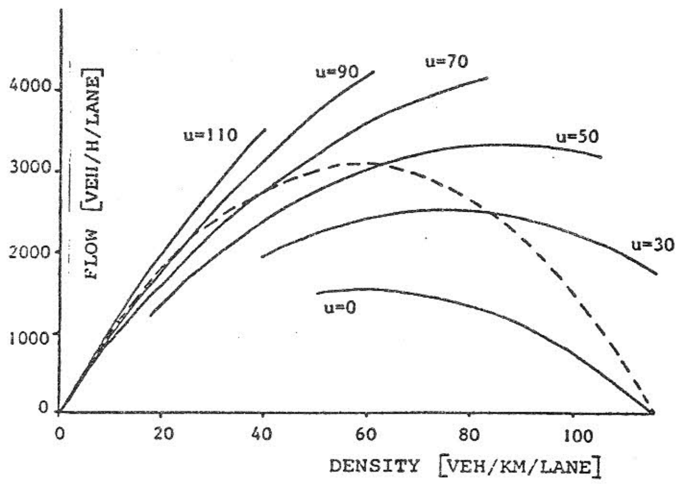

Fig. 5 Fundamental Traffic Diagram for Controlled Flow.