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The elementary theory of geometric constructions can be formalized in a natural deduction style system of introduction and elimination rules. We present a system within intuitionistic type theory in which geometric inference and construction are treated in a unified way. The composition of the basic constructions formally amounts to functional composition which gives a functional hierarchy. Solutions to construction problems are explicitly represented as effectively calculable functions that take the constructions assumed given in a problem into the construction sought for. This constructive character of the system stands in contrast to previous axiomatic treatments where existence is not based on construction. Our basic types are point, line, circle and ray. The definition of geometric concepts proceeds in a straightforward manner via the notions of product and sum of a family of types. The treatment of congruence is based on the notion of equality of objects of a given type, such equality being part of the definition of a type. The Euclidean system of construction postulates is notoriously insufficient for even the simplest construction problems. This is seen right from the first Euclidean construction problem in which an intersection point of two circles is needed. In our approach, conditions of intersection are set down pairwise for the basic types line, circle, and ray.

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Many-sorted equational logic became a central topic in algebraic semantics, giving the basis of the so-called initiality approach in the algebraic specification of abstract data types. In algebraic specification, however, several phenomena indicate that this logic encounters limitations in practice. We mention a few, the most interesting of these phenomena: partiality, type polymorphism, and dependent types.

To overcome these limitations *Equational type logic* (ET) has been introduced as an extension of (conditional) equational logic where elements and types are merged in a single framework (see [1]–[3]). This immediately introduces a great amount of flexibility and generality: several types may be assigned to an element, which may be also a type of another element, and operations may take type arguments or yield types.

Usual abstract data type specifications consist of two parts: a signature, where type constraints are specified, and axioms, where equality constraints are specified. An ET specification, in a more general way, merges in a single framework the type and equational constraints. ET formulas are conditional formulas where equations or type assignments can occur indifferently either in the premise or in the conclusion. Therefore, the type of a term is not a priori fixed, but is deducible from the axioms of the given ET specification. Models of ET are *type algebras*, viz. total one-sorted algebras equipped with a binary typing relation. Then, the partiality of an operation can be expressed by letting the undefined applications of the operation yield underdefined elements: such is considered any element of the carrier if no type is assigned to it nor is it itself a type of some element of the carrier. ET has a sound and complete calculus, and every ET theory admits an initial model.

The expressiveness of ET logic is assessed in comparison with other logics. We have translated order-sorted logic (classical many-sorted equational logic being a special case), logics of partial algebras, and Horn logic into ET logic. By means of these representations, complete calculi for those other logics become available. Finally, we are investigating how the notion of type algebra itself can be generalized in a fundamental perspective: we aim at constructive mathematical reasoning by type assignment; this program stimulates new reflections on the notion of mathematical structure.

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