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Sampling-free Linear Iterative Bayesian Updating of Non-linear Model States

W. van Dijk, W.B.J. Hakvoort, B. Rosic
Department of Mechanics of Solids, Surfaces, and Systems (MS3), University of Twente
P.O. Box 217, 7500 AE Enschede, The Netherlands
{w.vandijk/w.b.j.hakvoort/b.rosic}@utwente.nl

1 Introduction

For highly non-linear dynamics and/or non-Gaussian distributed states, standard filters for state estimation lose their optimality or in extreme cases, fail altogether. In this work, a sampling-free filter is proposed that iteratively linearises the non-linear observation function without explicitly deriving the Jacobian.

2 Iterative Bayesian updating

Assume a continuous non-linear dynamical system,

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, t), \tag{1}$$

which is observed at discrete time moments k with an observation function $h(\cdot)$,

$$\boldsymbol{y}^k = h(\boldsymbol{x}^k) + w^k. \tag{2}$$

A general version of the Kalman filter is based on the conditional expectation of the random variable $\boldsymbol{x}^k$ given a measurement $\boldsymbol{y}^k$ [1], and can be written as:

$$\boldsymbol{x}_a^k(\omega) = \boldsymbol{x}_f^k(\omega) + \phi(\hat{\boldsymbol{y}}^k) - \phi(\boldsymbol{y}(\boldsymbol{x}_f^k(\omega))), \tag{3}$$

in which the map ϕ approximates the previously mentioned conditional expectation, $\phi(\boldsymbol{y}) \approx \mathbb{E}(\boldsymbol{x}|\boldsymbol{y})$, and $\boldsymbol{x}_f^k(\omega)$ represents our prior knowledge (i.e. uncertainty) on the state estimate.

Further assumptions on the map ϕ, such as linearity, as well as possible approximations of the highly nonlinear functions $f(\cdot)$ and $h(\cdot)$ in equations (1) and (2) will result in different types of filters. Two special cases of this are the classical and the ensemble Kalman filters [1], although many more exist.

In this work, a linear approximation of the observation function is made using an ensemble, i.e. $\hat{H} \approx h(\cdot)$. Subsequently, this approximation is updated by iteratively adjusting its linearisation point such that an unbiased filter is obtained.

Furthermore, the sampling based approximation of $h(\cdot)$ can be replaced using polynomial chaos expansions [3]. This results in a sampling-free filter, suitable for non-linear dynamics and non-Gaussian distributed states.

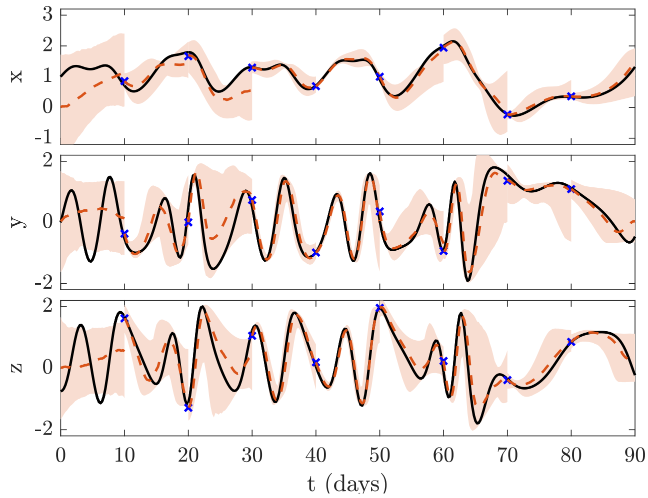

Figure 1: State estimation of the Lorenz 84 problem. (▬) Truth; (- -) Estimated mean; ( ) 90% confidence interval; (×) Linearisation point.

3 Results

The sample based iterative filter is applied on the Lorenz 84 [2] problem with nonlinear observations (Figure 1). As can be seen, the filter is able to track the the states of the system. Similar results are obtained for the sampling-free iterative filter, however, this filter requires more frequent measurements for accurate approximation of the observation function.

References

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[3] O. Pajonk, B. Rosić, H.G. Matthies, 2013, "Sampling-free linear Bayesian updating of model state and parameters using a square root approach", Computers & Geosciences, 55, (70-83).