Modelling rainfall-runoff relation by representative elementary watershed approach

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Introduction

The representative elementary watershed (REW) approach is a newly developed watershed modelling approach describing the relation between rainfall and runoff. In this approach a watershed is discretized into a number of sub-watersheds, called REWs, serving as computational units. The boundary of each REW coincides with the topographic divide delineating an area of the land surface that captures precipitation and delivers it to the river channel. Reggiani *et al.* (1998, 1999, 2000) describe watershed-scale balance equations for mass, momentum, energy and entropy that serve as a base for the mathematical model underlying our computer code. The governing equations constitute a system of coupled ordinary differential equations, leading to a drastic simplification of the mathematical model. Model equations are obtained by averaging the corresponding point-scale equations over the scale of a REW and as such, in the approach the model variables and parameters are spatially lumped over an REW. Therefore, compared to fully distributed, complex physically based modelling approaches (Rientjes, 1999), our approach has the advantage that much fewer parameters and thus less input data are required while yet, at the core of the mathematical model, the real world physics underlying watershed runoff dynamics is maintained.

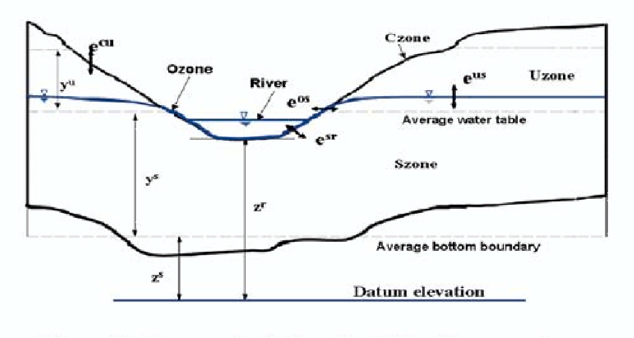

Figure 1. Cross-sectional vieuw of a REW with mass exchange terms and characteristic state variables added.

Model structure

REWs form the model building blocks and represent the spatial regions where water flows take place. REWs are organized around the tree-like structure of the channel network and are linked laterally through a subsurface flow model to allow for regional groundwater flow. Based on the characteristics of the runoff mechanisms as observed in the real world, a REW is sub-divided into 5 zones (Figure 1, Figure 2):

1 Unsaturated zone (Uzone);
2 Saturated zone (Szone);
3 Saturated overland flow zone (Ozone);
4 Concentrated overland flow zone (Czone); and
5 River channel.

Continuity equations of mass and momentum have been developed and govern the water movement and mass exchange across the zones and REWs. Mass balance equations for each zone are the core of the mathematical model and are listed here.

1 Unsaturated zone mass conservation:

$$\rho \frac{d}{dt}(\theta^u y^u \omega^u) = e^{us} + e^{uc} + e^u_{wg}$$

2 Saturated zone mass conservation:

$$\rho\varepsilon \frac{d}{dt}(y^s \omega^s) = e^{su} + e^{so} + e^{sr} + e^{s,bot} + e^{sA}$$

3 Saturated overland flow zone mass conservation:

$$\rho \frac{d}{dt}(y^o \omega^o) = e^{o,top} + e^{os} + e^{or}$$

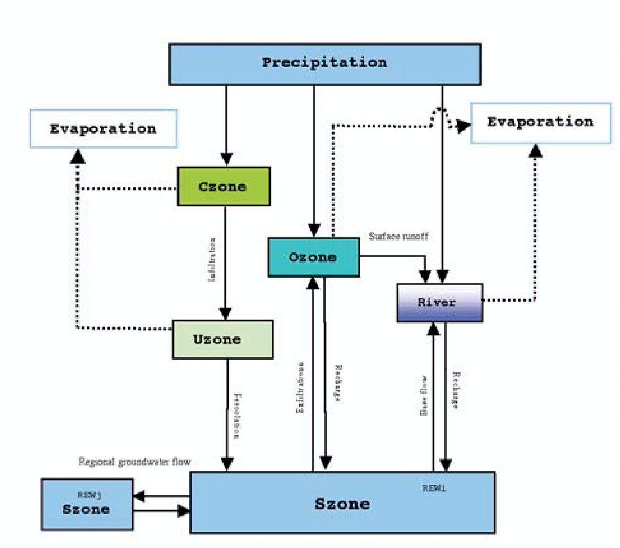

Figure 2. Flow chart of the REW model.

4 Concentrated overland flow zone mass conservation:

$$\rho \frac{d}{dt}(y^{c}\omega^{c}) = e^{c,top} + e^{cu}$$

5 River flow zone mass conservation:

$$\rho l \frac{dm}{dt} = e^{rA} + e^{r,top} + e^{rs} + e^{rs} + e^{ro}$$

where: ρ is water density, θ soil moisture content, y average depth of the zones (water depth for oveland flow zones), ω projected plannar area fraction covered by zones, *m* and *l* cross-section area and lenth of the river segement respectively. In the equations, superscripts are zone indices with exceptions that bot and A are for REW bottom and mantle boundaries respectively, *top* for either rainfall or evaporation. In the right hand side of the equations, *e* stands for flux exchange terms among zones. The momentum equations as well as model assumptions are described in Reggiani *et al.* (1998, 1999, 2000, 2001) and will not be repeated here.

Combined with momentum balance equations and constitutive geometric relationships, the final set of coupled non-linear odinary equations have been obtained through parameterization. In the numerical model, the computed hydrological variables are, e.g., a) soil moisture content, b) groundwater table, c) river discharge and d) overland flow zone area fractions. The dominant model parameters that need intensive calibration are saturated hydraulic conductivity, soil porosity and Manning roughness coefficient.

Application and results

We applied the REW approach to the Geer basin, located in Belgium to simulate the rainfall-runoff relation. The Geer basin, covering an area of 494 km2, is characterized as a "deep" groundwater system. The unsaturated zone is underlain by an aquifer that extends beyond the boundary of the catchment. Prior to a model simulation, TARDEM software (Tarboton, 1997) was applied to subdivide the watershed into a number of REWs (Figure 3) based on a specified Strahler order threshold value. By doing so the REW discretization and the extraction of geometric data were carried out. The numerical model was tested using a series of constant effective rainfall inputs and different parameter sets. The model testing results are presented in Figure 4 and show catchment outlet hydrographs that indicate the model is sensitive to different inputs. In order to analyse whether the physics of the catchment hydrological runoff behaviour is represented in the model calculations, also the behaviour of hydrological state variables such as soil moisture content, groundwater table depth, the size of saturated-excess flow area fractions are calculated throughout the simulation. A second test dealt with the application of measured daily rainfall and evaporation data to the model. We present here the hydrograph at the outlet of the basin (Figure 5) for the period 17-05-1994 till 25-08-1994. In the graph it can be observed that the base flow discharge is simulated appropriately

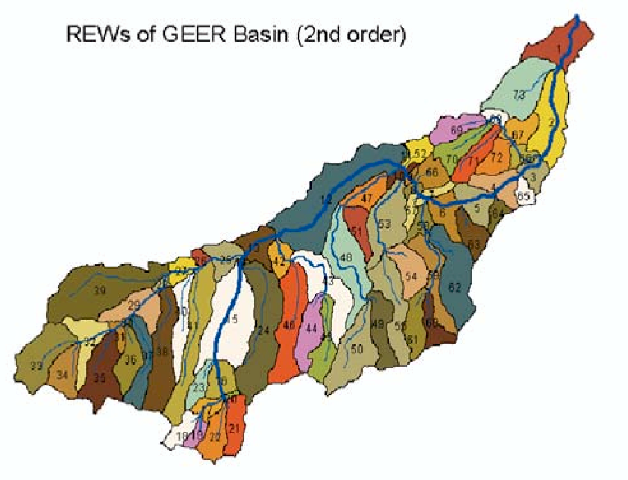

Figure 3. Identification of REWs for Geer basin.

while some peak flow discharges are over-estimated and others under-estimated. The effect of the time-varying rainfall inputs, however, is clearly represented. The results indicate that the rainfall-runoff relation can be simulated accurately by the REW approach. In order to be more conclusive on the performance of our model approach, the approach must be applied to catchments in different physiographic and climatic settings.

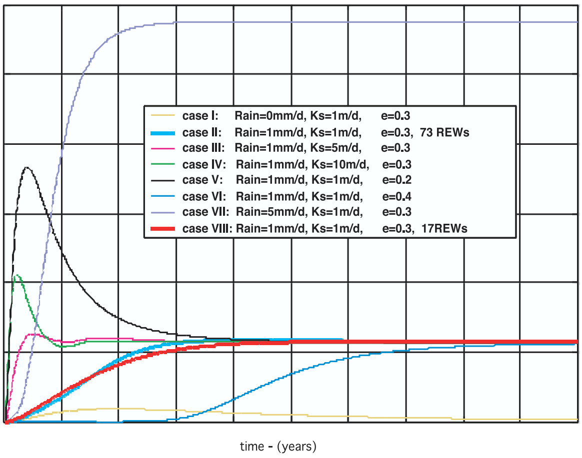

Figure 4. Hydrographs at the outlet of Geer basin with various rainfall rate and parameter sets.

Conclusions

REW approach is a semi-distributed and physically based approach. It forms an appropriate base for watershed hydrological modelling tool. REW approach also gives a prospective for runoff forecasting and prediction and possibly for analysis of effects of land use and climate changes.

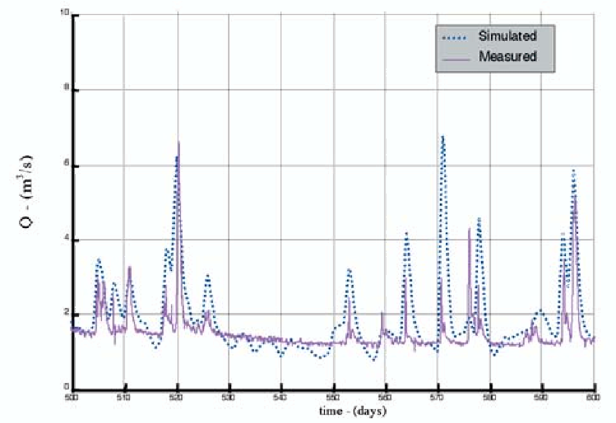

Figure 5. Discharge at the outlet of Geer river.

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