Dynamic access control
for two-direction shared traffic lanes

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Abstract

In specific traffic situations, a single lane is available for traffic from two directions. Examples are traffic accidents or road maintenance reducing the number of available lanes on a road or, as we faced in a project on underground freight transportation, construction of a single lane for two directions to reduce infrastructure investment. Access control rules are required to manage vehicle flows such that collision is avoided and waiting times are minimised. In contrast to standard traffic control at crossroads, these control rules should take into account significant driving times along the single lane (in our application up to 8 minutes). In this paper, we discuss several heuristic control rules for access control, both simple periodic rules and intelligent adaptive rules (look ahead heuristics, dynamic programming solutions). Numerical experiments show that intelligent control rules reduce waiting times by 10-25% compared to a straightforward periodic rule. The best adaptive control rule is a dynamic programming rule.

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1. Introduction
Traffic congestion is an increasing problem in the Netherlands, as in many countries, causing delay to both private and commercial road transport. The classic solution to the congestion problem, increasing the transportation capacity by an extension of the road infrastructure, is hard to accomplish in densely populated areas. Reasons can be found in a lack of space, high costs and environmental objections. As a consequence, the Dutch government and industry are forced to look for alternative ways to deal with the problem.

One of the possible answers to the congestion problem is to build underground transportation systems as an alternative transport mode. An example that is currently under consideration is the automated underground freight transportation system near Schiphol Airport (van der Heijden et al., 2000). This system focuses on transportation of time-critical goods like flowers and newspapers in unit loads (e.g. pallets) using Automatic Guided Vehicles (AGVs). Typically, this type of system consists of a number of terminals connected by tubes, which lengths may equal several kilometres. The realisation of the underground system requires high investments in both infrastructure and control systems. Dominant cost factors are the total length of the tubes and their diameter. To make the project feasible from an economical perspective, alternative configurations are studied, where terminals are connected by a single tube with a small diameter, allowing for traffic flows from one side at a time. However, such solutions may have a severe impact on logistics performance as AGVs are forced to wait at the tube entrances. Consequently, there is a need for intelligent control of the driving direction in order to guarantee acceptable order throughput times.

In the sequel, we will address a tube allowing for alternating traffic as a Bi-Directional Tube (BDT). We describe the BDT as a part of an underground automated transportation network, although the same setting applies to systems like bridges, tunnels or roadblocks caused by traffic accidents or road maintenance. As a starting point for our discussion we use Figure 1.

![Bi-directional tube](image_url)
The central part of Figure 1 is the BDT itself, where a single track is available for traffic in both directions. The AGVs arrive and depart on tracks, which only allow for one-way traffic. According to a control rule, AGVs arriving at the tube either may enter the tube immediately or queue until a signal is given that entrance is allowed. Each control rule generates the following system states in cyclic order:

1. AGVs from the left have to wait, but AGVs from the right are not yet able to enter the track, because it still contains AGVs driving from left to right; we shall use the term clearing the tube for this system state in the sequel;
2. AGVs from the right may enter and AGVs from the left have to wait;
3. AGVs from the right are stopped and AGVs from the left wait until the tube is cleared.
4. AGVs from the left may enter and AGVs from the right have to wait;

The time spent in the states 2 and 4 is called the green time (from the right and the left, respectively) and the time spent in states 1 and 3 is called the clearance time. The cumulative time spent in the four system states makes up the cycle time. The time spent in each system state is variable and depends on the driving time in the bi-directional tube, the arrival distributions at both sides and the control rule.

Our study of the freight transportation system around Schiphol raised the question which rule should be used for access control to the bi-directional tube in order to minimise delay. Although central scheduling of all activities, including the BDT, is theoretically possible for the closed Schiphol system, we chose to adopt a general framework for logistics and transport agents and their control for reasons of flexibility, robustness and extendibility (cf. van der Zee, 1997). This requires a control rule for the isolated subsystem, possibly taking into account information on the environment such as expected AGV arrival times.

A straightforward control rule is First Come First Served (FCFS), but this rule is only applicable in case of a very low traffic intensity and relatively small driving times along the two-way section (for example, cross-roads). This is not the most interesting situation and it is also not applicable to the Schiphol system that we have in mind. Another obvious option is switching the driving direction periodically. That is, the system spends a fixed time $P$ in both the system states 1&2 and in the states 3&4. So, the green time depends on the preceding clearance time. An alternative simple control rule could be to fix the green time instead of the cycle time. An obvious drawback of these simple rules is that they do not use information on AGV arrivals and queues at both tube
entrances. Hence, it is possible that AGVs have to wait at one entrance, while the tube is free and no AGVs are present or approaching the other entrance. Especially when the traffic intensities at both sides of the tube are different and/or fluctuate in time, a simple periodic control rule may lead to excessive waiting times. Therefore, we decided to develop several local control rules with increasing complexity and information usage. We focus on control rules that are suitable for on-line control, i.e., they have to be efficient in terms of computation time. The potential of the new control rules is demonstrated by an extensive simulation study based on independent Poisson arrivals of AGVs as is plausible in an open system. Also, we will discuss the implications of embedding the BDT in a closed transportation network. More specific, we will consider the effects of the convoys created at a BDT on terminal operation and other BDTs in the system.

The paper is structured as follows. In the next section a literature overview is given where we relate the studied systems to similar traffic and production systems. In Section 3, our model is defined in more detail. Section 4 discusses the control rules for a BDT. The design of the simulation study is described in Section 5. Section 6 presents the results of the simulation study, while Section 7 discusses the implications of a BDT in a closed system. Finally, Section 8 summarises the main conclusions and gives directions for future research.

2. Literature review

The problem of controlling a BDT appears to be at the crossing of a number of research fields. While the link to traffic literature is intrinsically clear, also other research is relevant. One may formulate the problem as a machine scheduling problem in which a planner has to decide when the next batch of products (= a convoy of AGVs) should be processed. Below we will relate the BDT control problem to these research fields.

Traffic literature

Linking BDT control with traffic literature seems natural as the system is intended to be an integral part of (future) traffic systems. Haight (1963) generalises the problem of controlling BDTs and similar systems as the control of a road section which allows for one-way traffic only. Given the arrival distributions and the allowed driving directions (either North-South or the other way around), formulas are derived which specify queuing behaviour. The BDT-control is also related to junction control supported by traffic signs. In the early days traffic signals were mostly scheduled according to a pre-determined scheme (cf. Bell, 1992). Such a scheme allowed traffic to be handled following a fixed sequence. A periodic control rule is a good example of such a
scheme. Mung, Poon and Lam (1996) elaborate on Haight's model and derive distributions of queue lengths at fixed time traffic signals. Heidemann (1994) derives analytical results on statistical distributions of queue lengths and delays at traffic signals, given Poisson arrivals and fixed-time control. The results are compared with several approximations (Webster, 1958; Miller, 1968). Nowadays, traffic signals are mostly controlled using adaptive rules, which take the actual traffic situation into account. Most traffic control at intersections is vehicle dependent: The arrival of one vehicle can switch the traffic light to green. Increasingly, these control rules become traffic dependent. A common objective is to minimise the average loss times, i.e. minimising average waiting times. For example, Robertson and Bretherton (1974) describe an optimal control policy of an intersection for any known sequence of vehicle arrivals. They compare the optimum policy with a fixed time control policy and responsive control policies, a "saturation flow" and "no-flow" policy. The responsive control policies cannot be used for a BDT due to the long clearance time of the BDT.

An essential difference between an automated transportation system and traffic systems, as discussed here, is that behaviour of AGVs can be directly influenced. Human drivers feel a need for a "fair" handling of traffic and therefore restrictions are set to maximum cycle times, which is roughly the maximum waiting time for a car. Above that, quite precise information is available on expected arrival times of AGVs at the tube entrance, also because the driving behaviour of AGVs is far more predictable than that of passenger cars. An important difference with traffic systems is the significant driving time in the BDT, several minutes. The clearance time of a traffic intersection is very short (a few seconds) and therefore it is normally considered fixed. The clearance time of the BDT is not fixed but depends on the last AGV that entered the BDT.

**Machine scheduling**

In machine scheduling literature, problems quite similar to the BDT problem are studied. Several batching strategies are developed within the context of deterministic scheduling, for overviews see e.g. Uzsoy et al. (1992, 1994), Webster and Baker (1995), and Potts (2000). An alternative approach is taken by an other group of authors (Glassey and Weng, 1991; Fowler et al., 1992; Weng and Leachman, 1993; Van der Zee et al., 1997), who study the batching problem in a dynamic context, where jobs have to be scheduled real time given the availability of information on (some) near future arrivals. Many of the studies mentioned focus at the control of oven systems found in semiconductor manufacturing and aircraft industry. Similarities between the BDT system and suchlike systems can be found in the fact that goods (AGVs) have to be
"batched", the processing time of a batch (= the driving time through the tube) and the dynamic nature of the problem, relating to the amount of information available. An essential difference can be found in the fact that for the BDT-system there is no fixed processing time. While for oven systems processing times relate to static product and process characteristics, the processing time of a convoy in a BDT depends on the time between the first and the last AGV in a convoy. Also oven systems set restrictions to the number of products in a batch, while there is no a priori limit set to the number of AGVs in a convoy.

Summarising, the most important differences compared to the existing literature are:
(a) Significant throughput time in the BDT (a few minutes)
(b) The clearance time is variable, depending on the time at which the last AGV entered the BDT
(c) No maximum batch size

3. Control rule concepts
In this section we characterise the rules for real-time BDT control in terms of information usage and decision structure. Further, we address the general assumptions and basic notation. For the detailed description of the control rules we refer to Section 4.

3.1. Characterisation of control rules
First, we describe the control options from a static perspective, i.e. the information available on AGV presence and future arrivals does not change in time. Hence, it is possible to construct an optimal schedule (cf. the concept of deterministic machine scheduling). In a dynamic situation however, information may change as a result of additional AGV arrivals or better forecasts of arrival times. This leads naturally to a rolling horizon approach. As a consequence, rescheduling is necessary. This implies that the rescheduling frequency appears as an additional parameter.

In a static context, we consider three typical situations as far as information on AGV arrivals is concerned:
(a) No on-line information on AGV queues and arrivals, i.e. only global information on average AGV arrival intensities is available
(b) Local information on queue lengths at the tube entrances
(c) Prior information, i.e. both on current queue lengths and on future arrivals within a certain information horizon H
Decision options open to the controller are a single decision on the timing of the next direction change, or integral scheduling of multiple direction changes within the information horizon $H$, i.e. all known AGVs (jobs) are included in the schedule. Now we can classify the control rules according to the information and decision options, see Table 1.

<table>
<thead>
<tr>
<th></th>
<th>no on-line information</th>
<th>local information</th>
<th>Prior information</th>
</tr>
</thead>
<tbody>
<tr>
<td>one change</td>
<td>periodic control</td>
<td>adaptive local control</td>
<td>adaptive look-ahead control</td>
</tr>
<tr>
<td>multiple changes</td>
<td></td>
<td></td>
<td>dynamic programming</td>
</tr>
</tbody>
</table>

Table 1: Classification of the control rules

If no on-line information on the AGVs waiting or arriving is available, a periodic control rule is the only realistic option. In that case, the only issue is to determine the fixed time interval $P$. This interval is chosen based on off-line optimisation considering the historical and/or expected traffic flows (cf. off-line junction control, Bell, 1992). The periodic control rule will be discussed in Subsection 4.1. The advantage of the periodic control rule lies in its simplicity: it requires no information on system status.

Information on the AGVs waiting or arriving can be used to improve system performance by developing control rules that use this information, so called adaptive control rules (see Subsection 4.2). When only local information on queue lengths is available, the possible decisions are clear:

1) The queue at the right first passes the tube after which the left queue passes the tube
2) The queue at the left first passes the tube after which the right queue passes the tube

It can easily be shown that any other option (e.g. the queue at the right is processed in two separate batches) is inferior because of the additional clearance time. For this reason, a dynamic programming approach that considers multiple changeovers naturally leads to either of the two solutions as mentioned above. Hence, adaptive control may be considered identical to dynamic programming for this particular case.

As can be seen from Table 1, we consider two decision options based on information on future arrivals. Generally, it only makes sense to change the tube direction directly after an AGV has entered the tube. The only additional decision moment relates to the situation where an AGV arrives at the empty tube. First, we developed a look-ahead rule for sequential direction change
Second, the dynamic programming rule (Subsection 4.4) computes an optimal sequence of direction changes.

The decision about changing the direction should be based on some optimisation criterion. As our focus is to minimise average order throughput times, we choose minimum average waiting time at the BDT as criterion. However, the basic ideas as presented in this paper can be applied to other criterions as well, such as minimum lateness.

In practice we seldom encounter static situations. In principle, we can apply the rules introduced above to the dynamic situation, where rescheduling can be initiated when information changes. Local information changes when an AGV arrives at the tube, whereas prior information changes when an AGV has approached the tube up to a time $H$, i.e. the information horizon. Rescheduling is possible at any of these moments. However, too frequent rescheduling may result in nervous schedules and/or excessive computation times.

3.2. Assumptions and notation
For reasons of clarity of understanding, and to keep the formulas simple, we make the following simplifying assumptions as far as the system configuration is concerned:

1) Queued AGVs accelerate instantaneously to their normal speed when activated
2) There are no restrictions concerning the minimum distance between AGVs
3) The length of an AGV is negligible

The last two assumptions imply that the interarrival time between two subsequent AGVs can be any real number $\geq 0$. In Subsection 6.5 we discuss the implications of assumptions 2 and 3 for the system performance. Note that we do not make an explicit assumption about the arrival processes as far as lot size is concerned. Both single and batch arrivals are considered.

To guarantee a clear understanding of terms and variables used in the construction of the control rules, the notation is explained:

$T$ = driving time of a single AGV for passing the tube
$t_0$ = the decision moment
$H$ = information horizon, i.e. at a decision moment $t_0$, AGV arrivals up to $t_0+H$ are known
\[ C(t_0) \] = the clearance time for the tube at \( t_0 \), i.e. the time needed for the last AGV to leave the tube, assuming that no additional AGVs enter the tube

\[ t'_i; t'_l \] = the i-th arrival at the right (left) side of the tube, where \( t'_i \) (\( t'_l \)) \leq 6 \) if an AGV has already arrived

\[ q'(t); q'(t) \] = the number of AGVs in queue at the right (left) at time \( t \) as far as known at \( t_0 \)

4. Control rules

In this section, we discuss the control rules in more detail, building on the notation and concepts introduced in the previous section. Successively, we address periodic control, adaptive local control, adaptive look-ahead control and dynamic programming.

4.1 Periodic control

As mentioned in Sections 1, 3, the Periodic Control Rule (PCR) bases its decisions to change tube direction on a fixed time interval \( P \) (or equivalently, it assumes a fixed switching frequency \( 1/P \)). The setting of the time interval is restricted by the need for clearing the tube every time the tube direction is changed. As a consequence, the minimum time interval \( P \) should exceed the maximum clearance time, i.e., the driving time in the tube \( T \). In this section, we focus on an expression for the mean waiting time as a function of the switching period \( P \). Using such an expression, we can find the optimal value of \( P \) using some standard procedure. For simplicity, we will focus on Poisson arrivals with AGV arrival rate \( \lambda \) at both sides of the tube.

The calculation of the mean waiting time as function of \( P \) has similarities with the analysis of a M/D/1 queue with vacations. The deterministic service time equals the minimum distance between successive AGVs expressed in time. The vacation periods in our case can be modelled as some random variable that depends on the switching interval \( P \), the driving time in the tube \( T \) and the AGV arrival process. However, we have not found an expression for the mean waiting time for our case in the literature on queuing systems with vacations.

If the minimum distance between successive AGVs can be ignored, a very accurate approximation for the mean waiting time can be derived, cf. Ebben (2001). Some numerical results showed that the optimal value of \( P \) decreases as traffic intensity increases. This is due to the assumption that a batch of AGVs can enter the tube in negligible time, irrespective of the batch size (compare Section 3). If the driving distance between AGVs is restricted by some minimum value, in order to prevent collision, an opposite effect may be expected. Hence, such a
result is only useful if the minimum driving distance between AGVs is negligible (e.g. if a train of AGVs can be constructed by magnetic coupling of AGVs while waiting for the tube entrance).

For the case that mutual distances between AGVs are significant, we may proceed as follows. We denote the minimum succession time between AGVs by $\delta$. In the case of heavy traffic (which is most interesting), it is reasonable to assume that the clearance time equals the driving time in the tube. Under this assumption, the queuing process can be modelled as an M/D/1 queue with fixed period vacations. The traffic literature contains an exact method (Heidemann, 1994) as well as several approximations (Webster, 1958; Miller, 1968) for the mean waiting time in such a system. The complexity of Heidemann’s exact method shows that for a seemingly simple periodic control rule, optimisation of the time interval $P$ is not straightforward. It includes the calculation of the zero-points of several non-linear functions that lie within the (complex) unit circle. This part can be avoided by using Miller’s (1968) approximation for part of the exact expression. This combination yields an easy and accurate expression for the mean waiting time:

$$E[W] = \frac{\delta}{2} + \frac{1}{2\lambda(1-\lambda\delta)} \left( \frac{2(P+T)R'(1) + \lambda(P+T)(\delta + P + T)}{2P} + \lambda\delta(1 + \lambda\delta) \right) \quad [1]$$

Where $R'(1)$ can be approximated according to Miller (1968) as

$$R'(1) \approx \exp \left[ -1.33 \frac{\sqrt{(P-T)\delta}}{2P\lambda\delta} - 1 \right] \frac{P-T}{2P\lambda\delta} - 1$$

This expression is only valid if the switching period $P$ is long enough to guarantee a stable system. A necessary and sufficient condition for stationarity is derived by Meissl (1963), which can be formulated in our notation as:

$$P > \frac{T}{1-2\lambda\delta} \quad [2]$$

Note that expression [1] overestimates the mean waiting time in our application. This is a consequence of the fact that the clearance time is less than or equal to the driving time in the tube, causing a longer effective green time. Comparison to simulation results revealed that the location
of the optimum switching period $P$ is not very sensitive to this approximation error (cf. Ebben, 2001).

As an example, we calculated the mean waiting time according to [1] as function of the switching period $P$ for various AGV arrival rates $\lambda$ (all time units in minutes). The driving time in the tube equals $T=7$ minutes and the minimum driving time between AGVs equals $\delta=3\frac{1}{2}$ seconds, which are realistic values for the Schiphol case that we have in mind. The results are shown in Figure 2.

![Figure 2: Mean waiting times as function of the switching period $P$ ($T=7$)](image)

As can be expected, the average waiting time increases with the traffic intensity. Also we see that the optimal switching period increases (and so does the green time) as traffic intensity increases. This is due to the fact that a longer green time is required to allow the AGVs waiting to enter the tube. The latter observation is further clarified in Figure 3, where the optimal switching period $P$ and the corresponding mean waiting time $E[W]$ are shown as function of the traffic intensity $\lambda$.

![Figure 3: Optimal switching interval $P$ and mean waiting time $E[W]$ as function of the arrival rate $\lambda$ ($T=7$)](image)
4.2 Adaptive local control

We consider the situation in which only the queue sizes at either side of the tube are known. Without loss of generality, we assume that the current driving direction is from left to right. In accordance with Section 3, we distinguish between two decision options: either the queue at the left first passes the tube or the queue at the right first passes the tube.

A direction change, meaning that the AGVs at the right side will first pass the tube, will induce waiting time for the AGVs in queue at the left. These AGVs are forced to wait at least for a period equal to the clearance time \( C(t_0) \) plus the driving time \( T \). The waiting time caused by a direction change is:

\[
W_{\text{change}} = q'(t_0)(C(t_0) + T) \tag{3}
\]

When the direction is not changed directly but after the AGVs from the left have entered the tube, all AGVs in queue at the right have to wait longer. The waiting time per AGV equals the difference between the clearance time before making a decision \( C(t_0) \) and the new clearance time caused by AGVs entering the tube from the left. The new clearance time is equal to the driving time \( T \). Therefore, the cumulative additional waiting time for all vehicles in the queue at the right \( q'(t_0) \) is:

\[
W_{\text{nochange}} = q'(t_0)(T - C(t_0)) \tag{4}
\]

The direction is changed immediately when \( W_{\text{change}} < W_{\text{nochange}} \). Otherwise, in a static situation the direction change is planned after all AGVs in the queue at the left entrance have entered the tube. Alternatively, in a dynamic situation it will be natural to postpone decision making until the next AGV arrives. Note that the waiting times related to the clearance time for those AGVs at the right side of the tube are regarded as sunk costs, i.e. the waiting time already incurred at the decision moment is not relevant.

4.3 Adaptive look-ahead control

Adaptive look-ahead control can be considered as an extension of adaptive local control, taking into account future arrivals within the information horizon \( H \). Let us consider the cost functions associated with the decision options: change the direction at the decision moment or after the \( i \)-th arrival from the left. The cost function for the case where tube direction is changed directly can be easily found by a straightforward extension of formula [3]:
\[ W_{\text{change}} = \sum_{t'_i < t_0 + T + C(t_0)} \min \left\{ t'_0 + T + C(t'_0) - t'_k, T + C(t_0) \right\} \] \[ [5] \]

The formula includes waiting time of arrivals \( (t'_i) \) at the left entrance of the tube up to \( t_0 + C(t_0) + T \), unless this information is not available given the restrictions set on the information horizon \( H \). Note how [5] also covers those situations in which decision making does not coincide with the arrival of an AGV (compare the second term).

The cost function associated with the alternative options boils down to scheduling \( i \) AGVs arriving at the left entrance and \( q'(t_t + T) \) AGVs queued at the right entrance. This means that the direction is not changed until AGV \( i \) has entered the tube. For each option costs are computed as:

\[ W_{\text{nochange}, i} = \sum_{t'_i < t'_i < 2T} (t'_i + 2T - t'_k) + \sum_{t'_i < t'_i + T} (t'_i + T - \max(t'_k, t_0 + C(t_0))) \] \[ [6] \]

The first term considers the waiting times of AGVs arriving at the left side of the tube within the interval \( <t'_i, t'_i + 2T> \), i.e., after \( i \) AGVs have entered the tube and before the scheduled AGVs from the right have passed the tube. The second term of [6] represents waiting times for AGVs in queue and arriving at the right side of the tube up to a time horizon \( t'_i + T \). Again, only arrivals within the information horizon \( H \) are taken into account. Remark how the choice of options may be uncoupled from the information horizon.

A straightforward comparison of the above options is not possible, as we compute waiting times for different schedules, each having their own planning horizon, i.e., the moment the last scheduled AGV leaves the tube. Hence, it seems no more than natural to facilitate comparisons by weighing waiting times for the planning period. Weighed costs associated with each option are therefore formulated as:

\[ WC_{\text{change}} = \frac{W_{\text{change}}}{T + C(t_0)} \]

\[ WC_{\text{nochange}} = \min_i \left\{ \frac{W_{\text{nochange}, i}}{t'_i + 2T - t_0} \right\} \] \[ [7] \]

The direction is changed immediately when \( WC_{\text{change}} < WC_{\text{nochange}} \). In the other case the direction change is planned after the \( i \)-th arrival which causes the minimum \( WC_{\text{nochange}} \) (static situation) or may be postponed (dynamic situation).
4.4 Dynamic programming

For those situations where multiple direction changes have to be scheduled within the information horizon $H$, we propose a Dynamic Programming approach (DP). Here the static situation will be considered. The results can also be used in a dynamic environment, by adopting a rolling horizon (see Section 3).

Figure 4: Sequencing of convoys

The scheduling decision comes down to finding the sequence of convoys from either side of the tube which minimises total waiting time (see Figure 4). Denoting by $N_L$ ($N_R$) the number of AGVs from the left (right) within the information horizon at $t_0$, we can formulate the objective function as:

$$MIN_{n=L,R}\{f_n(N_L,N_R,t^*)\}$$  \[8\]

The cost function $f_n(i,j,t)$ is defined as the minimum additional waiting time for all AGVs present at the BDT or arriving within the horizon $H$, if at time $t$ already $i$ AGVs from the left and $j$ AGVs from the right have passed the tube ($i = 0,1,..,N_L$, $j = 0,1,..,N_R$) and the last convoy passing the tube came from direction $n$ ($n = L,R$). Hence $t^*$ is a point in time at which all AGVs known at $t_0$ are processed. Note that $t^*$ is just an auxiliary variable for the recursion. Given the initial conditions at the decision moment ($t_0$):

$$f_n(0,0,t_0) = 0$$  \[9\]

the cost function $f_n(i,j,t^*)$ can be formulated as:
\[f_L(i, j, t') = \min_{k=1, \ldots, j} \{c[(i-k, j, t),(i, j, t')] + f_R(i-k, j, t)\}\]
\[f_R(i, j, t') = \min_{k=1, \ldots, j} \{c[(i, j-k, t),(i, j, t')] + f_L(i, j-k, t)\}\]

with:

\[c[(i-k, j, t),(i, j, t')] = \max_{y=i-k+1} \{t - t'_y, 0\}\]
\[c[(i, j-k, t),(i, j, t')] = \max_{y=j-k+1} \{t - t'_y, 0\}\]

The function \(c[(i-k, j, t),(i, j, t')]\) computes waiting times for a convoy of \(k\) AGVs that has entered the tube at time \(t\) and left it at time \(t'\), given that initially \(i\) and \(j\) AGVs were waiting at the entrances of the BDT.

5. Design of the simulation study

To demonstrate the potential of the new strategies for BDT-control, we set up an extensive simulation study. The design of the simulation study is based on case figures related to the Schiphol system. In this way, practical settings are guaranteed which may also be exemplary for future systems (see Section 1). In Table 2, experimental factors and their range are shown.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Range</th>
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<tbody>
<tr>
<td>1. Control rule</td>
<td>PCR, ARloc, ARla, DP</td>
</tr>
<tr>
<td>2. Average interarrival time (minutes)</td>
<td>0.5, 1, 1.5, 2, 4</td>
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<td>3. Arrival distribution</td>
<td>Poisson, Uniform</td>
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<td>4. Lot size</td>
<td>1, Uniform(3,9)</td>
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<td>5. Tube length (meters)</td>
<td>1000, 2000, 3000</td>
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</table>

Table 2: Experimental factors

For convenience, the control rules introduced in Section 4 are abbreviated as PCR (Periodic Control Rule), ARloc (Adaptive Rule with local control), ARla (Adaptive Rule with look-ahead), and DP (Dynamic Programming). The experimental factors concern the arrival patterns of AGVs (interval, distribution and lot size) and BDT characteristics (tube length). The driving speed of the AGVs is constant and equals 6 m/s. For each control rule average waiting time per AGV has been measured for the default settings (marked boldly). Alternative system configurations were chosen by changing the value for exactly one of the factors 2-5. For all control rules we consider a
dynamic setting where re-planning coincides with the moment one or multiple AGVs arrive at the tube and information changes or has changed. The fixed time interval for the periodic control rule is determined using the formulas derived by Ebben (2001). This results in change-over times of 6.3, 6.8, 7.1, 7.3 and 8 minutes corresponding to average interarrival times of 0.5, 1, 1.5, 2 and 4 minutes.

For the adaptive look-ahead rule (AR\textsubscript{LA}), we chose to consider all options for changing the driving direction within the interval \([t_0, t_0+T]\). Of course many choices are possible here. Our choice for the interval is motivated by two arguments. Firstly, considering system characteristics, the driving time in the tube forms a natural horizon. Secondly, as a result of simulation experiments, we found that the choice of the interval is an acceptable trade-off between the information requirement and the performance improvement. We assumed the information on AGV arrivals to be available for a period equal to \(H=3T\), i.e., three times the driving time for the BDT. This period logically follows from the above choice to consider all possible moments for changing tube direction up to once the driving time in the tube \((T)\). Given a maximum clearance time of \(T\), waiting times for AGVs may be influenced up to \(3T\). After all, after a change of tube direction they will not be able to enter the tube until the last AGV from the left and the AGVs queued at the right have passed the tube.

Next to the experiments mentioned in Table 2, we studied the practical use of a DP-approach in somewhat more detail by means of a sensitivity analysis. Computation times were registered and the effect of the planning frequency on system performance is studied. To get an idea of the effect of the planning frequency, we tested the DP-rule for a smaller planning frequency. Furthermore, we considered the effect safety precautions in terms of the required minimum distance to be kept between AGVs might have on system performance.

In our experiments we adopted the batch means method (cf. Law and Kelton, 1991; Hoover and Perry, 1989). Each batch equals one day. The first batch is discarded to account for any start up bias. We related the number of batches to the relative width \((\gamma=0.01)\) of the confidence interval for the average waiting time, where the significance level \(\alpha\) is set to 0.05. Uncorrelatedness of the batches has been tested using the runs test (cf. Hoover and Perry, 1989). It showed no significant correlation, given a significance level \(\alpha = 0.05\).
6. Analysis of simulation results

In this section, we analyse the outcomes of the simulation study for each control rule. First, we will present results for the default settings (Subsection 6.1). Next, we discuss how the choice for alternative system configurations in terms of arrival distributions and tube length may influence system performance (Subsection 6.2). Subsections 6.3-6.5 consider the results of the sensitivity analysis concerning the length of the information horizon, computation times and the possible need for safety precautions in terms of the intermediate distance required for AGVs. In this way additional insights are gained in the practical needs for successful application of the rules.

6.1. Default settings

In this section outcomes of the simulation study are presented for the default settings: single AGVs arriving according to a Poisson distribution at a BDT with a length of 2000 meters. In order to facilitate comparison of results, the performance for each setting is indicated in terms of normalised average delay, i.e., the average waiting time at the BDT divided by the driving time in the BDT (T).

![Figure 5: Normalised average delay for the default settings.](image)

In Figure 5 it is shown how the adaptive rules clearly beat the PCR-rule by percentages up to 25% at low arrival rates. Differences between the adaptive rules are smaller. The look-ahead rule performs 2-3% better than the local control rule, while the dynamic programming rule performs 5-8% better than the look-ahead rule. The outcomes clearly confirm the general proposition that the more information on future arrivals is included in decision-making, the better the results. Note that the performance differences presented in Figure 5 may all be considered significant, given a paired t-test with significance level 95%.
6.2. Alternative system configurations: tube length and arrival distribution

We found that relative performance of the rules is hardly influenced by tube length. The influence of the arrival distribution on system performance is studied in two ways. In a first series of experiments the Poisson distribution is replaced by a more “regular” Uniform distribution. Although most results are comparable to those found for Poisson arrivals, relative performance differences between the control rules tend to be much smaller, mostly less than 6% (see Figure 6). Secondly, we considered compound Poisson arrivals, where the lot size of arriving AGVs is uniformly drawn from the interval [3,9]. Note that we increased the average interarrival times (3, 6, 9 and 12 minutes) correspondingly. Two conclusions may be drawn from these series of experiments:

1. The larger the irregularity of the arrival pattern, the better the adaptive control rules perform compared with the periodic control rule. This is due to their responsiveness to system status.
2. More irregular arrival patterns lead to lower normalised average delay. More frequently, large time gaps occur between AGVs, which are efficiently exploited by the adaptive rules by changing the tube direction.

These conclusions are illustrated in Figure 6. The performance difference between the periodic control rule and the dynamic programming rule is much smaller for Uniform arrivals, compared with Poisson arrivals. We also see that normalised average delay is much smaller for Poisson arrivals, compared with Uniform arrivals (cf. DP Poisson and DP Uniform). Similar conclusions are also found by Glassey et al. (1991) in the context of dynamic machine scheduling.

Figure 6: Periodic control and dynamic programming for Uniform and Poisson arrivals
The above experiments started from the idea that traffic intensity is the same for both sides of the BDT. However, if the arrival rates from the left and right side differ, we expect that the periodic control rule performs worse. In an additional experiment this hypothesis was confirmed. Given an average interarrival time of 0.5 minutes from one side and 1 minute for the other side relative difference of the periodic control rule and the dynamic programming rule rises to 22% (compare Figure 5).

6.3 Sensitivity of the adaptive rules for information horizon and planning frequency

An interesting question from a practical perspective is how the performance for the new control rules relates to the availability of information on future arrivals. To answer this question, we study the relationship between the length of the information horizon and system performance. In this subsection, we express the information horizon in meters rather than in time, because this naturally relates to facilities for vehicle detection like sensors or inductive loops. The experiment includes the DP-rule and the ARLA rule. As far as the DP rule is concerned two scheduling frequencies are considered: per arriving AGV (DP as considered in Subsection 6.1) and per convoy (DPconvoy). The scheduling frequency for the DPconvoy rule is related to a planned direction change, i.e. no new decision is made before all AGVs, which have previously been scheduled to pass the tube have done so. Tube length is set to 2000 meters, while AGVs arrive one by one according to a Poisson distribution with average arrival intervals of 1 and 4 minutes respectively. Figure 7 shows the numerical results, where the average interarrival time in minutes is given between parentheses. We see that the dynamic programming rule (DP) and the look-ahead rule (ARLA) are not very sensitive to the length of the information horizon. An information horizon of 3000 meters already gives good results, while larger information horizons do not lead to considerable improvement. The fact that performance is most influenced at low arrival rates can easily be explained by the relative weight of information on arriving AGVs under these circumstances.
Another effect, which is clearly shown by Figure 7, is the effect of the re-planning frequency in a dynamic context; compare the results for both DP rules. Clearly a lower frequency may significantly reduce system performance. As may be expected, for longer information horizons, the effect is smaller.

6.4 Practical use of the DP-approach

The outcomes of the simulation study clearly point out dynamic programming as the best performing heuristic for BDT control. However, whether the dynamic programming approach is useful in practice also depends on the computational effort. A series of experiments indicated that even in the worst case evaluated in the simulation study (see Section 5; average interarrival time of 0.5 minutes) the dynamic programming rule only requires a few seconds per changeover using a Pentium III 500 MHz. Although computation times for the dynamic programming rules tend to increase exponentially for higher arrival rates, this outcome still leaves a lot of room for practical application. This especially is true when one considers the fact that for high arrival rates performance of the DP control rule is not very sensitive to the planning frequency (see Subsection 6.3) - which supplies us with an additional means to reduce computation times.

6.5 AGV length and safety precautions

So far we assumed that the minimum distance between two successive AGVs and the AGV length are negligible (compare Section 3). In this subsection we study the effect of both factors on system performance. The cost functions for the control strategies introduced in Section 4 do no longer apply to these situations; they have to be adapted, cf. Appendix 1. Parameter setting for the Periodic Control Rule is supported by expression [1] in Subsection 4.1. This results in a switching period $P = 8\frac{1}{4}, 7\frac{1}{2}, 7, 6\frac{1}{2},$ and $6\frac{3}{4}$ minutes corresponding to $\lambda$ equal to 0.5, 1, 1.5, 2 and 4
minutes. Figure 8 presents results for settings in which the required distance between AGVs is set to 15 meter and AGV length equals 6 meter (which is equivalent with 3.5 seconds driving time). The remaining settings correspond to the default settings. The results in Figure 8 are similar to those in Figure 5, although we clearly see the effect of safety precautions in terms of longer waiting times in case of high traffic intensities.

Figure 8: Intermediate distances and AGV length considered

7. A BDT as a part of a closed system
Till now we studied the BDT in isolation, being a part of an open system. As a consequence arrivals were considered independent of each other and an infinite population was assumed. An open system is a reasonable assumption in case of a road network or a large transportation network. In these cases the convoys that are formed by the BDT are dispersed due to different destinations. Also, the population is large compared to the traffic at the section under consideration. In a closed system, however, these assumptions may no longer be justified, making it necessary to study interaction effects with other elements of the network (compare the attention paid in traffic literature to co-ordinating traffic signs, see e.g. Bell, 1992). For example, let us consider the situation where the network at one side of the BDT is made up by a single terminal. The convoys that leave the BDT drive to the terminal, load/unload, and return probably soon to the same BDT. Decisions on BDT control may therefore have serious impact on future decisions, next to having a serious impact on total system performance. Similar remarks can be made in case of a second BDT having no crossings or intersections in between both tube sections. The convoys created can cause congestion problems at terminals and other BDTs. An interesting question therefore is how the local control rules effect total system performance. Below we try to answer this question by studying a case example.

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The Schiphol system

To examine whether our conclusions on the rules for BDT control in an open system are equally valid for a closed system, we embedded the rules in a simulation model for the Schiphol system, cf. Van der Heijden et al. (2000). The studied layout, see Figure 9, contains one bi-directional tube. At one side of the BDT there is only one terminal, so that the AGVs are less dispersed. At the other side the BDT is connected to several terminals. The bi-directional tube has a length of 1500 meter and for both sides accurate information on arrivals is available by registering AGV movements 1500 meters before the tube entrance. The required distance between two AGVs is 3.5 seconds. The control rules still aim to minimise the waiting time locally at the BDT, but we measure the total system performance in terms of the fill-rate, i.e. the percentage of transport orders that is handled before the due date.

![Figure 9: Layout of the Schiphol system](image)

The question is whether the control rules that perform best in minimising the average waiting time at the BDT also result in best system performance. The simulation results confirm the ordering of the control rules with respect to our performance measure, which can be seen in Table 3.

<table>
<thead>
<tr>
<th>Control rule</th>
<th>180 AGVs</th>
<th>190 AGVs</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCR (6.7 minutes)</td>
<td>90.2</td>
<td>99.7</td>
</tr>
<tr>
<td>AR_{LOC}</td>
<td>92.4</td>
<td>99.5</td>
</tr>
<tr>
<td>AR_{LA}</td>
<td>93.5</td>
<td>99.6</td>
</tr>
<tr>
<td>DP</td>
<td>99.1</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 3: Performance of the control rules in a closed system
Note that for a closed system as in our application, we may adjust the control rule, e.g. by including information on the situation at the surrounding terminals and BDTs. Also, we could use another objective function. Instead of minimising the average waiting time at the BDT, a local objective, we could minimise a systems related objective, for example the expected number of late AGVs.

8. Conclusions and directions for further research

In this article we addressed the issue of real time control of a tube allowing for bi-directional traffic on a single track. Several new control rules were defined as an alternative to the classic periodic control rule, which manages traffic flows through the tube by giving priority to either direction according to a fixed time interval. The new rules try to make good use of local information on queue lengths and look-ahead information on future arrivals in order to improve on system performance. Simulation results indicate that, depending on the amount of information available, adaptive control rules improve system performance up to 10% for high arrival rates and up to 25% for low arrival rates. Best performance is realised by a rule based on dynamic programming. While it is known that a dynamic programming approach may involve high computational costs, simulation results for a realistic large-scale transportation network indicated that computation times for the dynamic programming rules are acceptable for real-time applications.

In general, the performance of the rules is not much influenced by the length of the information horizon. Only in case of a low planning frequency (e.g. related to the planned change over times), the dynamic programming rule is sensitive for the information horizon length. As is to be expected, the effect is most noticeable at low traffic intensities. The introduction of safety precautions, such as the requirement that a minimum distance should be kept between AGVs to prevent collisions, does not significantly change relative performance of the rules. Nevertheless, it does seriously change the optimum changeover time for the periodic control rule. An experiment where the BDT was included in a closed network (the Schiphol system) showed that our conclusions remain valid for a closed system.

Interesting topics for future research are a more thorough examination of the interaction effects with terminals or other bi-directional tubes within an open or closed network. Furthermore, our research of the Schiphol system indicated that extensions including due date related performance measures would be worthwhile studying from a practical perspective.
References


Zee, D.J. van der, (1997) Simulation as a tool for logistics management, Ph.D. Thesis, University of Twente, the Netherlands.
Appendix 1. Extension of control rules to deal with safety precautions

In Section 3, we assumed that no minimum distance is required between two successive AGVs. Consequently, queued AGVs may enter a tube instantaneously. However, because of safety precautions, often a certain distance between two AGVs is to be kept. Given such a demand it may take a significant amount of time before a queue of AGVs has completely entered the tube (up to several minutes in the Schiphol case). In this appendix we consider the question how the control rules described in Section 4 can be extended to deal with suchlike situations. The extended rules have been tested by a simulation study, see Subsection 6.5.

We start from the assumption that the required time for a convoy for entering the tube is described by a function $g(x)$. For example, let us recall that we used a linear function $g(x) = 3.5(x-1)$ (in seconds) in Subsection 6.5, with $x$ the number of AGV in the convoy. Of course alternative choices for $g(x)$ are possible. Also other system characteristics like AGV length may be included in the function. Given the function $g(x)$, cost functions for each of the control rules can be found, cf. [A1]-[A8]. The extended cost functions correspond with expressions [3]-[10] respectively (see Section 4).

[A1] \[ W_{change} = q'(t_0)(C(t_0) + T + g(q'(t_0))) \]

[A2] \[ W_{nochange} = q'(t_0)[T - C(t_0) + g(q'(t_0))] \]

[A3] \[ W_{change} = \sum_{t'_i \leq t_0 + T + C(t_0) + g(q'(t_0 + C(t_0)))} \min \{T + T + C(t_0) + g(q'(t_0 + C(t_0))) - t'_i, T + C(t_0) + g(q'(t_0 + C(t_0))) \} \]

\[ W_{nochange,i} = \sum_{t'_i < \min\{t'_i, t_0 + g(i)\} + 2T + g(q'(\max\{t'_i, t_0 + g(i)\} + T))} \]

\[ + \sum_{t'_i < \max\{t'_i, t_0 + g(i)\} + T - \max\{t'_i, t_0 + C(t_0)\}} \]

\[ WC_{change} = \frac{W_{change}}{T + C(t_0) + g(q'(t_0 + C(t_0)))} \]

[A5] \[ WC_{nochange} = \min \left\{ \frac{W_{nochange,i}}{\max\{t'_i, t_0 + g(i)\} + 2T - t_0 + g(q'(\max\{t'_i, t_0 + g(i)\} + T))} \right\} \]
[A6] \[ \min_{n=L,h} \left\{ f_n \left( N_{L}, N_{R}, t^* \right) \right\} \]

[A7] \[ f_n \left( 0,0,t_0 \right) = 0 \]

\[
f_L \left( i, j, t' \right) = \min_{k=1,\ldots ,l} \left\{ c \left[ \left( i-k,j,t \right), \left( i,j,t' \right) \right] + f_R \left( i-k,j,t \right) \right\}
\]

\[
f_R \left( i, j, t' \right) = \min_{k=1,\ldots ,r} \left\{ c \left[ \left( i,j-k,t \right), \left( i,j,t' \right) \right] + f_L \left( i,j-k,t \right) \right\}
\]

with:

[A8] \[
c \left[ \left( i-k,j,t \right), \left( i,j,t' \right) \right] = \sum_{y=i-k+1}^{y=i} \max \left\{ t-t'_y + g(y-i+k-1), 0 \right\}
\]

\[
c \left[ \left( i,j-k,t \right), \left( i,j,t' \right) \right] = \sum_{y=j-k+1}^{y=j} \max \left\{ t-t'_y + g(y-j+k-1), 0 \right\}
\]

\[ t' = T + \max \left\{ t + g(q^i(t)), t^i \right\} \]