Active noise control using a modified filtered-error algorithm with frequency domain decoupling

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Abstract

This paper presents an algorithm for broadband active noise control, using a modified version of the filtered-error algorithm with frequency domain decoupling and preconditioning. The decoupling is done efficiently in the frequency domain without regard to causality, but the decoupling transfer functions can be made causal by adding a delay. The influence of this delay on the speed of convergence is eliminated by using the modified version of the filtered-error scheme. The control effort is reduced by adding a regularization part which is consistent with the decoupling approach. The system can be used for multiple error signals, multiple secondary sources, and multiple references. This paper presents simulations of the behavior of the algorithm based on acoustic transfer functions and signals, showing that the frequency domain decoupling approach using the delay compensation technique increases the speed of convergence.

1 Introduction

The motivation of this work is to provide a method for broadband control using a method which provides a feasible number of computations to obtain a feedforward controller for problems with many sources and sensors. In this work we focus on adaptive methods to provide the controller coefficients. An instanteneous least-mean-squares adaptation in which the reference signals are filtered with the transfer function between actuator and error sensor is the filtered-reference algorithm [1]. For multi-channel systems the computational complexity can be substantial [2] and the speed of convergence can be an issue as well [2]. A reduced computational complexity for multiple reference signals is provided by the filtered-error algorithm [3, 4]. Bai and Elliott [5] described a method to decouple the preconditioned filtered-error scheme in the frequency domain. Berkhoff and Nijsse [6] presented a method in which the post-conditioned filtered-error scheme was made consistent with a regularized secondary path and which also provides a method to compensate the delay for improved convergence. The present paper combines the work of Refs. [5] and [6]. A description of a full multichannel implementation including prewhitening and decorrelation of the reference signals can be found in Ref. [7].

2 Filtered-error algorithm and preconditioning

A block diagram of the filtered-error scheme [3] can be found in Fig. 1. Let z be the discrete-time transform-domain variable. In this scheme, the disturbance signal d at the error sensor from the primary path $P(z)$ is reduced by a contribution y from the secondary path $G(z)$ using secondary sources with driving signals u. The signals u are obtained from a feedforward controller W, which uses the reference signal x as input. In the filtered error scheme the error signal is filtered with the adjoint $G(z)^* = G(z^{-1})^T$, in which T denotes

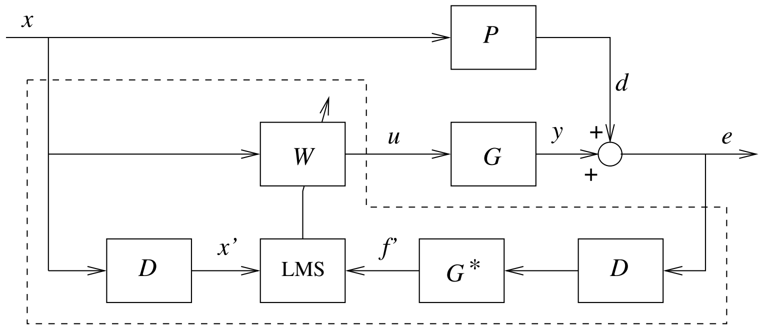

Figure 1: Filtered-error adaptive control scheme. The controller is within the dashed lines, physics is outside the dashed lines.

transpose, which is a time-reversed and transposed version of G. To make the adjoint causal, a delay D is needed, which also requires a delay D of the reference x as input to the Least-Mean-Squares (LMS) block. It is assumed that the controller W is updated at each time step n using an LMS update [8].

In Fig. 1, it can be seen that the adaptation loop, which is the transfer path from the output of W to the input of the LMS-block, contains a delay and frequency dependent transfer functions G and G^*, which lead to a reduced speed of convergence. Furthermore, in multichannel applications the coupling between the individual transfer functions of the secondary path G also cause a reduced convergence rate. Preconditioning of the secondary path using an inverse of the minimum-phase part of the secondary path eliminates the frequency dependence and the cross-coupling between the individual channels [8]. Using the minimum-phase inverse as a preconditioner, the adjoint of G has to be replaced by the adjoint of the all-pass part of G. In Ref. [6] it is shown that the effective adaptation loop is a delay D. To reduce the influence of this delay on the convergence, a modified scheme is presented in [6], in which we first subtract a delayed output of controller W and add the output of a second controller W of which the input is delayed. At each time step, the coefficients of the second controller are adapted and copied between to the first control block. This realizes a delay-free adaptation loop, which improves speed of convergence. However, tracking is still limited by the delay. Another aspect that is presented in Ref. [6] is the regularization of the secondary path, which reduces the output of the controller at frequencies where the secondary path has a low value, to prevent large amplifications. The decomposition of the secondary path in a minimum-phase part and all-pass part is based on an augmented plant $\bar{G}$

$$\bar{G}(z) = \begin{bmatrix} G(z) \\ G_{\mathrm{reg}}(z) \end{bmatrix}, \tag{1}$$

in which $G_{\mathrm{reg}}(z)$ is the regularizing transfer function. Both the preconditioner and the adjoint are based on the augmented plant, ensuring that the modified scheme is consistent with the regularization. Denoting $\bar{G}_i(z)$ as the all-pass transfer function and $\bar{G}_o(z)$ as the minimum-phase transfer function, the required factorization is

$$\bar{G}(z) = \bar{G}_i(z)\bar{G}_o(z), \tag{2}$$

in which the all-pass property holds:

$$\bar{G}_i^*(z)\bar{G}_i(z) = I_M, \tag{3}$$

as well as the minimum-phase inversion property

$$\bar{G}_o(z)\bar{G}_o(z)^{-1} = I_M, \tag{4}$$

with I_M the identity matrix of size M. The scheme can be found in Fig. 2. The minimum-phase/all-pass factorization is based on a state-space approach, using an inner-outer factorization [9]. The update rule for the controller is

$$W_i(n+1) = W_i(n) - \alpha e''(n)x'^T(n-i), \quad i = 0, \ldots, I-1, \tag{5}$$

in which i denotes the tap-delay of the Finite Impulse Response (FIR) coefficients, assuming FIR filters of length I, α is the convergence coefficient, and e'' is the error signal for the second control filter (see Fig. 2).

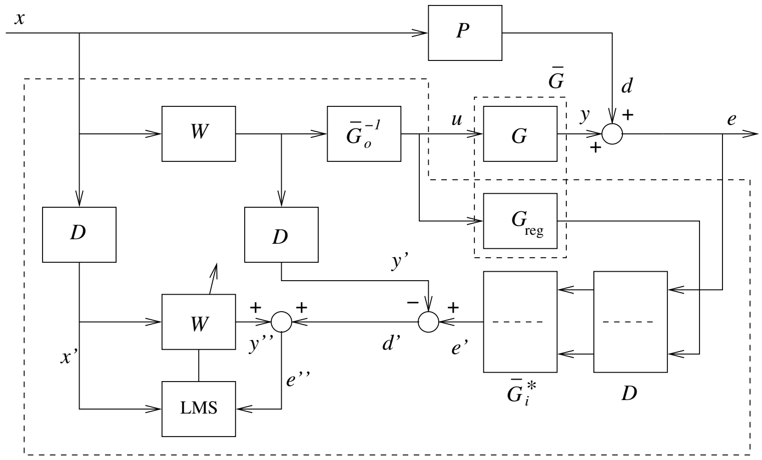

Figure 2: Regularized modified filtered-error adaptive control scheme [6].

3 Frequency domain decoupling

The use of inner-outer factorization is very costly for systems with a high number of sources and error sensors. Therefore, we try to use a decoupling approach in the frequency domain, substituting $z = e^{j\omega T}$, with unit imaginary number $j = \sqrt{-1}$, angular frequency ω and sampling interval T. The transfer functions evaluated in the frequency domain are the secondary path inverse $\bar{G}_{mi}$ and the adjoint $\bar{G}_{ai}$, which are related to $\bar{G}_o^{-1}$ and $\bar{G}_i^*$, respectively. This is identical to the method by Bai and Elliott [5], except that the present paper provides a different approach for regularization and provides a method to compensate the delay for improved convergence. For compactness, the frequency domain transfer functions are denoted by the argument ω instead of $e^{j\omega T}$. At each frequency, a singular value decomposition is applied to the secondary path:

$$\bar{G}(\omega) = \bar{U}(\omega)\bar{S}(\omega)\bar{V}(\omega)^H. \tag{6}$$

We also write

$$\bar{G}(\omega) = \bar{G}_a(\omega)\bar{G}_m(\omega), \tag{7}$$

with

$$\bar{G}_a(\omega) = \bar{U}(\omega), \tag{8}$$

and

$$\bar{G}_m(\omega) = \bar{S}(\omega)\bar{V}(\omega)^H. \tag{9}$$

The following properties can be used for decoupling:

$$\bar{G}_m(\omega)^H\bar{G}_m(\omega) = \bar{G}(\omega)^H\bar{G}(\omega), \tag{10}$$

and

$$\bar{G}_a(\omega)^H\bar{G}_a(\omega) = I_M. \tag{11}$$

It is noted that the transfer function $\bar{G}_a$ is not all-pass in general. Similary, the transfer function $\bar{G}_m$ in general is non minimum-phase. We define $\bar{G}_{ai}(\omega)$ and $\bar{G}_{mi}(\omega)$ by

$$\bar{G}_{ai}(\omega) = \bar{G}_a(\omega)^H = \bar{U}(\omega)^H, \tag{12}$$

and

$$\bar{G}_{mi}(\omega) = \bar{G}_m(\omega)^{-1} = \bar{V}(\omega)\bar{S}(\omega)^{-1}. \tag{13}$$

To make a scheme that can be implemented without wrap-around effects in case of the discrete Fourier transform, a delay $\Delta_M(\omega) = \delta(\omega)I_M$ has to be added to $\bar{G}_{mi}(\omega)$, which leads to $\tilde{G}_{mi}(\omega)$, in which $\delta(\omega)$

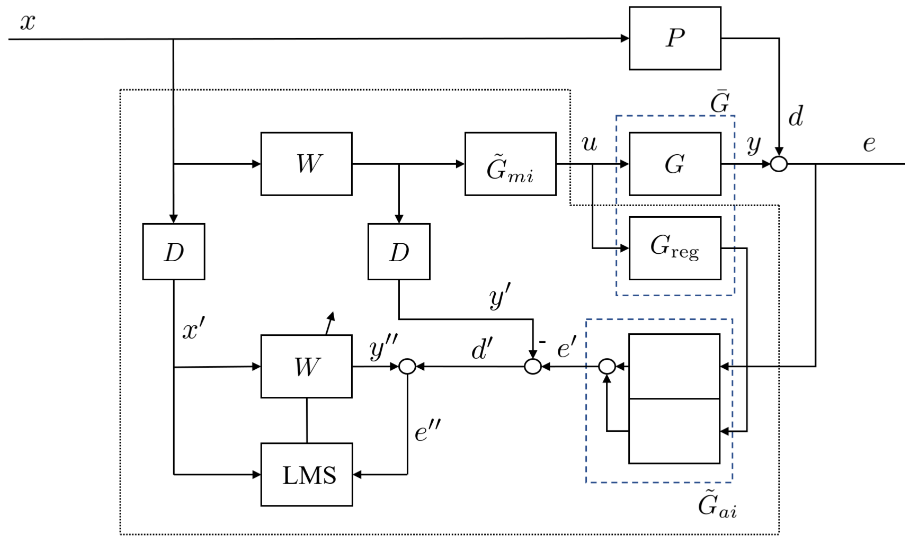

Figure 3: Regularized modified filtered-error scheme with frequency domain decoupling.

represents the phase-shift operator to account for the delay. We write

$$\tilde{G}_{mi}(\omega) = \Delta_M(\omega)\bar{G}_{mi}(\omega) = \Delta_M(\omega)\bar{V}(\omega)\bar{S}(\omega)^{-1}. \tag{14}$$

The transfer function $\tilde{G}_{ai}(\omega)$ should also incorporate a delay $\Delta_M(\omega)$ but then conjugated, since it is based on the adjoint - and the Hermitian in the frequency domain - of the product $\bar{G}(\omega)\tilde{G}_{mi}(\omega)$. Furthermore, the delay $D_M(\omega) = D(\omega)I_M$ should be incorporated in $\tilde{G}_{ai}(\omega)$ to ensure that it can be implemented causally, in which $D(\omega)$ represents the phase-shift operator to account for the delay. Then, we have

$$\tilde{G}_{ai}(\omega) = D_M(\omega)\Delta_M(\omega)^H\bar{U}(\omega)^H. \tag{15}$$

In this equation, $D_M(\omega)$ represents a positive delay, wheras $\Delta_M(\omega)^H$ represents a negative delay. In this procedure, we apply a delay $\Delta_M(\omega)$ to the secondary path. Therefore, in some applications it could be possible that the controller can not anticipate the disturbance and therefore a delay of the disturbance is necessary to get sufficient performance. After computation of $\tilde{G}_{ai}(\omega)$ and $\tilde{G}_{mi}(\omega)$ for all frequencies, an inverse Fourier transform is applied to go to the time domain, taking into account symmetry properties of the Fourier transform to obtain real-valued impulse responses. As can be seen in Fig. 3, the adaptation loop at the LMS block is delayless, which should have a positive influence on convergence. If the delays are used to prevent wrap-around effects of periodic Fourier transforms then it would also be possible to implement a circular shift in the time domain.

The full scheme is presented in Ref. [7]. This reference also includes an extension by prewhitening and decorrelating the reference signals, to improve the speed of convergence for the case that the spectrum of the reference signal is not flat or when the reference signals are correlated.

4 Results

4.1 Single-input single-output example

A simulation was performed with broadband noise with a single-input single-output system. The secondary path is a unit impulse at the 99th sample. The delay δ is one sample. The delay D is 99 samples. The unit variance primary disturbance was filtered with a Butterworth low-pass filter of 2nd order and cutoff-frequency being half the Nyquist frequency. The convergence coefficient is 0.025. The results of convergence are shown in Fig. 4. This the same example as in Ref. [6] in the sense that the total delay is the same. A difference is that a delay is added to the preconditioner. Comparing the results, it can be seen that the present

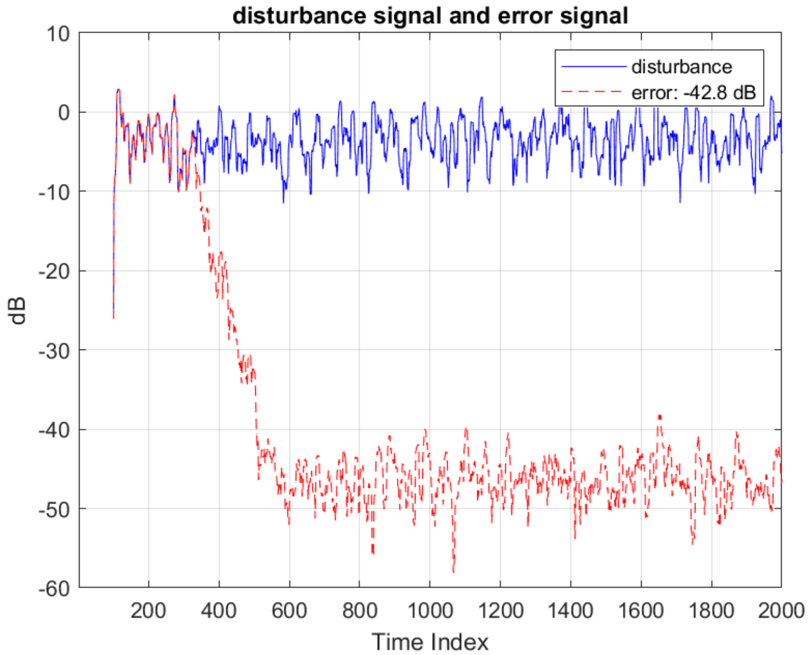

Figure 4: SISO convergence result.

algorithm has better convergence than a post-conditioned algorithm [6]. Convergence starts later (sample 300 vs. 200) than the convergence of the regularized modified filtered-error algorithm, but the final reduction of the error signal is obtained just as quickly (sample 650) with the frequency domain decoupling method of the present paper.

4.2 Multiple-reference multiple-output multiple-error sensor example

This section describes a result from a system with 4 references, 4 outputs and 4 error sensors [7]. The system demonstrates active reduction of reflections for an anechoic chamber at low frequencies. The system contains a prewhitening procedure for the reference signals, which is described in [7]. The control coefficients after $4 \cdot 10^4$ samples are given in Fig. 5. Fig. 6 shows the error signals after $4 \cdot 10^4$ samples, asssuming a stationary controller. Due to the delay that has to be added to the secondary path, the system benefits from a time advantage due to proper the placement of the transducers.

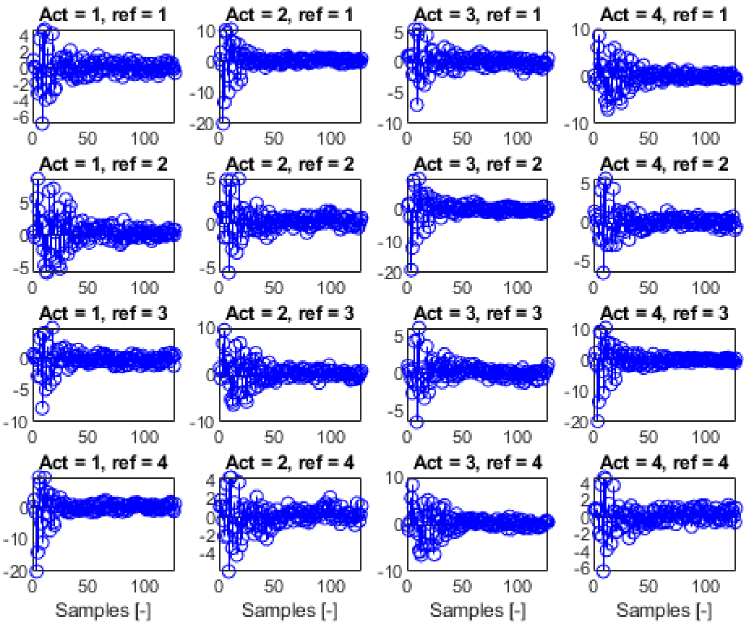

Figure 5: 4x4 Finite Impulse Response control filters for active reduction of reflection in an anechoic chamber.

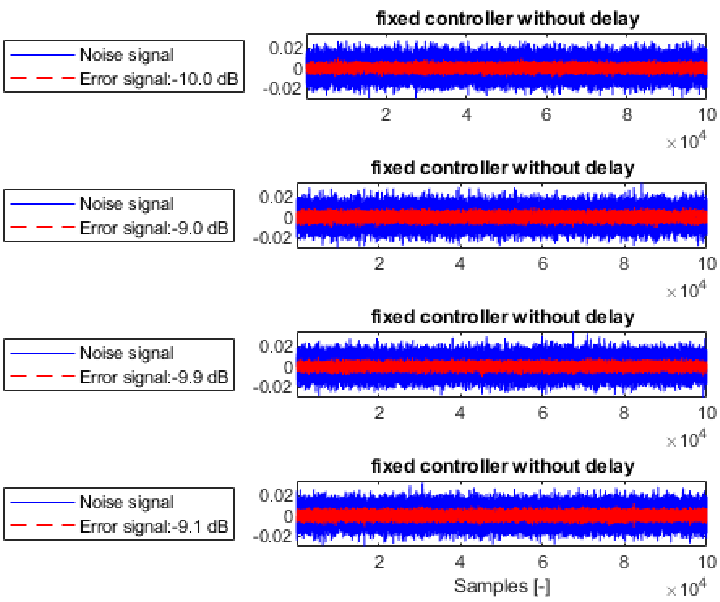

Figure 6: Disturbance signals and error signals for active reduction of reflection in an anechoic chamber.

Acknowledgements

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