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RESEARCH ARTICLE

Modeling variability of the electrical conductivity tensor for the induction welding of composites

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Abstract
Induction welding utilizes electromagnetic induction to heat carbon fiber reinforced thermoplastic (CFRTP) materials, enabling their fusion without direct contact. This heating process directly impacts material viscosity and bonding strength, with optimal temperatures ensuring proper fusion and strong adhesion between composite layers. The process is strongly influenced by material quality, as poor-quality materials can lead to uneven heating, incomplete fusion, and compromised structural integrity. To investigate the influence of the variation of CFRTP material properties on the generated heat, in this paper, we propose a novel way of modeling and propagating the stochastic electrical conductivity tensor through the physics based model describing the fusion bonding. The electrical conductivity of CFRTP is an anisotropic and inherently variable material property. To model this variation, we propose a stochastic model belonging to a family of positive definite tensors. In particular, we suggest the stochastic model in which both the scaling/strength and orientation of the tensor are separately modeled as stochastic. In this manner, we are capable of assigning a specific class of spatial symmetries and invariances to individual realizations of the stochastic tensor, all the while insisting that the average of the entire population adheres to a potentially 'higher' spatial invariance class. Finally, the proposed model is incorporated to the physics based model based on which the variation in the temperature field characterizing the fusion bonding process is quantified with the help of the sparse grid integration.

1 | INTRODUCTION

In an effort to reduce carbon emissions in the transportation industry, there is growing interest in replacing conventionally used metals with composite materials for load bearing components. Among possible alternatives, carbon fiber reinforced thermoplastics (CFRTP) offer a compelling solution due to their tailorable properties and advantages in reworkability, as remelting CFRTP offers possibilities for processing, repair and recycling [1]. Although promising, industrial application of CFRTP is hampered by difficulties in material modeling that would be used in the overall optimization of the manufacturing process of CFRTP parts. One example of such a processing technique is induction welding [2], a fusion bonding

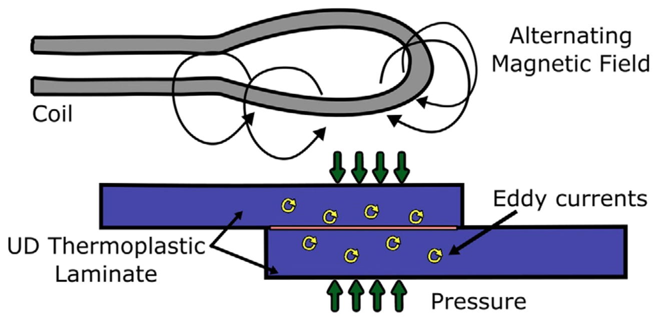


FIGURE 1 Schematic for the induction welding of CFRTP. CFRTP, carbon fiber reinforced thermoplastic.

process where the interface between two composite parts is heated, allowing the material to melt. Then the thermoplastic, under sufficient pressure, heals to form a seamless weld during the cooling down. During the process, the temperature field serves as an important indicator of the weld quality.

The temperature field during induction welding is strongly affected by variations in the electrical conductivity, as shown by ref. [3]. Although this paper depicts the variation of the temperature for three different values of electric conductivity (the average, low, and upper bound), the work does not offer an explicit material model which captures the variation in the material. The difficulty in material modeling comes from the anisotropic structure of the material. CFRTP, consisting of a fiber and thermoplastic material, exhibits anisotropy as the fiber direction is configured for specific load cases. Additionally, the manufacturing process introduces inherent variability in CFRTP, affecting its material properties. Specifically, electrical conductivity is strongly affected [4]. The modeling of the full anisotropic electric conductivity tensor is often seen in medical imaging, for example, [5]. However, for a varying electrical conductivity most examples are limited to the isotropic case, for example, see [6]. The challenge arises from the need to maintain the symmetric and positive definite properties of the tensor while introducing variation in the tensor. [7] formulated a reduced parametric approach to generate varying anisotropic tensors. The property of positive definiteness is taken into account by use of square root approach. Recently, ref. [8] has provided a parametric stochastic tensor model for thermal conductivity by use of an exponential map to uphold the positive definiteness, and allowing for a separate notion of variation in magnitude and orientation of the tensor by spectral decomposition.

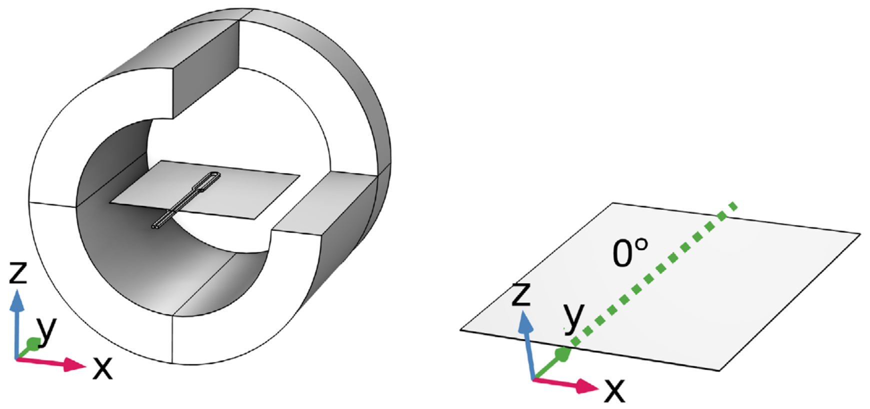
The main goal of this paper is to model the varying electrical conductivity by adapting the probabilistic description in ref. [8]. The conductivity tensor is then introduced as a tensor valued random variable. Furthermore, the model is introduced into the mathematical formulation of the induction welding process. The corresponding electromagnetic-thermal model is therefore reframed as a stochastic model, and the uncertainty in the electrical conductivity field is further propagated to the quantity of interest, that is, the temperature field, by means of sparse grid integration.

The paper is structured in the following way. In the second section, the induction welding process model is introduced by describing the electromagnetic and heat transfer equations. In the third section, a parametric stochastic model for the uncertain electrical tensor is formulated. In the fourth section, the governing equations are reformulated in stochastic setting and an example problem is shown. The paper ends with a conclusion and discussion on the model and results.

2 | INDUCTION WELDING MODEL

Induction welding works by generating an alternating current through a coil, see Figure 1. The current results in an alternating magnetic field, which further induces current in the conductive carbon fibers. In this manner, eddy currents are formed in closed loops of fibers. Heat is consequently generated in the material by the Joule heating effect [9]. Thus, one has to introduce a coupled electromagnetic-thermal model in order to represent the induction welding process in computer simulation.

To simplify the problem formulation, two assumptions are made with respect to the electromagnetic behavior (i) we assume magnetoquasistatic behavior, because the characteristic length of the induction welding setup is a lot lower than the proposed wavelength of excitation. (ii) The current that excites the coil is set to be harmonic. Hence, the electromagnetic part of the induction welding process shown in Figure 1 can be described by the magnetoquasistatic time harmonic



(A) Domain $\mathcal{G}$ including the air, coil and CFRTP

(B) Domain $\mathcal{G}_C$ which is the CFRTP plate and an indication of direction for the fibers

FIGURE 2 Simulations domains.

Maxwell equations:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon} \tag{2.1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2.2}$$

$$\nabla \times \mathbf{E} = -j\omega^f \mathbf{B} \tag{2.3}$$

$$\nabla \times \mathbf{B} = \mu^m \mathbf{J}. \tag{2.4}$$

Here $\mathbf{E}$ (V/m) is the electric field strength vector, ρ (C/m^3) is the electric charge density, ϵ (F/m) is the permittivity, $\mathbf{B}$ (T) is the magnetic flux density vector, ω^f (Hz) is the angular frequency, μ^m (N/m^2) is the magnetic permeability and $\mathbf{J}$(A/m^2) is the current density vector. In the previous formulation Equation (2.3) describes the inductive behavior of the magnetic field whereas Equation (2.4) describes the generation of magnetic field by a current. As both $\mathbf{E}$ and $\mathbf{B}$ are vectors, there are six dependent variables. The amount of dependent variables is reduced by introducing the magnetic vector potential $\mathbf{A}$ with Coulomb gauge as:

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \nabla \cdot \mathbf{A} = \mathbf{0} \tag{2.5}$$

inherently satisfying Equation (2.1). Furthermore, the coil and included current densities are additively decomposed by the use of structure equation:

$$\mathbf{J} = \mathbf{J}_s + \boldsymbol{C}\mathbf{E}. \tag{2.6}$$

Here, $\mathbf{J}_s$ is the source current density and $\boldsymbol{C}$ is the electrical conductivity tensor (S/m). Using the previous relation, Equations (2.3) and (2.4) are now reformulated to:

$$\nabla \times \frac{1}{\mu^m} \nabla \times \mathbf{A}(\boldsymbol{x}) = \mathbf{J}_s - j\omega^f \boldsymbol{C}\mathbf{A}(\boldsymbol{x}) \text{ with b.c. a.e. on } \mathcal{G}, \tag{2.7}$$

in which, $\mathcal{G} \subset R^3$ is the bounded domain of interest in Euclidean space. The domain is shown in Figure 2A with the geometry of the coil, the CFRTP plate and the surrounding air. To introduce the coupled thermal effect, the Equation (2.7) is weakly coupled to the heat transfer equation:

$$\rho c_p \frac{\partial T(\boldsymbol{x}, t)}{\partial t} = \nabla \cdot (\mathbf{k} \nabla T(\boldsymbol{x}, t)) + Q_e(\boldsymbol{x}) \text{ with b.c. a.e. on } \mathcal{G}_C \times \mathcal{T} \tag{2.8}$$

describing the heat conduction. Here, ρ (kg/m^3) is the density of the material, c_p (J/[kg*K]) is the heat capacity, T (K) is the temperature, $\mathbf{k}$ (W/[m*K]) is the thermal conductivity and Q_e (J) is the heat generated. The last term is given by $Q_e = \frac{1}{\boldsymbol{C}}|\mathbf{J}_e^2|$, which is the coupling term to the electromagnetism model given in Equation (2.7). The heating problem is only defined on the subdomain $\mathcal{G}_C \subset \mathcal{G}$, shown in Figure 2B and for the time interval $\mathcal{T} = [0, t_e] \subset R_+$. Solutions for the weakly coupled problem are obtained by use of the finite element method (FEM) procedure described in ref. [10].

3 | MODELING PARAMETRIC UNCERTAINTIES

The model presented in the previous section is deterministic, and assumes that all model parameters are known. However, due to variations in the manufacturing process the electrical conductivity $\boldsymbol{C}$ of the thermoplastic composites is known to be intrinsically varying, that is, uncertain. The uncertainty is reflected in both the material symmetry and the values of the tensor. To account for this variation, one has to extend the deterministic coupled model given in Equations (2.7)–(2.8) to a stochastic model.

We assume that the material is homogeneous, and is thus modeled by an effective material tensor $\boldsymbol{C}$, whose symmetry and the material constant values are not known. The material tensor is thus modeled by the random variable valued tensor $\boldsymbol{C}(\omega)$ in a probability space defined by a triplet $(\Omega, \mathcal{F}, \mathbb{P})$ in which ω is an outcome, Ω is the sample space, $\mathcal{F}$ represents the set of all possible events, and $\mathbb{P}$ is the probability measure. The proposed stochastic tensor $\boldsymbol{C}$ has to satisfy the following properties [8]:

1. the tensor has to be symmetric positive definite:

$$\boldsymbol{C}(\omega) \in \mathrm{Sym}^+(d) \quad \forall \omega \in \Omega, \tag{3.1}$$

   in which,

$$\mathrm{Sym}^+(d) := \boldsymbol{C} \in (\mathbb{R}^d \otimes \mathbb{R}^d) \mid \boldsymbol{C} = \boldsymbol{C}^T,\ \boldsymbol{z}^T\boldsymbol{C}\boldsymbol{z} > \mathbf{0},\ \forall \boldsymbol{z} \in \mathbb{R}^d \backslash \mathbf{0}, \tag{3.2}$$

2. the tensor is invariant under group transformation $G \subseteq O(\mathbb{R}^d)$:

$$\boldsymbol{R}^T\boldsymbol{C}(\omega)\boldsymbol{R} = \boldsymbol{C}(\omega) \quad \forall \boldsymbol{R} \in G, \quad \forall \omega \in \Omega; \tag{3.3}$$

3. and the mean of the stochastic tensor is invariant to a larger group transformation $G \subseteq G_m \subseteq O(\mathbb{R}^d)$:

$$\overline{\boldsymbol{C}}_\vartheta := \mathbb{E}_\vartheta(\boldsymbol{C}(\cdot)) \in \mathrm{Sym}^+(d) \text{ has to be invariant in } G_m, \tag{3.4}$$

   in which G_m represent larger group transformations such as inversion, scaling or frame transformation.

To facilitate these requirements, the model for $C(\omega)$ is adopted from ref. [8], and is based on the eigenvalue decomposition of the deterministic conductivity tensor, that is:

$$\boldsymbol{C} = \boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{Q}^T \text{ with } \boldsymbol{Q} \in \mathrm{SO}(d) \text{ and } \boldsymbol{\Lambda} \in \mathrm{Diag}^+(d). \tag{3.5}$$

Each column in $\boldsymbol{Q}$ represents an eigenvector, which signifies the orientation associated with one of the eigenvalues of the tensor. These eigenvectors are orthogonal to each other, belonging to the special orthogonal group SO(d), which is a Lie group that represents proper rotations. The diagonal matrix of $\boldsymbol{\Lambda}$ represents the magnitude or strength of the corresponding eigenvectors. They are elements of $\mathrm{Diag}^+(d) \subset \mathrm{Sym}^+(d)$, a positive definite diagonal matrix, which is a commutative Lie group under normal matrix multiplication. The split between magnitude and orientation provides a choice to either model $\boldsymbol{\Lambda}$ as $\boldsymbol{\Lambda}(\omega)$, $\boldsymbol{Q}$ as $\boldsymbol{Q}(\omega)$ or both at the same time to form a stochastic description. However, it is not trivial to directly manipulate either $\boldsymbol{\Lambda}$ or $\boldsymbol{Q}$ due to the properties of their respective Lie group. It is established that an exponential map can be used to lift a Lie algebra to a Lie Group. Working from a Lie algebra perspective is advantageous as one works with an unconstrained linear vector space, providing freedom for introducing stochastics. The Lie algebra associated with Diag^{+} is denoted as $\mathfrak{diag}(d) \subset \mathrm{Sym}(d)$, and the exponential map for diagonal matrices involves a straightforward element-wise computation. For $\mathrm{SO}(d)$, the associated Lie algebra consists of the skew-symmetric matrices $\mathfrak{so}(d)$.

As shown by ref. [8], this connection with the Lie group not only allows us to satisfy the first two requirements given in Equations (3.1) and (3.3) but also allows the definition of Riemannian structures, and thus an appropriate definition of distance hereby satisfying Equation (3.4). The complete mapping as provided by the above discussion is then given by:

$$\begin{aligned} &\mathfrak{diag}(d) \times \mathfrak{so}(d) \xrightarrow{\exp\times\exp} \mathrm{Diag}^{+}(d) \times \mathrm{SO}(d) \quad \longrightarrow \mathrm{Sym}^{+}(d), \\ &(\boldsymbol{Y}, \boldsymbol{W}) \longmapsto (\boldsymbol{\Lambda}, \boldsymbol{Q}) = (\exp \boldsymbol{Y}, \exp \boldsymbol{W}) \longmapsto \boldsymbol{C} = \boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{Q}^{T} \end{aligned} \tag{3.6}$$

where $\boldsymbol{Y} \in \mathfrak{diag}(d)$ and $\boldsymbol{W} \in \mathfrak{so}(d)$ are introduced as matrices representing the eigenvalues and eigenvectors on the corresponding Lie algebra. With the three requirements satisfied, one may model a stochastic conductivity tensor by distinguishing both scaling and rotations. For example, one may model scaling by introducing uncertainty in the magnitude of $\boldsymbol{C}$ by modeling $\boldsymbol{\Lambda}$ as uncertain in the following manner:

$$\boldsymbol{C}(\omega) = \overline{\boldsymbol{Q}}\boldsymbol{\Lambda}(\omega)\overline{\boldsymbol{Q}}^{T} = \overline{\boldsymbol{Q}} \exp\left(\boldsymbol{Y}(\omega)\right)\overline{\boldsymbol{Q}}^{T} \quad \forall \omega \in \Omega \tag{3.7}$$

where the overline indicates a component of the reference tensor. The stochastic eigenvalue matrix $\Lambda(\omega)$, according to the principle of maximum energy, is further modeled as:

$$\boldsymbol{\Lambda}(\omega) = \ \mathrm{diag}\left(\exp(y_i(\omega))\right) \quad \text{with} \quad y_i(\omega) \sim \mathcal{N}(\mu_i, \sigma_i), \quad i = 1, \dots, d \tag{3.8}$$

where the subscript i refers to the ith diagonal term of the eigenvalue matrix. Here, one immediately notices that this equates to modeling of a lognormal distribution directly on the diagonal of the matrix $\boldsymbol{\Lambda}(\omega)$. Here, for, $\mathbb{R}^3$ it is possible to define three different symmetries for realization of the second order conductivity tensor. The first is isotropy, having only one distinct eigenvalue. The second is planar isotropic symmetry with two distinct eigenvalues. Lastly, a fully orthotropic tensor can be modeled with three distinct eigenvalues. Thus, the amount of distinct eigenvalues governs the amount of required random variables.

To model the directional uncertainty in the conductivity tensor, one may introduce uncertainty in the orientation of the eigenvectors. To achieve this, the reference eigenvector $\overline{\boldsymbol{Q}}$ is rotated by the stochastic rotation matrix $\boldsymbol{R}(\omega)$ in the following manner:

$$\boldsymbol{C}(\omega) = \boldsymbol{R}(\omega)\overline{\boldsymbol{Q}}\,\overline{\boldsymbol{\Lambda}}\,\overline{\boldsymbol{Q}}^{T}\boldsymbol{R}(\omega)^{T} = \ \exp\left(\boldsymbol{W}(\omega)\right)\overline{\boldsymbol{Q}}\,\overline{\boldsymbol{\Lambda}}\,\overline{\boldsymbol{Q}}^{T} \exp\left(\boldsymbol{W}(\omega)^{T}\right) \quad \forall \omega \in \Omega. \tag{3.9}$$

The stochastic eigenvector matrix $\boldsymbol{Q}(\omega)$ is modeled as a transformation from Euler vector $\boldsymbol{w}$ along $\boldsymbol{w}/||\boldsymbol{w}||$ by the Rodrigues' rotation formula:

$$\boldsymbol{R} = \exp(\boldsymbol{W}) \text{ where } \boldsymbol{W} = \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix} \quad \text{with} \quad \phi = ||\boldsymbol{w}|| \tag{3.10}$$

with ϕ being the rotation angle of $\boldsymbol{R}$. The Euler vector or random angle variables $\boldsymbol{w}$ are drawn from a von Mises Fisher distribution (vMF). vMF is a generalization of a Gaussian distribution to the unit sphere, and also adheres to the principle of maximum entropy. In $\mathbb{R}^3$, there are two possible symmetries given random orientation, the first is planar isotropic symmetric, realized with two random angle variables $\boldsymbol{w} = [w_1\ w_2]$ and the second is orthotropic symmetry which is realized with three distinct random angle variables $\boldsymbol{w} = [w_1\ w_2\ w_3]$. Finally, when there is both uncertainty in the magnitude and orientation, one may model the stochastic tensor by:

$$\boldsymbol{C}(\omega) = \ \exp\left(\boldsymbol{W}(\omega)\right)\overline{\boldsymbol{Q}} \exp\left(\boldsymbol{Y}(\omega)\right)\overline{\boldsymbol{Q}}^{T} \exp\left(\boldsymbol{W}(\omega)^{T}\right) \quad \forall \omega \in \Omega. \tag{3.11}$$

TABLE 1 Deterministic process parameters where $\mathbf{k}_1$ is the thermal conductivity along the fiber, $\mathbf{k}_2$ is transverse to the fiber, and $\mathbf{k}_3$ is in the out-of-plane direction.

Parameter	Value (unit)	Parameter	Value (unit)
$\mathbf{J}_s$	200 (A)	c_p	946 (J/[kg*K])
ω^f	324 (kHz)	ρ	1601 (kg/m^3)
μ^m	1 (N/m^2)	$\mathbf{k}_1$	6.4 (W/[m*K])
L × W × H	300 × 300 × 0.14 (mm)	$\mathbf{k}_2$	0.81 (W/[m*K])
Layup	$(-45,45)_s$	$\mathbf{k}_3$	0.74 (W/[m*K])
		t_e	10 (s)

TABLE 2 Reference tensor $\overline{\boldsymbol{C}}$ with mean and standard deviation, directions are the same as that of the thermal conductivity [4].

Parameter	μ_T (S/m)	σ_T (S/m)
c_1	38 200	1600
c_2	12.86	2.2
c_3	1.99	0.5

$$\overline{\boldsymbol{C}} = \begin{bmatrix} \bar{c}_1 & 0 & 0 \\ 0 & \bar{c}_2 & 0 \\ 0 & 0 & \bar{c}_3 \end{bmatrix}$$

4 | STOCHASTIC MODEL FOR THE INDUCTION WELDING OF CFRTP

Introducing the previous formulation of stochastic conductivity tensor to Equation (2.7) one obtains:

$$\nabla \times \frac{1}{\mu^m} \nabla \times \mathbf{A}(\boldsymbol{x}, \omega) = \mathbf{J}_s - j\omega^f \boldsymbol{C}(\omega)\mathbf{A}(\boldsymbol{x}, \omega) \text{ with b.c. a.e. on } \Omega \times \mathcal{G} \tag{4.1}$$

where the magnetic vector potential $\mathbf{A}$ is now expressed as a function of both spatial coordinates $\boldsymbol{x}$ and the outcome ω. Similarly, the heat transfer model given in Equation (2.8) is reformulated to:

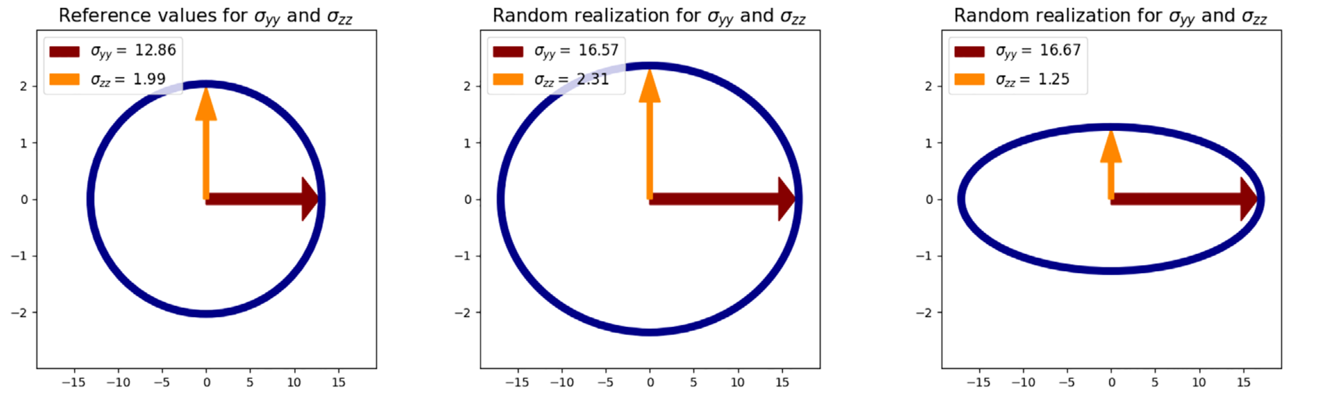
$$\rho c_p \frac{\partial T(\boldsymbol{x}, t, \omega)}{\partial t} = \nabla \cdot (\mathbf{k}\nabla T(\boldsymbol{x}, t, \omega)) + Q_e(\boldsymbol{x}, \omega) \text{ with b.c. a.e. on } \Omega \times \mathcal{G}_C \times \mathcal{T} \tag{4.2}$$

where the heat source Q_e and consequently the temperature T is now a function of ω. Both reformulations also affect the solution procedure due to stochastic dependence. As we are only interested in the statistics of the quantity of interest such as $\mathbf{A}$, Q_e, and T, an integration method is further employed. Namely, we estimate the mean value by:

$$\mathbb{E}(T(x, t, \omega)) = \int_{\Omega} T(x, t, \omega) d\mathbb{P}(\omega) \approx \sum_{i=1}^{N} T(x, t, \omega_i) p_i \tag{4.3}$$

in which $T(x, t, \omega_i)$ is solved by FEM in the integration point corresponding to the tensor sample ω_i and affiliated weight p_i. The integration over the stochastic domain is done with the univariate rule of third order Genz-Keister 24 quadrature and further reduced by Smolyak sparse integration [11].

The numerical results are further computed for the model described by parameters shown in Table 1. The process parameters, which are assumed to be deterministic, for induction welding are the coil current, frequency, permeability, thermal properties, and dimensions of the CFRTP plate. Finally, Table 2 shows the stochastic electrical conductivity, the reference tensor $\boldsymbol{C} \in \mathrm{Sym}^+(d)$ is displayed alongside its mean and standard deviation, where the statistical values are given in the fiber direction, transverse to the fiber and out of plane direction. $\boldsymbol{C}(\omega)$ is modeled with Equation (3.7), only looking at variation in the magnitude. Each of the three diagonal elements in $\boldsymbol{Y}(\omega)$ is modeled as a distinct eigenvalue, requiring three Gaussian variables. To obtain μ_i and σ_i, we transform the mean μ_T and standard deviation σ_T of the reference tensor

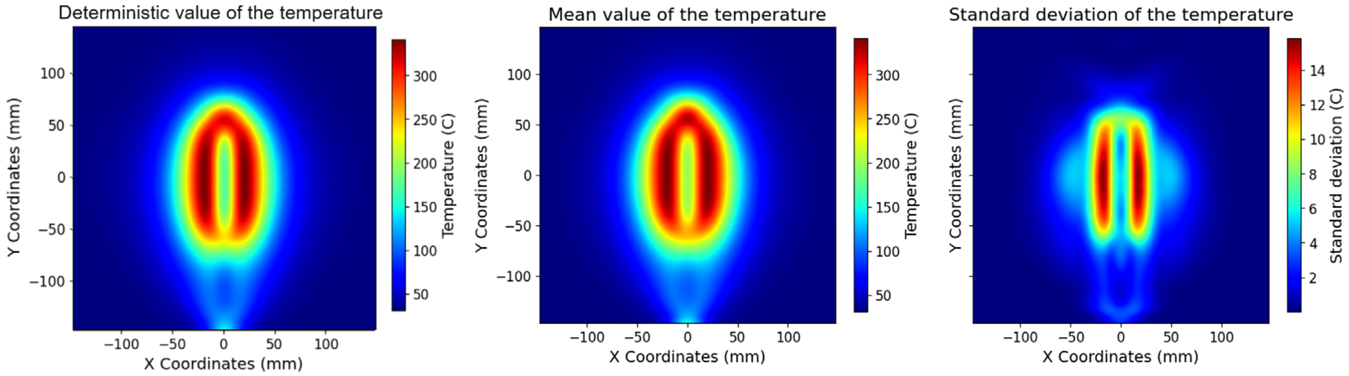


(A) Visualization of the reference tensor in YZ plane.

(B) Random realization of the tensor in YZ plane.

(C) Random realization of the tensor in YZ plane.

FIGURE 3 Visualization of the reference tensor and two random realizations.



(A) 2D Temperature field of a deterministic simulation with Eq. (2.7) and Eq. (2.8).

(B) 2D Temperature field of Eq. (4.1) and Eq. (4.2).

FIGURE 4 Deterministic and stochastic temperature field after 10 s of heating.

on $\mathrm{Sym}^+(d)$ to the mean and standard deviation of $\boldsymbol{Y}(\omega)$ on the Lie algebra by:

$$\mu_i = \ln\left(\frac{\mu_T^2}{\sqrt{\mu_T^2 + \sigma_T^2}}\right), \quad \sigma_i^2 = \ln\left(1 + \frac{\sigma_T^2}{\mu_T^2}\right). \tag{4.4}$$

A 2D visualization of the eigenvalues in the transverse and out of plane directions is presented in Figure 3. The leftmost figure depicts the reference tensor $\overline{\boldsymbol{C}}$ with the mean eigenvalues. Subsequently, Figure 3B,C display two random realizations of the tensor. These illustrate the potential range of electrical conductivity values across different material samples.

The weakly coupled electromagnetism and heat transfer equations are solved by COMSOL Multiphysics [12], a commercial finite element software. The temperature for the purely defined deterministic simulation, where only the mean electrical conductivity is used, is visible in Figure 4A. Here, the coil geometry dictates the heating pattern, generating most of the heat at the shortest distance from the coil. The stochastic result is obtained with 147 samples from the FEM solver using the aforementioned sparse grid Genz-Keister integration implemented with the chaospy library in Python [13]. The mean and standard deviation are plotted in Figure 4B. The mean, as expected, follows a similar pattern as the deterministic result. The standard deviation shows a less obvious result, depicting most uncertainty being present along the long edges of the coil. Further, a very high uncertainty in the temperature field is observed.

5 | DISCUSSION

The stochastic tensor model presented in this paper proposes the mathematical way of including randomness in the magnitude of the electrical conductivity tensor, as is visible in Equation (3.11) and Figure 3. The inclusion of stochastic electrical conductivity affects the temperature field for induction welding, as shown by Equations (4.1) and (4.2). To showcase the effect on the output of a simulation, a deterministic and stochastic simulation result are shown next to each other in Figure 4. By adopting a stochastic framework, one may further investigate the influence of uncertainty in the induction welding process of CFRTP.

The stochastic tensor model defines a parametric probability distribution in $\mathrm{Sym}^+(d)$. The application is not limited to electrical conductivity and can be directly employed to model the thermal conductivity in the same process model. A severe limitation is that the stochastic tensor is assumed to be homogeneous throughout a single specimen, which is often untrue. Just as material properties vary from specimen to specimen, they also vary from location to location within a specimen. Future work will thus focus on extending the homogeneous formulation and providing a description for heterogeneous materials to obtain a stochastic tensor field that accurately reflects the variability across different regions of the material.

For the induction welding process, the extra information obtained by not only generating a mean and standard deviation, but also a probability distribution for the interface temperature can be very valuable for optimization and decision making. However, the sparse grid method is limited to linear problems and does not provide a direct way to obtain a probability density function. Monte Carlo simulations should be performed to not only estimate the density but also to allow a nonlinear description of temperature dependent material parameters, which are apparent in CFRTP. To reduce the computational load of Monte Carlo simulations, a surrogate model that approximates the induction welding process will be created.

Understanding and characterizing stochastic variations is important for predicting the performance and behavior of composite materials in processing and manufacturing applications. The stochastic technique presented in this work offers a model which takes into account the relevant symmetries and allows for stochastic strength and orientation with respect to the reference values. This method should not only be applicable to the induction welding of CFRTP but can be used to investigate the influence of uncertainty in a plethora of engineering applications.

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