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Role of analogies with classical physics in introductory quantum physics teaching

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In this study, we explore the use of analogies with classical physics in introductory quantum physics (QP) teaching. This work is part of a larger project in which a learning sequence covering three introductory QP topics was designed, tested, and evaluated in collaboration with high school teachers and physics education researchers. Based on the theoretical foundations of guided inquiry-based learning and collaborative learning, three lessons covering the topics of photoelectric effect, wave-particle duality, and tunneling were developed. For each lesson, a digital inquiry learning space (ILS) was created which implemented a digital laboratory for performing experiments with specific interactive assignments and multimedia material. For this study, each ILS was adapted into two distinct versions, to be applied with students in two experimental conditions: one in which analogies with classical physics were present in the lessons, and one in which such analogies were avoided. Six classes of Dutch upper-secondary students ($n = 89$) were assigned to one of the conditions and followed the respective version of the learning sequence during a period of approximately 6 weeks. Students' answers to ILS activities were logged and used for a qualitative analysis. Once the learning sequence was finished, students took a post-test covering all three QP topics and scores were compared between the analogy and no analogy conditions. After correcting for students' overall performance in physics, we found that students in the no analogy condition scored significantly higher on the post-test than the analogy condition for two out of the three QP topics (wave-particle duality and tunneling). An accompanying qualitative analysis suggests that the use of analogies with classical physics might hinder QP learning, as some students manifested difficulties in grasping the limitations of the classical analogies that were used in the materials.

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I. INTRODUCTION

Introducing new concepts to students by making references to already known ones is an acknowledged instructional tactic which is also often used in physics teaching [1]. Treagust *et al.* [2] assert that analogies concern "… a relation between parts of structures of two conceptual domains…" (p. 413). Well-known examples of analogies used in physics teaching are water flowing in pipes as an analogy to electricity flow and water wave patterns as an illustration of interference of electromagnetic waves. By using analogies, a link is made between something students already know and what they need to learn.

On the one hand, there are several studies that show the advantages of using analogies, for science, technology, engineering, and mathematics (STEM) subjects in general and physics in particular. For example, Podolefsky and Finkelstein [3] conducted a study in which students who learned about electromagnetic waves using analogies with oscillating strings and sound waves were compared to students who learned the same topic without using these analogies. The students using analogies made a greater shift by giving a fully correct answer to a deep conceptual question than students who were not exposed to analogies. Ugur *et al.* [4] had students learn electricity concepts with the help of different analogies (trains and water circuits), which were either demonstrated in a physical way or presented on a blackboard. Compared to students who followed the same lessons without analogies, the analogy group outperformed the no

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analogy group at a post-test measuring conceptual knowledge about electricity.

On the other hand, some studies found that analogies may not work as expected. For example, analogies may cause too much cognitive processing which may be especially true for weaker students [5]. Another problem may be that students fail to see the analogy between situations. Lin and Singh [6], for example, provided students with analogical problems about tension in a rope and frictional force and asked them to indicate the analogies between the two problems. Only 35% of the students were able to do this. In the field of mathematics, Richland *et al.* [7] analyzed the way teachers used analogies in the classroom and made a comparison between the U.S. and high-achieving Asian regions. Their analysis revealed that in the Asian regions, students received more cognitive support (hints, prompts, and elaborations) than the U.S. students, which made active reasoning by the latter group less likely to happen. Their study also emphasized that the source (analogy) must be familiar to students in order to play a fruitful role in learning. Learning by analogy can also be hindered by superficial characteristics of the analogy. Therefore, Kurtz *et al.* [8] recommend supporting students in making (jointly) deep comparisons between the two structures involved in the analogy.

The controversy we have sketched here regarding analogies in STEM teaching is even stronger when it concerns quantum physics (QP) instruction. That is because when learning QP, students are exposed to a new, probabilistic way of thinking, which clashes with the deterministic reasoning from classical physics that they are familiar with. This deterministic reasoning is precisely what challenges the use of analogies in QP instruction, because while QP teaching exposes students to a probabilistic view of physics, the analogies available to teach QP use classical physics concepts.

Therefore, the aim of this study is to explore the role of analogies with classical physics in introductory QP teaching. The introductory part of this article focuses on detailing relevant literature findings about the use of such analogies in QP teaching, as well as describing the learning materials used in this study and how they were adapted into two different conditions (with and without analogies). Then, the research questions and methodology of the study are described, followed by a quantitative and qualitative analysis of the findings. Finally, such findings are discussed and possibilities for future research are proposed.

A. Analogies in quantum physics teaching

Even though analogies have been successfully used in physics teaching, their role in QP instruction is still being debated. More specifically, researchers diverge about whether making analogies with classical physics phenomena is beneficial to learners when explaining QP topics. A well-known example of analogy between classical and quantum physics consists of comparing the quantum tunneling phenomenon to a classical ball having to overcome a physical barrier, such as a wall. In this analogy, the phenomenon of quantum tunneling is compared to an impossible scenario in which the ball penetrates the barrier and is found on the other side, as if there was a tunnel inside the barrier. Still concerning tunneling, Brookes and Etkina [9] identified that physicists employ different analogies (or metaphors) to reason about the phenomenon in a productive way. In these metaphors, classical elements are present, such as the definition of a “perfectly rigid box, represented by a rectangular potential well with infinitely high walls” (p. 8) to treat the potential energy. Using these analogies, physicists also used deterministic verbs such as contain, trap, bounce, and reflect. However, when turning to how students use analogies, Brookes and Etkina [9] found that students show misconceptions which seem to be derived from the very metaphors used during instruction: “The metaphorical language, grounded in the classical world, may encourage students to associate extra classical properties with the QM system. These overextensions of the representation seem to be the source of their difficulties.” (p. 13).

In this line of reasoning, Greca and Freire [10] propose not to use analogies when teaching QP when they write: “… We have chosen a didactic strategy that privileges the phenomenological-conceptual approach, with emphasis upon quantum features of the systems, instead of searching for classical analogies...” (p. 541). Instead, these authors chose to confront students with QP phenomena right from the start and obtained favorable learning outcomes. Contrary to that, Baily and Finkelstein [11] wrote in favor of drawing analogies with classical physics, as long as students are aware of the limitations of such analogies. Their study also reported increased understanding from students at the end of instruction. While many studies [10,12–14] have actively used or avoided classical analogies in their instructional approaches and provided insight about such decision, none has focused on the specific role of analogies in students’ reasoning with QP. Considering the arguments pro and con the use of analogies in teaching quantum physics, we designed an experiment to compare both approaches.

B. Analogies in our learning materials

The current study is part of a larger project in which a learning sequence about introductory QP was designed, implemented, and evaluated with Dutch high school students [15]. This learning sequence is designed to be used in upper secondary level classrooms and is made up of three units, covering the photoelectric effect, wave-particle duality, and tunneling topics. Each of these units takes approximately two 50-min lessons to be completed and is predominantly student centered, with a teacher-led discussion taking place at the end of each unit. The student-centered portion of a unit consists of a collaborative inquiry-based investigation, designed to be taken by students in pairs. This investigation is structured in an online module using the Go-Lab ecosystem [16]. Such online modules are also known as inquiry learning spaces or ILSs [17]. In this section, we provide an overview of the different theoretical frameworks used in our design and a brief description of the structure of the ILSs used in this study, as well as the differences between the ILSs with and without analogies.

The student-centered investigations in the ILSs are based on the principles of collaborative and guided inquiry-based learning. In guided inquiry-based learning, students build knowledge by investigating a phenomenon using principles of scientific inquiry—that is, by formulating hypotheses, testing them, drawing conclusions, and sharing their findings and ideas with each other. This translates to our ILSs' structure comprising five phases: orientation, conceptualization, investigation, conclusion, and discussion, resulting in what is called the inquiry cycle [18]. The word "cycle" implies that inquiry-based lessons should allow students to go back and forth between the phases of scientific inquiry. For example, a student should be able to revisit their hypotheses after conducting an experiment or after interpreting data. In the orientation phase, students activate the necessary prior knowledge through multiple-choice questions with immediate feedback and are introduced to the lesson's topic. The conceptualization phase prompts students to write hypotheses about the phenomenon at hand and provides examples of well-formulated, measurable hypotheses. The investigation phase guides students to test their hypotheses using virtual labs that simulate the respective QP phenomenon. The virtual labs used in this study were designed by our research team and were based on the QP simulations developed by the PhET group [19]. During the investigation phase, students are supported by investigation assignments, displays of collected data in graphs and tables when applicable, and a field to add written observations while the virtual labs are explored. Then, students move on to the conclusion phase, where they first answer questions about possible conclusions and receive immediate feedback on their correctness. If answered incorrectly, the feedback will instruct the student to return to the virtual lab and repeat the respective investigation phase.

Finally, the discussion portion of the ILS is an "ongoing process" (p. 57) [18] and is therefore present throughout the whole ILS in the form of collaborative learning. Collaboration between students was promoted through computer-supported collaborative learning (CSCL) tools and strategies [20]. In practice, students access the ILSs individually in their own laptops and communicate with their partners using a chat tool, which is part of the digital environment of the ILSs. Furthermore, the virtual labs allow students to see their pair's cursor and its interactions with the simulation. That means that pairs share the same virtual lab environment but can enter their own answers individually in other activities of the ILSs. Regarding CSCL strategies to support collaboration [21], our ILSs use scripts, which are a set of instructions on how students should collaborate with each other [22].

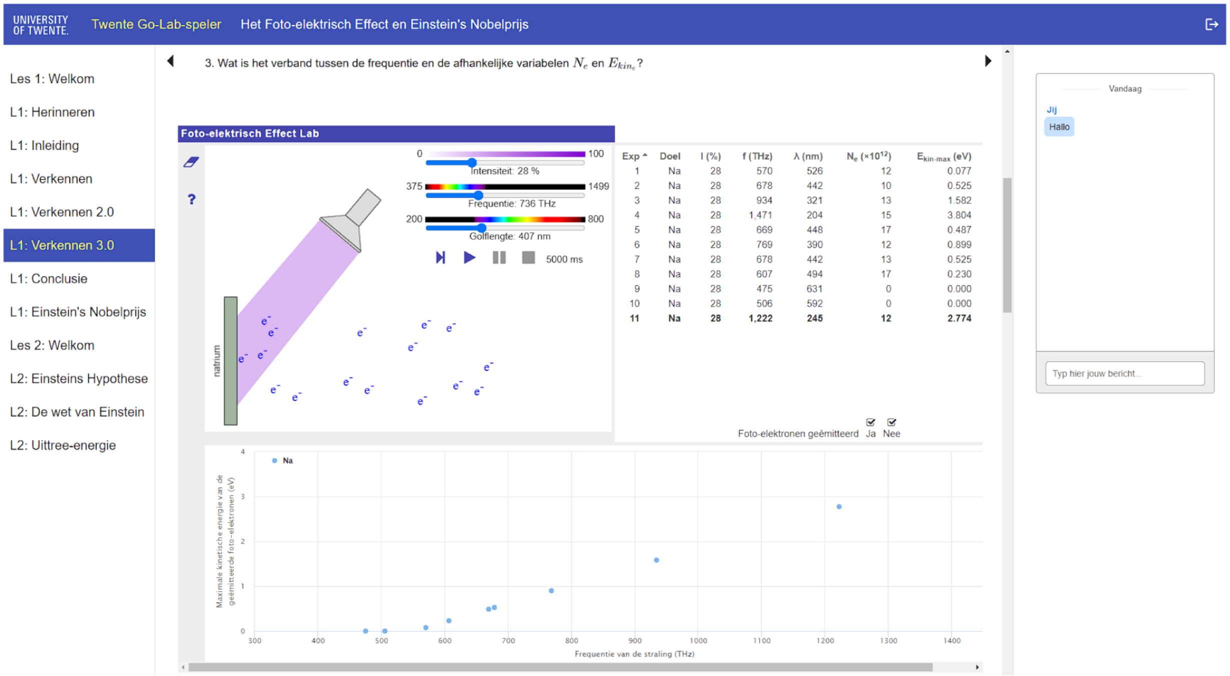
In the following sections, we detail the contents of each ILS, based on the phases of the inquiry cycle, and specify the differences between the ILSs with and without analogies.

1. Photoelectric effect unit (ILS 1)

The photoelectric effect unit is the first part of our learning sequence. The learning goals for this unit were based on the [19] work of McKagan *et al.* and state that, by the end of the lesson, students should be able to (a) list the circumstances under which the photoelectric effect is observed (that is, when the frequency of light is above a certain cutoff value, assuming an intensity greater than zero); and (b) explain why the photoelectric effect is evidence that light is quantized, using the concept of photons or light quanta.

Regarding the ILS developed for this unit, Fig. 1 displays a screenshot of the digital learning environment as experienced by students. The ILS begins by recalling prior knowledge about electromagnetic waves and by introducing the photoelectric effect in a broad and descriptive way, without further details on how or why the effect takes place. These specific details and conditions are the aim of a series of investigations which follow this introductory part of the ILS. To conduct such investigations, students use a virtual lab, displayed in Fig. 1.

In addition, the ILS implements a series of scaffolding tools and provides guidance to students on how to explore the variables involved in the photoelectric effect. After the investigations are completed, students are introduced to the concept of a quantum of light as a possible explanation for the phenomenon and develop this concept further with exercises and an explanatory video. Finally, regarding the use of analogies in this ILS, Table I details the various instances in which the ILSs differ between conditions. More specifically, such differences are present in the ILS's electron representation, description of the photon-electron interaction, and the explanation of Einstein's law. Regarding the representation of electrons in the virtual lab, electrons are depicted as spheres in the analogy condition, similar to PhET's virtual lab [19]. In the no analogy condition, we opted to represent electrons with the symbol e^-, in an attempt to avoid the link between the particle and a macroscopic object. Concerning the description of the interaction between photons and electrons, the analogy condition compares such interaction with a collision. More specifically, the photon is described as if it collides with an electron, transferring energy that could result in the ejection of the electron. In the no analogy condition, however, no such comparison is made, and the



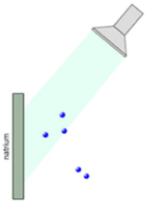
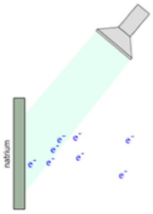
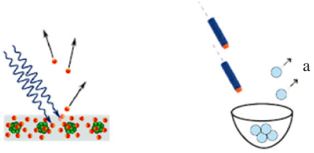
3. Wat is het verband tussen de frequentie en de afhankelijke variabelen N_e en E_{kin_e}?

Exp	Doel	I (%)	f (THz)	λ (nm)	N_e ($\times 10^{12}$)	$E_{\text{kin-max}}$ (eV)
1	Na	28	570	526	12	0.077
2	Na	28	678	442	10	0.525
3	Na	28	934	321	13	1.582
4	Na	28	1,471	204	15	3.804
5	Na	28	669	448	17	0.487
6	Na	28	769	390	12	0.899
7	Na	28	678	442	13	0.525
8	Na	28	607	494	17	0.230
9	Na	28	475	631	0	0.000
10	Na	28	506	592	0	0.000
11	**Na**	**28**	**1,222**	**245**	**12**	**2.774**

Foto-elektronen geémitteerd ☑ Ja ☑ Nee

FIG. 1. ILS 1 interface with virtual lab as experienced by a student.

TABLE I. Differences between analogy and no-analogy conditions in ILS 1 (photoelectric effect). [a]Image on the left retrieved from [23].

Topic	Analogy	No analogy
Representation of electrons	Electrons as spheres (such as in PhET).  	Electrons represented as a letter. 
Interaction between photons and electrons	Comparison with collision: “Think as if each photon collides with an electron, exchanging energy. If that energy is high enough, the electron will be emitted.”	No comparison with collision, only mention of an interaction between photon and electron.
Explanation of Einstein’s law	Ping pong balls and bowl analogy, as proposed by Choudhary *et al.* [14]. Excerpt from the ILS and illustration: “Imagine the electrons as ping pong balls in a deep bowl (the metal). You can shoot these electrons with soft rubber bullets, which represent the photons. A bullet comes with energy (hf) which is transferred to a ping pong ball in the form of energy to exit the bowl (W) and kinetic energy once it exits the bowl (K_{max}).”  	Description of Einstein’s law as a law of conservation of energy, where the energy provided by the photon is converted into energy for the electron to leave the metal and gain speed.

exchange between photons and electrons is simply described as an interaction. Finally, regarding Einstein's law of the photoelectric effect, the analogy condition illustrates the relationship between the different energies with an analogy involving ping pong balls and a bowl, based on the work of Choudhary *et al.* [14]. In this analogy, electrons are represented by ping pong balls in a deep bowl, and photons are represented as soft rubber bullets, which are shot and hit the ping pong balls. The energy from each bullet is analogous to the photon's energy, which is converted into energy for the ball to escape the bowl (representing the work function) and kinetic energy. Conversely, the no analogy condition presents Einstein's law directly as an energy conservation law, without a concrete analogy.

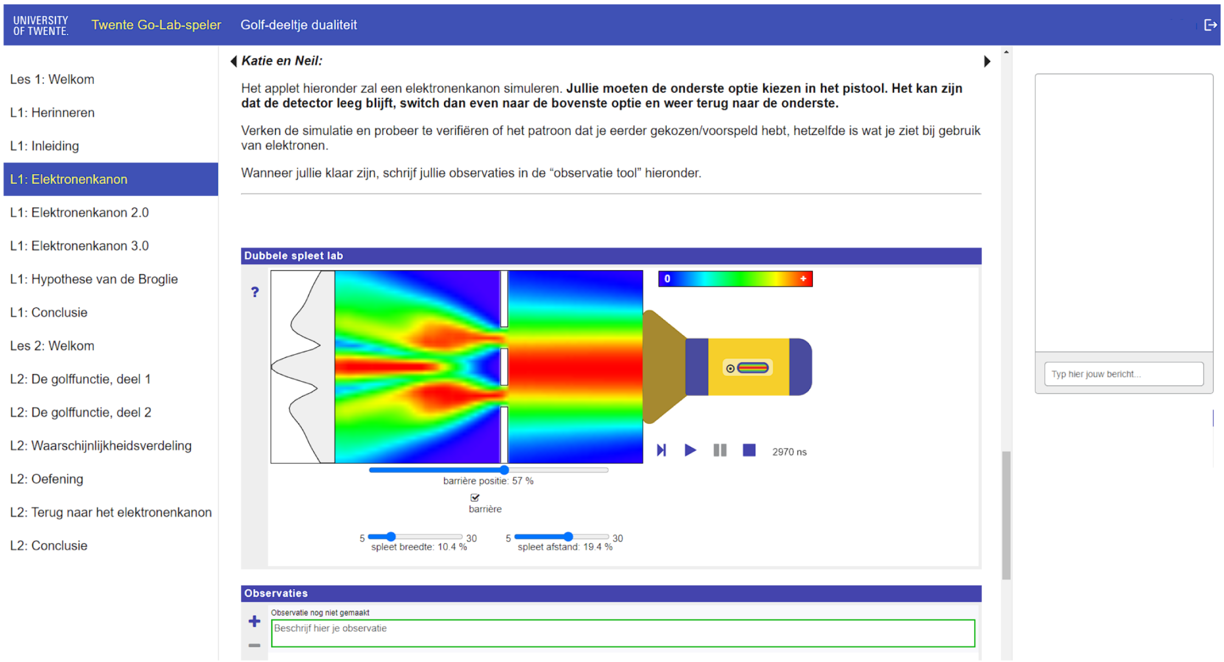
2. *Wave-particle duality unit (ILS 2)*

In the wave-particle duality unit, students investigate the dual behavior of quantum entities and are introduced to the concept of probability distribution. The unit's learning goals state that students should be able to (a) apply the wave-particle duality concept to explain the display of wave phenomena with particles; (b) make calculations with the de Broglie wavelength and explain it; and (c) explain the concept of probability distribution.

The second ILS is structured as follows: After answering prior knowledge questions, students are introduced to the concept of wave-particle duality through a comparison between the different kinds of light behavior. Students are also introduced to the concepts of locality and nonlocality in physics and how they are intrinsically related to the distinct behavior of particles and waves. Then, the question of whether the wave-particle duality can be observed with other entities as well is proposed to students. Such a question is the focus of the ILS's first guided investigation, where students use a virtual lab to perform a double-slit experiment with a beam of electrons and draw preliminary conclusions. Figure 2 displays a screenshot of the learning environment including the virtual lab. Next, a different setup of the double-slit experiment is proposed, now with electrons being shot one at a time. Students are asked to hypothesize which kind of pattern they expect to see in the detector but do not test these predictions in the virtual lab this time. Rather, they are shown a video of a similar experiment conducted by Tonomura *et al.* [24], which shows how an interference pattern builds up out of single electrons being shot at a biprism. Students are also asked to speculate on a possible explanation for the results shown in the video.

This first investigative phase of the lesson is followed by the presentation of new concepts through videos, images, text, and knowledge-building exercises. These concepts are the de Broglie wavelength, wave function, and probability distribution. At the end of the lesson, students perform one more guided investigation in the virtual lab, this time conducting the double-slit experiment with individual electrons. The ILS ends with wrap-up questions with feedback covering all the concepts introduced in the lesson.

Regarding the use of analogies in ILS 2, Table II details the differences between the analogy and no analogy conditions for this ILS. Such differences are found in (a) how the trajectory of an electron is represented; (b) the use of classical patterns to provide examples for the outcomes



Katie en Neil:

Het applet hieronder zal een elektronenkanon simuleren. **Jullie moeten de onderste optie kiezen in het pistool. Het kan zijn dat de detector leeg blijft, switch dan even naar de bovenste optie en weer terug naar de onderste.**

Verken de simulatie en probeer te verifiëren of het patroon dat je eerder gekozen/voorspeld hebt, hetzelfde is wat je ziet bij gebruik van elektronen.

Wanneer jullie klaar zijn, schrijf jullie observaties in de "observatie tool" hieronder.

2970 ns

barrière positie: 57 %

☑ barrière

5 30 spleet breedte: 10.4 %

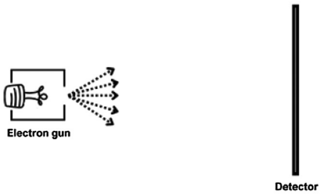
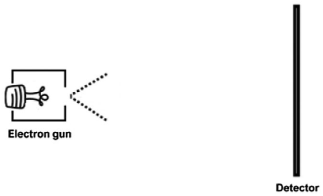
5 30 spleet afstand: 19.4 %

Observatie nog niet gemaakt

Beschrijf hier je observatie

FIG. 2. ILS 2 interface with virtual lab as experienced by a student.

TABLE II. Differences between analogy and no-analogy conditions in ILS 2 (wave-particle duality).

Topic	Analogy	No analogy
Representation of trajectory	Illustration for thought experiment with electron gun [25] has arrows, in analogy to a classical particle.  	Illustration for thought experiment with electron gun is less suggestive of a definite trajectory. 
Examples of patterns	When asking students to hypothesize about the pattern they expect to see in a double-slit experiment with electrons, we add the following: “For example, do you think the pattern would be more comparable to the one seen when shooting bullets through the slits, or when sending water waves through the slits?”	No extra example or prompt is added to the hypothesis activity.
Word choice when talking about single-electron experiment	When explaining a double-slit experiment with single electrons (i.e., sent individually), it is explained that such experiment was done to rule out the possibility of electrons colliding with each other.	The word “colliding” is replaced by “interacting.”
Example of de Broglie wavelength	When explaining about the de Broglie wavelength, we use the example that a billiard ball with speed v has an associated de Broglie wavelength.	No mention of a classical particle having a de Broglie wavelength in the explanatory text about the topic.

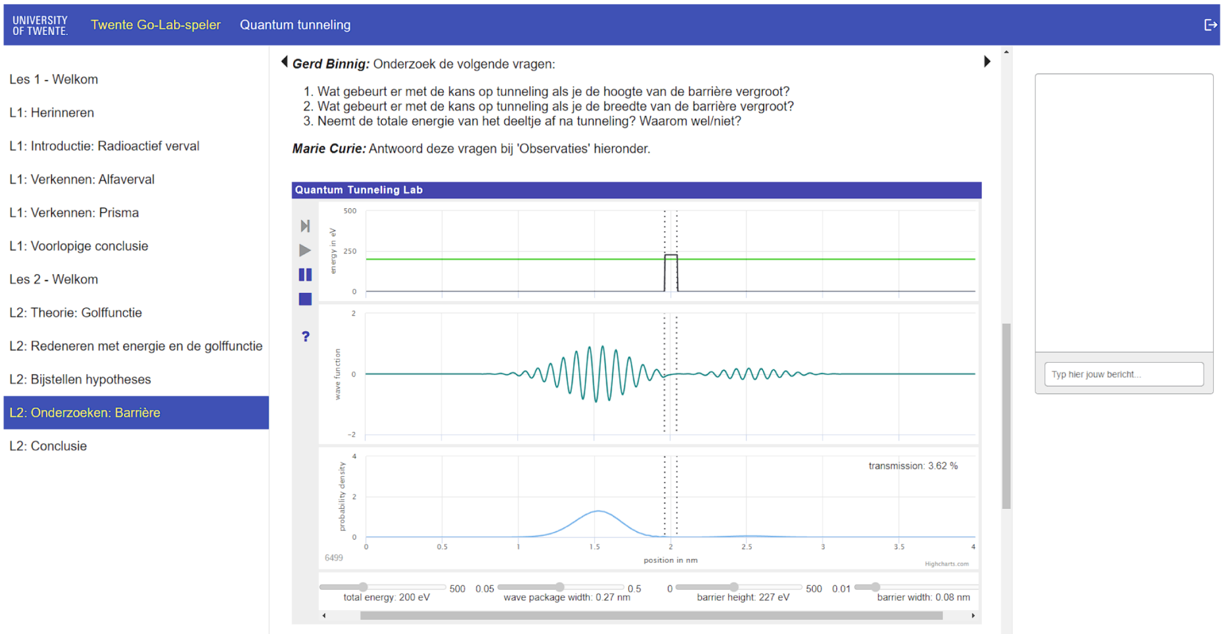
of the double-slit experiment; (c) the choice of terminology between ILSs; and (d) the explanation of the de Broglie wavelength. First, when proposing the thought experiment with an electron gun, the illustration of hypothetical trajectories of electrons leaving the electron gun is illustrated with multiple arrows in the analogy condition. In the no analogy condition, the illustration is less suggestive of a definitive trajectory. Second, when guiding students to hypothesize about the expected pattern in a double-slit experiment with electrons, the analogy condition provides examples to illustrate possible outcomes of the experiment. More specifically, students are asked if the resulting pattern would resemble bullets or water waves passing through slits. However, this comparison with classical patterns is omitted in the no analogy condition. Third, when proposing the double-slit experiment with electrons being shot individually, each condition employs different words. In the analogy condition, the experiment is proposed to rule out the “collision” between electrons, while in the no analogy condition, the word “interaction” is used instead. At last, to introduce the concept of de Broglie wavelength, the analogy condition includes the example of a billiard ball having an associated de Broglie wavelength. In contrast, the no analogy condition does not provide such example.

3. Tunneling unit (ILS 3)

In the tunneling unit, students apply the newly learned concepts of wave function and probability distribution to investigate the behavior of a quantum object when faced with a finite potential energy barrier. The learning goals for this unit state that students should be able to (a) describe the tunneling effect using a simplified model of a particle’s wave function and a potential energy barrier and (b) correctly relate the height and width of the potential barrier with the particle’s probability of tunneling.

Students start ILS 3 by first answering questions to recall the necessary prior knowledge. Subsequently, the introductory part of ILS 3 provides students with a real-life example in which tunneling takes place: alpha decay. Such an example is used in the first part of the lesson to (a) introduce the roles that probability and chance have in radioactive decay; (b) present and explain the graphic representation of the energies involved in the decay process; and (c) relate this graphic representation to prior knowledge of potential wells. In this ILS’s first guided investigation, students use PhET’s “alpha decay” virtual lab to explore such energies and how they influence the probability of decay. As an additional real-life example of tunneling, the phenomenon of frustrated total internal reflection is also presented to students.

Before further exploring the energies involved in the tunneling process, the ILS provides a review of the theory about wave function and potential wells. Then, students are presented with new content about how the wave function behaves in a finite potential well and in a potential energy barrier. Thereafter, students conduct another guided investigation, focusing on how a potential energy barrier



Gerd Binnig: Onderzoek de volgende vragen:

1. Wat gebeurt er met de kans op tunneling als je de hoogte van de barrière vergroot?
2. Wat gebeurt er met de kans op tunneling als je de breedte van de barrière vergroot?
3. Neemt de totale energie van het deeltje af na tunneling? Waarom wel/niet?

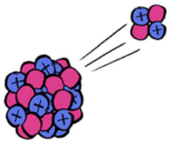
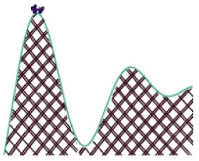
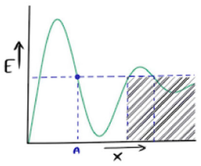
Marie Curie: Antwoord deze vragen bij 'Observaties' hieronder.

total energy: 200 eV
wave package width: 0.27 nm
barrier height: 227 eV
barrier width: 0.08 nm

FIG. 3. ILS 3 interface with virtual lab as experienced by a student.

influences the probability of tunneling. This investigation is conducted in a virtual lab made by our research group (see Fig. 3). The concluding part of the ILS poses wrap-up questions to assess students' conclusions about this final investigation. The lesson ends with a summary of tunneling and its related concepts. The differences between ILS 3 for each condition (analogy and no analogy) are detailed in Table III. Both conditions differ in their representation of radioactive decay, the use of the "particle in a box" example, and the use of a roller coaster analogy. Regarding the representation of radioactive decay, in the analogy condition, an original illustration of alpha decay used in the ILS's pilot was kept, providing a visual representation of the decay process. In the no analogy condition, however,

TABLE III. Differences between analogy and no-analogy conditions in ILS 3 (tunneling).

	Analogy	No analogy
Representation of radioactive decay	An original illustration from the ILS's pilot version was kept in the explanatory part about alpha decay. 	The illustration was replaced with a radioactive decay equation. $U_{92}^{238} \rightarrow Th_{90}^{234} + He_2^4$
Use of particle in a box analogy.	When explaining about potential wells and the behavior of particles in them, we use the well-known analogy of a particle in a box.	No mention to the particle in a box analogy is done, we simply refer to the potential and total energies without comparisons.
Explanation of total and potential energy	When explaining the relationship between potential and total energy in alpha decay, a comparison with a roller coaster is done to illustrate the conservation of energy.  	No comparison is done.

this illustration is replaced by a radioactive decay equation. Concerning the particle in a box example, the analogy condition employs such example when treating the topic of potential wells and how particles behave in them. Conversely, the no analogy condition omits the particle in a box example. It is important to point out that the particle in a box model is part of the Dutch curriculum and was taught to students outside of our experiment. Finally, when discussing the relationship between energies involved in the alpha decay example, the analogy condition includes a roller coaster analogy, comparing changes in total and potential energy in alpha decay to these same energies in a roller coaster. The no analogy condition explains the relationship between total and potential energy without such analogy, focusing on the concept of conservation of energy only.

C. Research questions

Considering whether the use of analogies would facilitate or hinder the understanding of QP topics introduced in each of the units, we investigated the use of two versions of the ILSs described above, looking for differences in test results as well as for clues in student answers and argumentation. Therefore, the main research question guiding our study is the following: To what extent do analogies with classical physics influence students' reasoning with introductory QP topics? This overall question can be further specified as follows: To what extent do analogies with classical physics influence students': (a) post-test scores and (b) written answers to ILS activities?

II. METHODOLOGY

A. Research design

This study used a mixed-methods, between-group design. All participants were taught three QP topics through our learning sequence and were divided between two conditions: analogy and no analogy. The main differences between the learning sequences used by each condition are detailed in Sec. I B. Apart from these changes, the learning sequences were the same and followed the aforementioned inquiry-based, collaborative structure, with student- and teacher-centered parts, and three units covering the photoelectric effect, wave-particle duality, and tunneling topics, respectively. In the student-centered part of a unit, students took the unit's ILSs in pairs, in a collaborative setting. In the teacher-centered part, which took place after all students had finished the ILS, a whole-class discussion was conducted by the teacher. Once a classroom completed all units of the learning sequence, a post-test was applied.

Students' post-test scores were compared between the analogy and no analogy conditions. In addition, an in-depth qualitative analysis of students' written input in specific ILS activities was performed, to further understand eventual differences between the conditions.

B. Participants

Six classes from four different teachers participated in this study. Three out of these six classes were from the same school and were assigned to the same condition (analogy), which was chosen randomly. The six classes had a total of 112 students. Out of these 112 students, 89 took the post-test (41 males, 37 females, and 11 who chose not to say). These students were aged between 16 and 19 years old ($M = 17,2$, $\mathrm{SD} = 0.6$) and were following preuniversity science classes. Out of the 89 participants who took the post-test, 43 students were in the analogy condition and 46 were in the no analogy condition.

C. Instruments

1. Post-test

The post-test was composed of 17 questions, out of which 4 were taken from the Quantum Mechanics Conceptual Survey [26], 7 were taken from the Quantum Physics Conceptual Survey [27], and 6 were taken from our own question database, used in previous QP teaching projects. Table IV displays two examples of such questions. All post-test questions were multiple choice, with three to five alternatives. Out of the 17 questions, 4 were about the photoelectric effect, 10 were about the wave-particle duality, and 3 were about tunneling.

The post-test was pilot tested with 60 first-year students of Applied Physics at the University of Twente. Data analysis from this pilot provided a Cronbach's alpha of 0.467, which can be considered low.

TABLE IV. Example questions from the researchers' database used in the post-test.

A particle with a certain energy has a chance to tunnel through a potential energy barrier. Then, the potential energy of such barrier is increased. What is the effect of such an increase on the particle?

a) The kinetic energy of the particle decreases.
b) The particle's chance of tunneling decreases.
c) Alternatives a) and b) are correct.
d) None of the above.

Why can we use the photoelectric effect as an argument for the quantization of light?

a) Because the number of emitted electrons depends on the intensity of the light.
b) Because a minimal amount of energy is required to release electrons from a metal.
c) Because electron emission depends on the frequency of the light.
d) Because electron emission depends on the intensity of the light.

2. Dutch central physics examination

Students' scores in the Dutch central physics examination were used to assess and compare participants' overall performance in physics. This central examination is a standardized test elaborated yearly by the Dutch Central Institute for Test Development. The test is 3 h long, contains 25 questions, and covers all physics topics from the Dutch upper secondary curriculum.

3. ILS activities

ILSs provide a variety of ways to collect data. In this study, we focused on analyzing students' input in ILS activities which requested students to (a) enter a hypothesis; (b) write relevant observations while using a virtual lab or change a previously written observation; and (c) answer conceptual questions relating the content of the investigation to the theory of QP behind it.

D. Data analysis

1. Post-test

Student scores on the post-test were calculated per ILS topic. For each topic, a student's score was calculated by simply adding up the number of the student's correct answers for that topic and dividing it by the total number of questions measuring that topic. Thus, in the relevant tables, we present students' scores as fractional scores (percentages). In addition, a Mann-Whitney U test was conducted to compare the mean scores between groups.

2. Dutch central physics examination

Students' grades in the central examination were used as a covariate in a nonparametric analysis of covariance, performed using Quade's test [28]. This analysis was performed to compare post-test means, controlling for students' performance in the central examination.

3. ILS activities

To gain further insight into the role of analogies in students' understanding, a qualitative analysis of students' input in ILS activities was conducted. In this analysis, we investigated whether students themselves had made use of analogies with classical physics in their answers. First, open coding [29] was used to assess students' answers regarding (a) correct and incorrect lines of reasoning and (b) the explicit use of analogies (e.g., the roller coaster analogy described in ILS 3) and/or words related to classical phenomena (e.g., "collide" and "bounce"). Illustrative quotes were selected to build a picture of students' misconceptions, proper lines of reasoning, and use of classical physics elements and/or explicit analogies throughout the lessons. This analysis was done by the main researcher in consultation with one of the coauthors.

E. Procedure

We have been conducting research with the learning sequence used in this study since 2019 [15]. Such sequence was designed in collaboration with teachers, who also aided in data collection by applying the learning sequence with their students. Out of the six teachers who participated in the design of the learning material, two took part in this study. The other two participating teachers were new to the project and thus were not involved in the design process of the learning sequence or in previous studies. Therefore, the researchers provided additional instruction to these teachers on how to navigate the online learning environment and a description of the study's procedure. In addition, all teachers received instructions about the experiment and how to lead the classroom during it. Special attention was given to the use of analogies: teachers in the no analogy condition were instructed to avoid analogies with classical physics as much as possible in their explanations. They were also provided with examples of such analogies and of words to avoid. In contrast, teachers in the analogy condition were given a list of analogies and words to be used, which were the same as the ones included in the ILSs.

The procedure was the same for each classroom. Prior to the experiment, teachers were asked to assign their students to pairs. No specific criteria were given for teachers to follow when making the pairs. On the day of the experiment, the researchers were first introduced to the students and handed out consent forms. Thereafter, students were informed by the teacher about how the lesson would take place and were shown a sheet with their assigned pairs. Afterward, the students were instructed to log in to the online learning platform and follow the instructions therein. After completing all three units, students were required to take a post-test digitally in a testlike environment. The post-test was applied using the platform Qualtrics.

The implementation of all three units required a total of six 50-min lessons, in addition to a 30-min session to complete the post-test, spread over a period of 6 to 7 weeks per classroom. In the case of a student not being present in one of these lessons, the pair of the missing student was coupled with someone else.

III. RESULTS

In this section, we first present the results of quantitative comparisons between post-test scores from students in the analogy and no analogy conditions. Then, results from an in-depth qualitative analysis of student input in the online environment from both conditions are also reported.

A. Quantitative analysis

Before performing comparisons between conditions in this study, a Shapiro-Wilk test was conducted to evaluate whether our data could be considered normally distributed. The Shapiro-Wilk test of normality was conducted for the

TABLE V. Shapiro-Wilk statistics for normality distribution check of post-test scores (complete sample) and central physics examination scores (complete and reduced samples).

		Analogy			No analogy		
		W	d.o.f.	p	W	d.o.f.	p
Post-test scores ($N = 89$)	Photoelectric effect	0.890	43	0.001	0.901	46	0.001
	Wave-particle duality	0.924	43	0.007	0.877	46	<0.001
	Tunneling	0.841	43	<0.001	0.837	46	<0.001
Central physics examination scores	Complete sample ($N = 89$)	0.988	43	0.872	0.974	46	0.323
	Reduced sample ($N = 56$)	0.944	21	0.256	0.962	35	0.262

complete sample size of 89 students (for post-test and central examination scores) and for a reduced sample size of 56 students (for central examination scores). This reduced sample size concerns students who identified themselves in the post-test, allowing us to match their post-test and Dutch central physics examination scores in an analysis of covariance later on. Results of the Shapiro-Wilk test are presented in Table V.

The results presented in Table V show that post-test scores are not normally distributed. Central physics examination scores can be considered normally distributed for both complete and reduced samples. The non-normal distribution of post-test scores means that nonparametric tests should be used when conducting statistical analyses with this variable. Therefore, to compare how students in the analogy and no analogy condition performed in the post-test, a Mann-Whitney U test was performed. Results from this test are presented in Table VI.

The differences between conditions were significant for all topics, in favor of the no analogy condition. To evaluate whether such difference between conditions could have been influenced by students' overall performance in physics, we included scores on the Dutch central physics examination in the analysis. First, we present the average scores in the central physics examination for both conditions in Table VII, along with the outcomes of an independent samples t test and the related effect sizes.

Second, a new comparison of students' post-test scores, correcting for their overall performance in physics, was performed. We opted for Quade's test as a nonparametric alternative to an analysis of covariance [28]. Quade's test was performed with the reduced sample of 56 students who identified themselves in the post-test, allowing us to match only these students' post-test and central examination scores. The results from this analysis are presented in Table VIII.

In this comparison, the significant differences between conditions on the wave-particle duality and tunneling scores remained, in favor of the no analogy condition, after adjusting for students' overall performance in physics. However, no significant difference between conditions was found on the photoelectric effect post-test scores after adjusting for overall performance in physics.

B. Qualitative analysis

To deepen the understanding about the differences between both analogy and no analogy conditions, a qualitative analysis of students' logged answers to ILS

TABLE VI. Post-test scores (as fractions) from students in the analogy and no analogy conditions and Mann-Whitney U test statistics, per ILS topic.

	Analogy ($N = 43$)		No analogy ($N = 46$)					
	M	SD	M	SD	U	Z	p	η^2
Photoelectric effect	0.43	0.24	0.59	0.30	665.0	−2.75	0.003	0.08
Wave-particle duality	0.48	0.14	0.59	0.16	542.5	−3.72	<0.001	0.15
Tunneling	0.36	0.34	0.67	0.31	523.5	−3.95	<0.001	0.16

TABLE VII. Scores in central physics examination scores and t-test results, per condition.

	Analogy			No analogy					
	N	M	SD	N	M	SD	t	p	Cohen's d
Complete sample	43	6.59	1.38	46	7.38	1.21	−3.11	0.001	1.30
Reduced sample	21	6.92	1.28	35	7.35	1.18	−1.27	0.104	1.22

TABLE VIII. Post-test scores (as fractions) of the reduced sample of students and Quade's test results for the analogy and no analogy conditions.

	Analogy ($N = 21$)		No analogy ($N = 35$)				
	M	SD	M	SD	F	p	η^2
Photoelectric effect	0.44	0.26	0.58	0.30	1.23	0.272	0.02
Wave-particle duality	0.48	0.14	0.61	0.13	10.84	0.002	0.17
Tunneling	0.40	0.36	0.69	0.29	8.44	0.005	0.14

activities was conducted. In this exploratory analysis, we looked through students' answers to activities and focused on students' use of language referring to classical physics, such as deterministic verbs and explicit analogies. Results from this analysis are presented per ILS.

1. Photoelectric effect (ILS 1)

Each ILS started by recalling prior knowledge required for understanding the topic at hand. In ILS 1, this recall was done through multiple-choice questions about electromagnetic waves. In addition to these questions, an open-ended question was added to assess student knowledge about photons. The addition of this question was based on input from previous implementations of the ILS in classrooms. In these previous iterations, we noticed that some students had already learned about photons before taking ILS 1, usually when learning about radioactive decay. Even though prior knowledge about photons is not needed for the ILS, we deemed it necessary to add a question about the topic to assess what students eventually knew about it before taking the ILS. This question was the following: "Have you heard about photons? If yes, what do you know about them?". Analysis of students' input revealed that students in the no analogy condition had more elaborate and/or correct answers:

> Photons are 'packets' of energy without mass.
> A photon is a particle with a lot of energy and no charge and photons are sent through gamma radiation.
> Yes, photons are released through collisions and therefore the decay of nuclei. After that, two photons are released through the annihilation of a particle with an anti-particle.

Answers from the analogy condition were, in comparison, shorter and contained more incorrect information:

> They are light particles that move in waves.
> Yes, it is a very small particle, which travels with the speed of light and carries a (very small) mass.

Proceeding with the analysis of ILS 1 activities, we focused on students' answers to a conceptual question, posed halfway through the ILS, after students had conducted investigations in the virtual lab and were introduced to the concept of light as a particle and how that explained the occurrence of the photoelectric effect. The question read: "While many light phenomena can be explained using waves, the photoelectric effect is only explained through the particle character of light as photons with $E_{\text{photon}} = hf$. Explain in your own words why." Answers from both conditions suggested that students could reason with the concept of photon as intended for this stage of the ILS. However, when analyzing the wording from both conditions, we noticed that some answers from the no analogy condition made use of the words "interact" or "interaction" when referring to an action between photons and electrons. Example of such answers from the no analogy condition are as follows:

> Because that explains why there is a cutoff value of the frequency, which must be higher than a certain value to free electrons. If the frequency of the photons is lower, then the energy is lower, so the electrons don't get enough energy from the interaction between the two to be released.
> Because photons have a one-to-one interaction with electrons.
> If you consider light as a wave, it is difficult to understand how the photons interact with the electrons.

In comparison, the analogy condition often uses verbs that suggest a physical, deterministic action between the particles, as illustrated in the following example:

> An electron can only leave if a particle (photon) hits it and the particle has enough energy. It is difficult to explain how an electron escapes when it is hit by a wave.

As detailed in Sec. I B, two differences between conditions in this ILS were the wording and the use of the analogy of a collision between the photon and electron in the analogy condition. More specifically, the ILS in the analogy condition contained the text: "Think as if each photon collides with an electron, exchanging energy. If that energy is high enough, the electron will be emitted," whereas in the no analogy condition, there was only mention to an interaction between the particles. Furthermore, the analogy of a collision between photon and electron was repeated in later stages of the ILS used with the analogy condition. Based on student answers from both conditions, it seems that the wording in each ILS and the presence of the collision analogy might have influenced the way students view the interaction between photon and electron in the photoelectric effect. While the no analogy condition used more neutral words such as "interact," the analogy condition reproduced the collision analogy in their answers or used verbs that alluded to a physical collision.

All in all, this analysis suggests that students in both conditions were able to reason correctly about the photoelectric effect, but students' word choices seemed to vary between conditions. More specifically, students in the no analogy condition used neutral terms like "interaction" to describe the relationship between photons and electrons, while those in the analogy condition tended to describe the interaction in more deterministic terms such as "collide."

2. Wave-particle duality (ILS 2)

In the first investigative part of ILS 2, students used a virtual lab to visualize the behavior of a beam of multiple electrons being shot at a double slit, as well as the pattern formed by the electrons on a detector behind the slits. Students were instructed to enter observations in the ILS as they explored the virtual lab and were also posed the following question: "How do you explain the pattern formed by the electrons on the detector?". Upon analysis of students' entries to these ILS activities, we found both similarities and differences between students' answers in the two conditions. More specifically, in the no analogy condition, some students resorted to the deterministic concept of trajectory to explain what they had observed in the lab:

> I mean the electrons are going to change each other's course so that they don't hit the wall only in front of the holes.
> By deflections of the direction in which the electrons fly.
> It seems that the electrons do not choose a straight path but diffract (just like waves do). This creates a pattern, where both constructive (the peaks) and deconstructive (sic) interference (the valleys) take place.

In addition to the use of the trajectory concept, the first two examples above also imply the idea of an interaction between electrons or between the electrons and the barrier. The use of the verb "hit" in the first example could be considered a deterministic verb to describe the interaction between the electrons and the wall. In addition to the concepts shown in the examples above, other students in this condition used the concept of waves in their answers:

> The waves coincide in some places, causing interference and therefore a peak pattern on the detector.
> Apparently, electrons can take on wave properties, they can diffract, and they can interfere with each other.
> The waves merge or break up and electrons move as waves.

In the analogy condition, students also made use of the wave concept in their input:

> We thought that 2 stripes would be visible, because electrons would be shot through 2 holes. However, it appears that the electrons here behave more like a wave.
> Electrons behave like a wave.
> The electrons are waves and they bend around something, creating intervention (sic) patterns.

However, when manifesting deterministic ideas, the analogy condition did not refer to the electrons' trajectories, as done in the no analogy condition. Rather, students in the analogy condition resorted to the ideas of electrons being held back by the slits and of electrons colliding with each other or with the slits:

> There were 2 openings and the electrons passed through them, the rest got stuck on the wall that held them back.
> The deflection to the center is probably because they don't go straight through the slit, maybe they hit the side and then go to the center.
> The electrons collide with each other.

Finally, we point out that some students in the analogy condition used the idea of electrons "coming together" as an attempt to explain their observations:

> After a while, electrons come together again.
> The electrons end up directly behind the hole, this creates two peaks, and they come back together where the middle part of the slit is.
> Because the photons collide and continue together.

All in all, when confronted with the phenomenon of multiple electron interference, students in both conditions resorted to classical physics concepts or recognized the possibility that electrons display wave behavior. In general, the analogy condition incorporated more deterministic descriptions in their answers.

After exploring the behavior of multiple electrons being shot at a double slit, students were asked to hypothesize about the outcomes of a similar experiment, but this time conducted with single electrons, shot one at a time.

Students were asked to enter which pattern they expected to see in the detector in this case. In the no analogy condition, some students predicted that an interference pattern would still be observed:

The same pattern,... they will choose the same path in a beam, individually (this is purely a matter of chance, and the number of electrons at a time has no influence on it).

However, these students were exceptions, as most participants in the no analogy condition hypothesized in fact that a pattern with two slits would be seen in the detector:

> I think you will see a pattern where more electrons arrive right after the holes, and fewer and fewer electrons arrive the farther you get from the holes.
> 2 slits because electrons are localized.
> No deflection takes place because they are fired one by one and thus end up on the detector.

Such pattern was also the predominant hypothesis from students in the analogy condition. In this condition, however, students did not explain their reasoning behind the hypotheses as much as students in the other condition, and most answers assumed:

> 2 peaks because the particles do not collide.

Still, some students provided more extensive answers:

> If one electron is fired at a time, then they will not collide with each other and will therefore not interfere. If the electrons fall on the wall, nothing will be detected. If the electrons fall through the slit, something will only be detected at that spot.

After formulating their hypotheses in the ILS, students were shown a video of the buildup of an interference pattern by single electrons and were asked to reason about how such pattern was formed. In the no analogy condition, most students reasoned that the electron displayed or had wave behavior.

> Each electron behaves like a wave, so constructive and destructive interference patterns are visible.
> It has properties of a wave. It does not matter whether it is an individual particle or a group of electrons.

Some students also resorted to the idea of collision to explain the video or displayed the well-known misconception of electrons having a wavelike trajectory [30]:

> The electron collides with something.
> I think electrons collided with each other, creating an interference pattern.
> Electrons move in a wave pattern.

Concerning the analogy condition, a minority of students also reasoned using the idea of electrons having wave behavior:

> One electron creates a wave at both slits.

In addition, many students also reproduced the misconception that electrons would move in a sinusoidal, wavelike trajectory:

> They move in waves around the barrier.
> The electrons have to go around the barrier in a wave motion, so there are certain places that the electrons cannot reach at all.

What differentiates the analogy condition from the no analogy condition in this particular activity is that some students from the former resorted to the idea of electrons splitting up:

> I think each electron splits up.
> The electrons are split up. One goes to the left and the other to the right.

It is noticeable that substantial effort was put by the students into explaining observations that did not fit with their hypotheses. We also point out that, at this point in the lesson, students were not yet introduced to the concepts of wave function or probability distribution as an explanation for the observed behavior of the electrons. Therefore, the reasonings presented above are a manifestation of students' own presuppositions in an attempt to explain electron interference.

After having explored the occurrence of electron interference and reasoned about it, students proceeded with the ILS and were introduced to fundamental concepts of QP, which were the focus of the rest of the lesson. After this theoretical part of the lesson, students underwent one last investigation with the virtual lab, this time shooting one electron at a time and being shown the visualization of the probability density of the electron and its behavior in the double-slit experiment. Such probability density was depicted in the simulation as a colorful spot, whose colors corresponded to higher or lower probabilities of detection of the electron. This correspondence between colors and probability was displayed as a horizontal scale in the virtual lab (see Fig. 2). As an activity, students were asked to answer what the colorful spot in the simulation represented and what happened to the electron after passing through the slits. Starting with the no analogy condition, many students reasoned with the concept of probability (distribution):

> The spot is the probability where the electron is located. Red is the highest probability and blue the smallest. Then the position of the electron becomes more uncertain because there is a larger surface area where the electron can be located.

Some students, however, entered the incorrect idea that the spot represented many particles:

> The spot is the electrons. Most of the electrons are in the red part, because electrons are more likely to come there. The particle is deflected after the slit.
> The spot represents the number of electrons, red is a lot, green is little. The electron is deflected, bounces, and interference patterns are created.

We point out that, in the answers above, students first referred to the electrons in the plural, but when describing what happens after the slits, they referred to a single particle. The same kind of incorrect interpretation of the simulation was seen in answers from the analogy condition as well:

> The spot is a beam of electrons. They pass through the slits and then come back through magnetism.
> The spot shows where the many electrons ended up. The moment the particle passes through the slit, the particle will divide into three pieces.

Nevertheless, answers from students in theanalogy condition also reasoned with the concepts of chance and probability, albeit less thoroughly than the no analogy condition:

> The spot represents a shot electron, the redder the spot, the more likely an electron is to follow that trajectory.
> The redder the color, the greater the chance that the particle will be there. Conversely there is a small chance in the blue areas. So the particle goes to a place based on the probability it has.

Finally, we see that the notion of the electron splitting up was present once more in answers from the analogy group in this stage of the lesson:

> The spot represents an electron. As it passes through the slits it is divided.
> That spot represents an electron. When it passes through the slits, it is divided into several particles.

It appears that students from both conditions could reason with the concepts of chance and probability in a correct way. In addition, both conditions had students with the misconception that the colors of the spot represented the number of electrons in the spot. Concerning the analogy condition specifically, we see that the idea of the electron dividing itself is again present in their answers, as seen in previous answers from that same condition about single-electron interference. The visualization provided by the virtual lab might have strengthened the idea of the particle splitting up.

In sum, the qualitative analysis of this unit's answers revealed that students in both conditions seemed to have developed their reasoning about the wave-particle duality in a similar way. More specifically, students from both conditions used deterministic ideas and resorted to well-known misconceptions throughout the initial part of the ILS. Nevertheless, the findings suggest that several students from both conditions managed to shift to more probabilistic ideas toward the end of the unit. Despite this similar development of ideas, some differences between conditions seemed to be present as well. First, students in the analogy condition often resorted to the idea that an electron can split up when going through the slits. Such a conception was present in answers throughout the whole ILS, and not only at the beginning, where misconceptions and deterministic ideas occurred more often. Second, there was a difference between conditions concerning the deterministic concepts used by students to reason about the double-slit experiment with electrons. While students in the no analogy condition focused on the electron's trajectory, those in the analogy condition focused on the electron's interactions with each other or the barrier, using words such as "collide" and "hit."

3. Tunneling (ILS 3)

In the last ILS, quantum tunneling is first introduced to students as a model for explaining alpha decay, a topic that Dutch students learn before moving on to QP. The remaining part of the ILS is dedicated to building a model of tunneling which includes the representations of the potential energy barrier and the wave function. To introduce students to a graphical representation of such a model, the following image (Fig. 4) is used for a knowledge-building exercise, in which students have to judge statements as true or false and explain their reasoning. One of the such statements was "The particle can only be detected in area A." In both no analogy and analogy conditions,

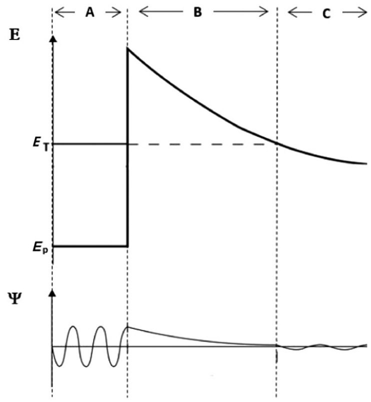


FIG. 4. Graphical representation of a model for quantum tunneling.

some students correctly judged the statement as false, but the reasonings behind the answers differed between conditions. For example, in the no analogy condition, students wrote:

> In the lower graph the amplitude is not zero, so there is still a probability that the particle is there.
> An energy spectrum can still be seen in areas B and C (the peaks), which therefore indicates that the particle can also be located there.
> I think [the statement is] incorrect because the barrier is thin so there is a small chance that it is also on the other side instead of just at A.

In the first two examples, we see that students were able to reason with the graphical representation of the wave function, despite it being called "energy spectrum" by the student from the second example, suggesting that the student recalled the interpretation of the wave function's graph but did not know what it referred to. In the final example, however, the student provided an incorrect, classical, reasoning. Students in the analogy condition often resorted to classical ideas when answering this same question:

> The particle can also go 'out' of the box, to B.
> The particle then 'jumps' over the barrier.

The potential energy can be higher than the total energy (e.g., on a roller coaster) and then the particle can move over the barrier.

We point out that students in the first two examples wrote specific words between quotes, indicating that they could have been aware that such words alluded to classical concepts. In the third example, we noticed that the student mentioned a roller coaster, probably referring to the roller coaster analogy used earlier in the lesson (see Sec. I B).

Another statement that students had to evaluate was "The particle can only move to region C if its energy temporarily exceeds the energy of the barrier." At this stage of the lesson, students had not yet explored the tunneling virtual lab and, therefore, had not yet investigated the effects of the different energies in depth. Thus, students' answers could be interpreted as students resorting to their own presuppositions. Examples of answers from the no analogy condition are as follows:

> Yes, because the particle must at least have an energy higher than the energy of the barrier.
> Correct, otherwise the particle will not decay.
> Incorrect, because the particle does not have to cross the barrier, there is just a chance that the particle is on the other side of the barrier.

While few students used the concept of chance, such as in the last example, most students' answers were similar to the first example and mentioned that such an increase in energy is necessary because the particle should go over the barrier. We also point out that the example of alpha decay, used in the introductory part of the lesson, is recalled by the student in the second example. In the analogy condition, most students also judged the statement as correct:

> Yes, as the energy increases it can go over the 'hill' and then to C.
> Yes, only then the particle can escape from the potential well.
> The particle collides in the barrier against the sides, which creates kinetic energy, which does not happen in region C.

We see that students' answers are similar to the ones by the no analogy condition and contain classical words such as "hill" and "collides." All in all, many students from both conditions displayed the known misconception that the particle's total energy must be higher than the potential energy of the barrier for tunneling to occur [30].

At the end of the ILS, students use a virtual lab to explore the graphical model of tunneling. They are instructed to investigate in the lab the effects of varying the barrier's height and width on the particle's wave function. More specifically, students are presented with three questions to guide their investigation: (1) "What happens with the particle's chance of tunneling if the barrier's height is increased?" (2) "What happens with this same chance if the barrier's width is increased?" (3) "Does the particle's total energy decrease after tunneling? Why?". Students were also guided to enter observations in the ILS environment as they used the virtual lab. While students in both conditions provided a similar number of observations, participants in the no analogy condition provided more extensive answers, as seen below:

> (1) When the barrier is raised, the wave function no longer passes through the barrier, which happened a bit with a lower barrier. (2) With a wider barrier, the wave function passes through the barrier later. (3) The green line in the top graph remains the same, so the total energy remains the same.
> … (2) That chance gets smaller and smaller because the probability of that particle also gets smaller. (3) I think so because a small part of the wave function goes to the other side of the barrier.
> … (2) When the barrier widens, the chance of finding a particle behind the barrier becomes smaller because a greater energy is required. (3) No, [the energy] is only distributed over the 2 sides of the barrier because there is a conservation of energy.

Regarding the examples above, we first see that students mixed classical and quantum concepts in their answers.

For instance, in the first example, the student mentioned that the wave function “passes through” the potential energy barrier, which could allude to the classical idea of a particle going through a material barrier. Nevertheless, quantum reasoning was still present in students’ answers, such as in the second example, in which the concept of probability was used. It is likely that the student in this example resorted to the graphical representation of the wave function, displayed in the virtual lab, and used it to reason about the particle’s probability. Second, we point out the variety of students’ lines of reasoning about what happens to the particle’s energy after tunneling. While in the first example the student correctly resorted to the energy graph, displayed in the virtual lab, and drew a correct conclusion, the second and third examples illustrate that students also had incorrect ideas about the energies involved in the process. Perhaps students from these last two examples interpreted the wave function on both sides of the barrier as a “division” of the particle. This speculation is similar to what we hypothesized at the end of the qualitative analysis of ILS 2, when students in the analogy condition reasoned that the electron divided itself. In that occasion, we argued that the visualization of the electron’s probability density in the virtual lab could have strengthened the idea of the particle splitting up.

Moving on to the analogy condition, students’ answers were less extensive, as seen below:

> The greater the barrier height becomes, the less likely tunneling becomes.
> With a higher barrier height, the probability in the barrier becomes higher. With a larger barrier width, tunneling is present. The total energy does not decrease.
> If the barrier becomes wider, the chance of tunneling decreases; The higher the barrier becomes, the smaller the chance of tunneling; Energy is lost through tunneling.

In this condition, we find shorter answers, some correct observations, and some references to probability when referring to chances. Again, the energy conservation issue is not grasped by all. Related to the short answers, the analogy students’ answers show little reasoning with respect to the explanations for their observations.

In conclusion, for this ILS, differences between conditions were more noticeable in the qualitative analysis. Overall, students in the no analogy condition provided more thorough responses and integrated both quantum and classical concepts. In contrast, students in the analogy condition provided less developed explanations and did not elaborate on their ideas as extensively. In addition, this same condition resorted to classical physics concepts more often (e.g., the idea that the particle “can go over the ‘hill’”), as also observed in the analyses of the previous ILSs.

IV. DISCUSSION

In this article, we explored the impact of the use of analogies with classical physics when teaching introductory quantum physics topics (photoelectric effect, wave-particle duality, and tunneling) to high school students. Using a learning sequence previously designed by our research group, we applied two different versions of digital learning materials (inquiry learning spaces or ILSs) with participants in two conditions: analogy and no analogy. Outcomes on a post-test were compared between conditions (quantitative analysis), as well as written answers entered by students in the digital learning environment (qualitative analysis). These analyses were used to answer this study’s main research question: “To what extent do analogies with classical physics influence students’ reasoning with introductory QP topics?”.

An initial comparison showed that the difference in post-test scores between the analogy and no analogy conditions was significant for all three quantum physics topics in favor of the no analogy condition. After a correction for students’ scores at the Dutch central physics examination (a measure of their overall physics performance), a significant difference for two out of three QP topics still was present (wave-particle duality and tunneling). In addition to this quantitative analysis, we examined students’ written answers to open-ended questions in the learning environment. The following paragraphs discuss the findings of the qualitative analysis of students’ answers and relate them to the results of the quantitative analysis, per topic.

We start by analyzing the photoelectric effect topic. When comparing post-test scores between the analogy and no analogy conditions, corrected for students’ central examination scores, no significant differences were found between them. Nevertheless, an analysis of students’ written input in the ILS provided further insight into each condition’s views about the effect. As pointed out in the Results section, when assessing students’ prior knowledge of relevant concepts for the lesson, we noticed that students in the no analogy condition displayed more correct and complex ideas about the photon. In contrast, the analogy condition’s prior knowledge about the photon was more often incorrect or basic. Given the higher level of prior knowledge from the no analogy condition, it could be expected that its students would perform better in the lesson; however, that was not what we observed. Ultimately, students in both conditions reasoned correctly with the corpuscular character of light in the photoelectric effect throughout the lesson, which was later reflected in the post-test scores showing no significant difference between conditions. What did differ between the conditions was the kinds of words used by students to describe the interaction between the photon and the electron. While the no analogy condition used the words “interact” or “interaction” more often, the analogy condition resorted to words that allude to a collision, such as “hit.” This difference between written answers could be caused by how the ILSs in each condition deliberately used different words to present information and included (or not) an analogy with a collision between the photon and the electron. We argue that the wording in students' answers could be a reproduction of how information was presented to them in each condition. This argument is further explored when discussing the results of the tunneling lesson.

Next, we cover the findings related to the wave-particle duality lesson. Post-test scores were significantly higher for the no analogy condition on this topic. Despite this significant difference, the qualitative analysis of students' answers in the ILS showed that both conditions developed similar ideas about the topic. More specifically, we observed that both conditions built their reasoning throughout the ILS in a similar way: at first, they hypothesized about multiple-electron interference taking place through changes in the particles' trajectories, collisions between the particles or with the slits, or due to wave behavior; then, when hypothesizing about single-electron interference, most students from both conditions believed no interference would take place; next, when asked to explain a video showing that single-electron interference does take place, students from both conditions reasoned that the electron moved in a wavelike trajectory, collided on its way to the detector, or displayed wave behavior; and finally, after learning about the concepts of wave function and probability distribution and interacting with a virtual lab displaying single-electron interference, many students from both conditions correctly applied the concepts of chance or probability distribution to interpret what was being investigated in the virtual lab. All in all, we observed that students from both conditions used deterministic concepts and manifested well-known misconceptions [30] throughout the initial part of the ILS but were still able to shift to more probabilistic ideas and apply them toward the end of the lesson.

The main differences between conditions throughout the wave-particle duality ILS were the analogy condition's answers often resorting to (a) the idea that an electron can split up when going through the slits and (b) vocabulary related to deterministic behaviors or characteristics of the electron. This last difference was recurrent in all three ILSs and will be discussed in more detail in the next paragraph. Concerning (a), the idea of division was occasionally accompanied by the notion of electrons "coming together" after having split up. In a study about the productive use of analogies in physics teaching, Podolefsky and Finkelstein [3] suggested that representations of physical phenomena have an important role in how analogies are interpreted by students. That is because representations direct learners to focus on specific characteristics of the physical phenomenon, which helps to correctly map the relations between the different domains involved in the analogy [3]. However, the authors add that students tend to interpret analogies literally, which can result in their unproductive use. In our study, answers from the analogy condition about electrons splitting up and coming together could be an example of students interpreting the virtual lab's representation of electron interference literally, which in turn resulted in an incorrect idea about the phenomenon. Such literal interpretation might have strengthened deterministic ideas already present in students' frameworks (manifested throughout the ILS), leading to an unproductive use of the analogies.

Finally, we analyze the outcomes of the third lesson, covering the tunneling topic. Similar to what was observed in the previous analysis, the no analogy condition scored significantly higher than the analogy condition in the post-test for this topic. In addition, the differences between conditions observed during the qualitative analysis were more noticeable in this ILS. In general, the no analogy condition presented more extensive answers with detailed reasoning and often resorted to a mix of quantum and classical concepts, while students in the analogy condition did not develop their ideas as much. When analyzing students' input about the true-false questions in the ILS, we observed that some students from the no analogy condition correctly interpreted the wave function graph, even if using incorrect terms to refer to it (e.g., "energy spectrum"). However, in the analogy condition, we observed more classical reasoning and an unproductive use of the roller coaster analogy, which was mentioned by a student to illustrate that a particle can be detected outside of the barrier only if its energy is greater than the potential barrier. Also, as observed in the previous lessons, the analogy condition resorted to classical physics vocabulary more often than the no analogy condition when answering these true-false questions. As mentioned in the discussion about ILS 1, such difference in wording between conditions and the presence of analogies in the analogy condition's responses could be a result of the different approaches applied in our study for each condition. We hypothesized that the wording in students' answers could be a reproduction of how information was presented to them in the ILSs. This relates to findings reported by Hoehn *et al.* [31], who investigated students' dynamic use of ontologies in quantum physics. These researchers found that students' use of ontologies in written answers was influenced by the wording and framing of the questions posed to them, so when a question used words leaning toward a specific ontology, such ontology was present in students' answers more frequently. As pointed out by Brookes and Etkina [9], language plays an important role in (quantum) physics learning and can influence how students turn concepts into ontological categories. In our study, it seems that the language expressed in the ILSs and the language used to convey analogies influenced the words and analogies present in our participants' answers. Based on these findings for all three lessons, we argue that using analogies (or not) might have influenced how students framed their ideas and reasoning, which in turn can affect how they build new concepts in quantum physics.

Another noteworthy finding from the qualitative analysis of ILS 3 came from the investigation of how students interpreted the tunneling virtual lab. Students in the no analogy condition made correct interpretations of various outputs from the virtual lab, even though a mix between quantum and classical concepts was present in their answers. However, these students' reasoning about the particle's energy after tunneling raised the hypothesis that they interpreted the wave function on both sides of the barrier as a division of the particle. This, in turn, could be another instance of how representations of a phenomenon influence students' interpretations of such phenomenon, as observed and discussed in the section about the wave-particle duality lesson.

Regarding the findings of all three ILSs, we consider that students from both conditions developed their reasoning and ideas presenting a mix of quantum and classical concepts. Such blend between classical and quantum concepts has also been reported in various studies (see Ref. [30] for a review). Despite this overall similarity between the conditions, the quantitative analysis showed that for two out of the three topics being taught, the analogy condition performed significantly worse than the no analogy condition on the post-test. In this respect, it is noticeable that on a more detailed level, the analogy condition students used more classical words and concepts in their answers and made unproductive use of some analogies. Such unproductive use might be related to (a) students taking the analogies literally, as suggested by Podolefsky and Finkelstein [3], and (b) the increased cognitive processing that analogies may cause, especially for weaker students, as pointed out by Alexander and Kulikowich [5]. Therefore, when answering the main research question of this study, we believe that analogies with classical physics can negatively impact student understanding of introductory QP by increasing the subjects' cognitive processing and by requiring students to understand the limitations of the classical analogy for the quantum scenario, so as to not take the analogies literally.

Finally, we point out how student outcomes in this study reinforce the potential of student-centered, guided inquiry-based learning as a productive means for QP instruction, as suggested by previous studies [32,33]. We believe such potential should encourage researchers to further develop and implement inquiry-based strategies for QP and other complex STEM subjects.

Some considerations about this study's limitations need to be mentioned. These limitations include (a) the fact that students were not randomly assigned to one of the conditions; (b) the post-test's low Cronbach's alpha, suggesting a reduced reliability of the obtained scores; (c) the exclusion of part of the student sample from Quade's test, due to the lack of identification in the post-test, impacting the validity of the results; and (d) the fact that not all deterministic wording could be avoided in the ILSs used by the no analogy condition. More specifically, much of the vocabulary used in quantum physics is inherently deterministic or analogylike, such as potential "well" or "barrier," "wave-particle" duality, among others. We opted to keep such terms in the no analogy ILSs as these words are standard quantum physics terminologies.

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