

# A Score Function for State-of-Charge Profiles for Rechargeable Batteries

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**Abstract**—We propose a new score function to compare and evaluate the relative impact of state-of-charge profiles on overall battery lifetime. Our score function, based on a discrete Fourier transform of the state-of-charge profile, formalizes and generalizes earlier ideas found in the literature, and can form an important help in optimizing overall life time for battery powered systems. In this paper we introduce and illustrate the method, and discuss its merits as well as open issues and related literature.

## I. INTRODUCTION

Battery powered devices are ubiquitous: You can find them in embedded contexts such as satellites, sensor networks, and pacemakers, in laptops/tablets/smartphones, and electric cars are also becoming popular. But while batteries provide portable power, they only do it for a limited period of time, whether it is a day or several years. Nonrechargeable (primary) batteries by definition need replacement when they run out, but even rechargeable (secondary) batteries deteriorate with time and use due to various unwanted chemical reactions that accompany the desired reactions that bind and release the chemically stored energy.

For systems that are not easily serviceable such as unmanned spacecraft and sensors embedded in bridges and buildings, predicting the wear on secondary batteries is a central part of predicting the total system lifetime. For serviceable systems, prediction can be part of calculating the maintenance cost of the complete battery powered system.

As part of design space exploration, a system designer may propose a set of possible system designs that (among other things) use the battery differently. To help evaluate these designs, he may consult battery documentation and data sheets but will often find that the manufacturer has only included limited performance and endurance data.

Testing proposed designs in experiments with physical batteries can be prohibitively expensive and slow, even with accelerated tests that wear out the battery faster at artificially high temperatures. Instead, we propose a scoring function that takes as input a battery usage profile (state-of-charge timeseries) obtained from, e.g., system simulation. To be able to analyze complicated workloads that cannot easily be decomposed into alternating phases of discharging and fully recharging, we analyze the usage profile in the frequency domain. The advantage of our approach is that we provide

a fully model-based evaluation approach for the performance and lifetime of battery-powered systems.

This paper is organized as follows. Section II introduces batteries and battery degradation, and Section III introduces our new score function, Section IV discusses limitations and assumptions, and Section compares to related work.

## II. BATTERY CONCEPTS

Batteries store chemical energy and are able to release it as electrical energy. In primary batteries the reaction irreversibly changes the chemical composition of the battery, but in secondary batteries it can be reversed, converting electrical energy back to chemical energy.

A fundamental concept for batteries is the **state of charge (SOC)**. Using a car analogy, a full tank corresponds to 100% SOC, and an empty tank corresponds to 0%. But while a fuel tank is a simple concept, batteries are more complicated. In a car, the distance you can drive depends on the speed, but the chemical *energy* you can get out of the fuel tank is proportional to the amount of fuel you put into it. On the other hand, the amount of energy a battery can deliver before running dry depends strongly on the usage pattern. This is due to the *rate/capacity effect* [4]. Furthermore, a battery that runs dry is not really empty because the *recovery effect* [4] means that it will slowly regain some charge while resting. Last but not least, a battery can be charged above the design capacity if a higher voltage is applied (at the cost of faster wear of the battery).

Since both an empty and a full battery are not easily defined in practical usage, we refer to the battery datasheets to define the SOC. The battery is full when it is charged at the design charge voltage. The battery has reached a 0% SOC when it has delivered the nominal capacity.

Some batteries are used for backup power and spend most of their lifetime near full SOC. In this work, we are interested in secondary batteries used in the typical cycling between discharging and recharging.

**Depth of discharge (DOD)** is defined here as  $1 - \text{SOC}$  and is a concept that is often used in discussion of battery wear. A **cycle** consists of discharging the battery and then recharging it to its full capacity. These concepts are illustrated in Fig. 1. The term DOD is often used in the sense of maximal DOD. For example, “cycling at 80% DOD” means to repeatedly discharge to 80% DOD and recharge to 100% SOC (0% DOD).

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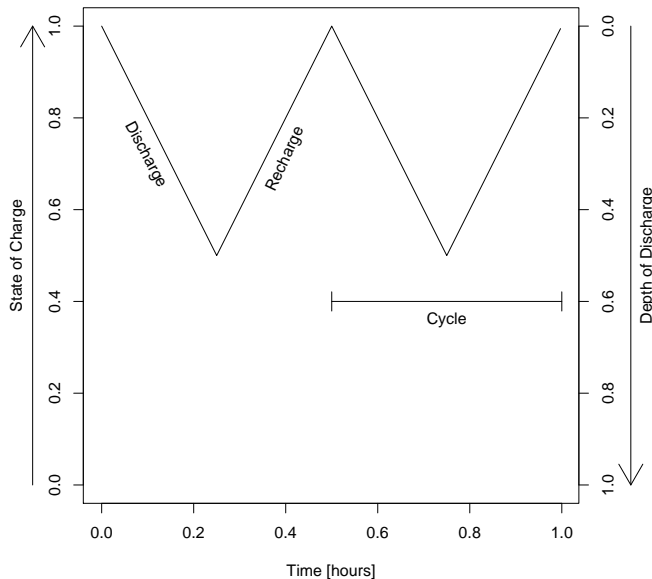


Fig. 1. Illustration of key battery concepts: State of charge (SOC), depth of discharge (DOD), discharge and recharge phases, and cycle.

Batteries consist of one or more electrochemical cells connected in series, for increased voltage, or in parallel for increased capacity. Small variations among the connected cells affect the performance and degradation of the overall battery and complicate charging and discharging procedures. Batteries can in turn be connected into battery packs with the same challenges. In this paper, we limit our attention to (single) batteries consisting of only one electrochemical cell.

#### A. Battery Wear

The chemical reactions that store and release energy, are, unavoidably, accompanied by other, unwanted chemical reactions and processes that slowly destroy the reactants or the electrodes. The main performance consequence is that with time and use, the capacity of the battery fades.

The rate of deterioration depends, among other things, on the maximal DOD reached, the rate of charge and discharge, temperature, dwelling at high and low SOC, and overcharging [2]. The maximal DOD that is discharged to is especially important, and can be the only focus of battery manufacturers' datasheets. For example, one battery is expected to reach end-of-life (80% capacity remaining) after 350 cycles at 100% DOD, 1000 cycles at 50%, and 1700 cycles at 25%<sup>1</sup>. This kind of data can be good enough for simple workloads and system designs, but is not enough for advanced workloads, which motivates our approach.

### III. A NEW SCORE FUNCTION

For a complicated workload, defining a cycle is not straightforward. Or at least, the mathematical definition of a cycle is not the most interesting. A periodically repeating

pattern is by definition cyclic, but each cycle may contain many alternating discharging and recharging phases. One example is SOC profile F in Fig. 3, where each cycle consists of three discharge-recharge phases. Another example is a battery that is discharged to, say, 20% SOC, then charged to 90%, and then discharged again to 20% before being recharged fully and starting over. It's only one cycle but from a battery application point of view, it is close to two "cycles". Not only is this completely likely usage of a battery difficult to discuss, it is also unlikely that a battery data sheet will say anything about the expected battery lifetime for this type of load, further making a score function desirable.

We propose to examine the SOC profile in the frequency domain to sidestep this issue. Using the discrete Fourier transform (as computed by a Fast Fourier transform (FFT) algorithm), we convert the SOC timeseries into a frequency spectrum containing all component frequencies and their magnitudes. The FFT output is a sequence of complex numbers, the moduli of which correspond to the magnitudes of the component frequencies. If the SOC profile is a sequence  $S = s_1 s_2 \dots s_n$  of SOC values sampled at frequency  $f$ , its score is

$$\text{score}(S, f) = \frac{2f}{n} \sum_{i=0}^{\lfloor n/2 \rfloor} i |\mathcal{F}(S)_i|^2 \quad (1)$$

where  $\mathcal{F}$  is the FFT function and  $|\cdot|$  the modulus. Because the input consists only of real numbers, the values in the second half of the FFT output (above the Nyquist frequency) are a mirror image of the first half, so we "fold it back" by multiplying the first half by two and discarding the second half (indexes above  $\lfloor n/2 \rfloor$ ). The fraction  $if/n$  is the frequency corresponding to the FFT magnitude at index  $i$ . We devised the score function such that an SOC profile with a lower score is better for the longevity of the battery.

Fig. 2 illustrates four different SOC profiles and an intermediate result of the score calculation. The FFT-like plots in fact show the calculation of (1), but with the calculation done element-wise on the FFT output sequence before the summation. In other words, each plot is the sequence  $2fi|\mathcal{F}(S)_i|^2/n$ , with modulus, exponentiation, and multiplication applied element-wise on the sequences. In the plot, the frequency axes are truncated to "zoom in" on the interesting harmonics.

Normalized to the first score, the four scores are 1, 2, 4, and 8. Comparing the profiles A and B, we see discharging to the same DOD (50%), but a doubling of the charge and discharge rates as well as number of cycles that can be completed in the same time frame. This also doubles the score. The same applies when comparing C and D. In these two simple comparisons, the score is proportional to the number of cycles/time, which matches well with the general idea that a battery can sustain a fixed number of cycles at a given DOD.

Comparing profiles B and C, we see that the same amount of charge is delivered — equivalent to three full capacities are discharged in six hours. However, profile C discharges to twice the DOD while doing it. This also increases the score, because a higher DOD wears out the battery faster, even though the same charge is delivered. This matches what [2] cites as

<sup>1</sup><http://www.gomspace.com/documents/gs-ds-batteries.pdf>

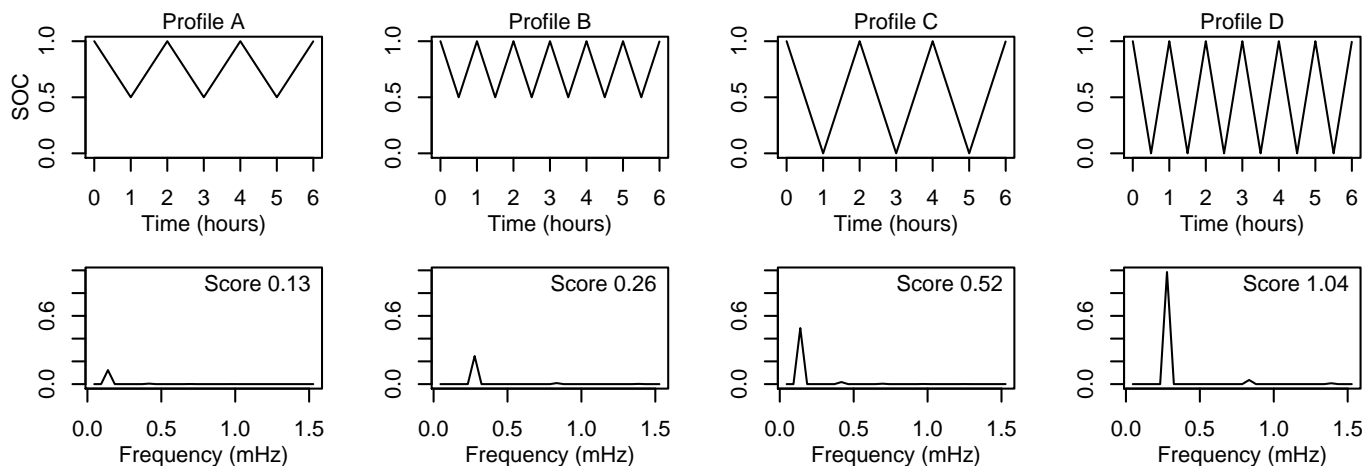


Fig. 2. The top row shows four example SOC profiles. The bottom row shows the corresponding FFT spectra with all the modifications done before the summation (see the text).

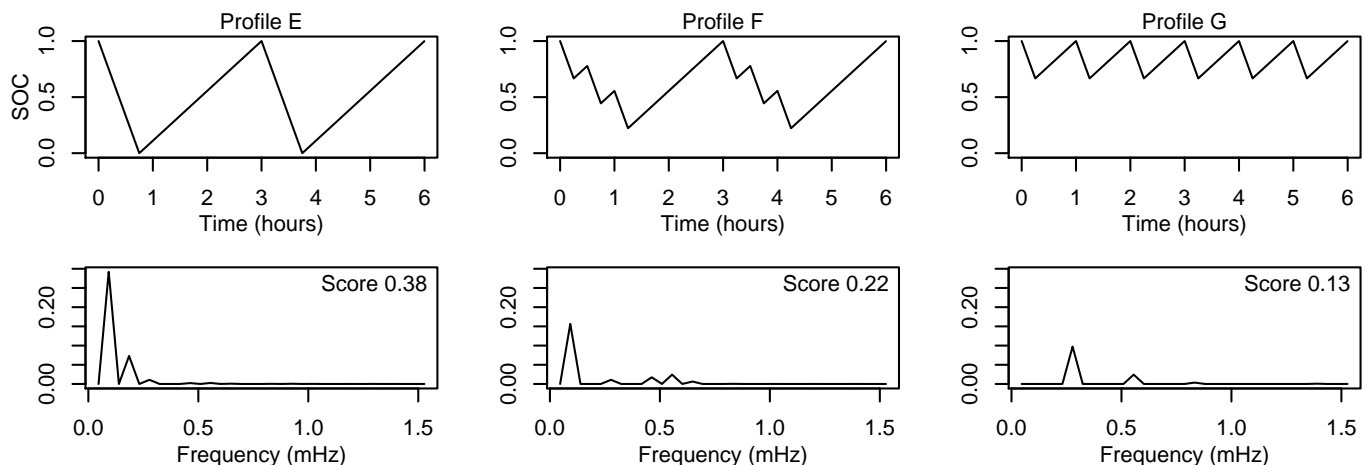


Fig. 3. Three SOC profiles that show different schedulings of 15 minute tasks with load  $-4$  and a constant recharging at  $+1$ .

Symon’s [6] Premise 2: “The charge life of the cell will always [...] be greater than [the rated charge life] when the battery is cycled less deeply.” In other words, shallow DOD cycling improves the total amp-hour throughput in the lifetime of a battery.

In Fig. 3, a battery powered system is supplied with a constant charge rate of  $+1$  current unit, but every three hours it has to perform three sequential tasks, each affecting the system with  $-4$  current units for 15 minutes. In profile E, they are run back to back, after which the battery is recharged fully. In profile G, the tasks are equally spaced in time. Profile F is an example of a “non-simple” workload in between the two others. The charge delivered and the charge and discharge rates involved are the same, but the DOD reached, as well as the score, is in between the two others’.

#### IV. DISCUSSION

Our prediction method is generic and must be fitted to a specific battery technology based on measurements from the manufacturer and/or the system designer. The method is valid for battery technologies where charging and discharging can be considered to have symmetric effects on wear. With

some battery technologies, this is not always the case for high currents, see, e.g., [1]. Drouilhet and Johnson [2] also cite a work saying that high charge rates at low and intermediate SOC may *increase* battery life, but they do not consider the evidence conclusive.

As we saw when comparing profiles B and C, the score doubled with a doubled DOD. This tendency is correct but also generic. In practice the wear may not be directly proportional to the DOD. At the moment, the score function can be minimized to, e.g., schedule a set of tasks in a way that is beneficial to the battery. To more exactly predict battery lifetime, the approach will have to be targeted to a specific battery technology.

However, obtaining wear data from battery manufacturers’ datasheets can be very challenging. Petricca et al. [5] report that “from an extensive survey of more than 100 datasheets of commercial battery of different chemistries, sizes, energy and form factors, we did not find a single datasheet that report information about the following characteristics altogether in the same document: battery behavior due to constant current, pulse current, and aging effects.”

We speculate that with enough data available for fitting, the following generalized form of (1) parameterized on  $p$  and  $q$  could be relevant.

$$\frac{2f^p}{n} \sum_{i=0}^{\lfloor n/2 \rfloor} i |\mathcal{F}(S)_i|^q \quad (2)$$

For our score function, we assume that the proposed SOC profile is thought to be repeated indefinitely. Therefore it should start and end at the same SOC to prevent an assumed discrete jump between SOC's when the profile is repeated. We also assume that the profile is of a short duration wherein the battery wear can be ignored, i.e., the capacity remains close to constant. Such a profile could be on the order of days or weeks rather than months or years.

The sampling/simulation parameters  $f$  and  $n$  are chosen according to the speed of changes in the SOC. The sampling frequency  $f$  should be large enough that the oscillations of interest are slower than the Nyquist frequency  $f/2$ . Higher harmonics are negligible at realistic uses of batteries because they would not support sustained high currents (changes in SOC), which in turn give rise to strong high frequency harmonics. The sampling window  $n/f$  affects the lowest observable frequency, which is its reciprocal  $f/n$ . The lowest observable frequency should be low enough to observe the oscillations of interest.

It seems to be not very well studied what happens when batteries are used in ways that are not simply repeating "charge fully, then immediately discharge to some depth". Drouilhet and Johnson [2] mention dwell time at low and high states of charge as a contributor to wear, implying that a medium SOC could be good for battery life. Similarly, the end of charge voltage, which also affects the SOC to which the battery is charged, is said to influence battery life<sup>2</sup>. Here, the DC component of the Fourier transform could be relevant to explore even though it is ignored in (1) and (2) due to being multiplied by zero.

Finally, we assume a constant temperature for the battery.

## V. RELATED WORK

Drouilhet and Johnson [2] in the context of energy storage describe a battery life prediction method that takes into account DOD and discharge rate. They propose a function for each of these to which manufacturer data can be fitted. Combining the two expressions, the effective discharge affecting the battery with respect to wear can be computed from a user-prescribed discharge profile consisting of a series of discharge events. The battery is seen as having a fixed charge life (lifetime Ampere-hour throughput until end-of-life), but relative to "effective" discharge, which depends on DOD and discharge rate. They apply their method to a case study of peak shaving in an Alaskan village powered by wind energy. By predicting the lifetime of different sizes of NiCd and VRLA batteries, they find the most cost effective battery technology and size for the given application. In our approach, we try to generalize from the focus on simple workloads described as a sequence of

discharge event (specified only with average discharge current), between which full a recharge is assumed.

Petricca et al. [5] describe an electrical circuit model of capacity fading due to cycling, as well as the increase of the internal resistance due to cycling. What is needed to build the model is manufacturer's data on capacity fading due to cycling at different temperatures and discharges rates (C-rates), and data on increase of the internal resistance at a reference discharge rate and for various DODs. It is questionable whether this data is always available to the user. As mentioned in Section IV, the authors had a hard time obtaining it. The capacity loss is based on an equation that takes as input the number of cycles. Again we encounter the concept of cycles that only works for simple workloads, where each discharge phase starts at 100% SOC.

Guená and Leblanc [3] in the context of backup power experimentally examine how DOD affects the cycle life of lithium-metal-polymer batteries. They test at 0.6%, 50%, 60%, 70%, 80%, and 100% DOD and find that reduced DOD improves cycle life and total charge throughput. This matches our expectations. They also test micro cycling (to 0.6% DOD) and find it to have no effect on cycle life, i.e., the micro cycled cell had the same capacity fade as one with a float charge (no cycling). Unfortunately the sample sizes are too small to say anything conclusive (one cycled cell compared to three floating cells), and more information would be interesting to have.

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<sup>2</sup>[http://batteryuniversity.com/learn/article/how\\_to\\_prolong\\_lithium\\_based\\_batteries](http://batteryuniversity.com/learn/article/how_to_prolong_lithium_based_batteries)