Battery Modeling

M.R. Jongerden and B.R. Haverkort

Abstract

The use of mobile devices is often limited by the capacity of the employed batteries. The battery lifetime determines how long one can use a device. Battery modeling can help to predict, and possibly extend this lifetime. Many different battery models have been developed over the years. However, with these models one can only compute lifetimes for specific discharge profiles, and not for workloads in general. In this paper, we give an overview of the different battery models that are available, and evaluate these models in their suitability to combine them with a workload model to create a more powerful battery model.

1 Introduction

Portable devices often rely on battery energy to work. The energy stored in these batteries is limited. So, it is important to use this energy as efficiently as possible, to extend the battery lifetime. In this paper, we define the lifetime as the time one can use the battery before it is empty. Note that, for rechargeable batteries, this is not the same as the time one can use the battery before it stops working properly.

The battery lifetime, of course, mainly depends on the rate of energy consumption of the device. However, lowering the average consumption rate is not the only way to increase battery lifetime. Due to nonlinear physical effects in the battery, the lifetime also depends on the usage pattern. During periods of high energy consumption the effective battery capacity degrades, and therefore the lifetime will be shortened. However, during periods without energy consumption the battery can recover some of its lost capacity, and the lifetime will be lengthened.

Energy consumption of wireless devices has been studied using performance models. These models describe the various states a device can be in, and the energy consumption rate in these states. However, typically these models only take the energy consumption into account and do not deal with the effects of the usage pattern on the battery lifetime. To be able to do this we have to extend the model, by combining it with a battery model.

In the literature, many battery models can be found. Different approaches have been used to model the battery properties, varying from very detailed electro-chemical models to high level stochastic models. In this report an overview of all these different battery models is given. These models are evaluated for their suitability to combine them with a performance model. One of the models that turns out to be well suited for this purpose

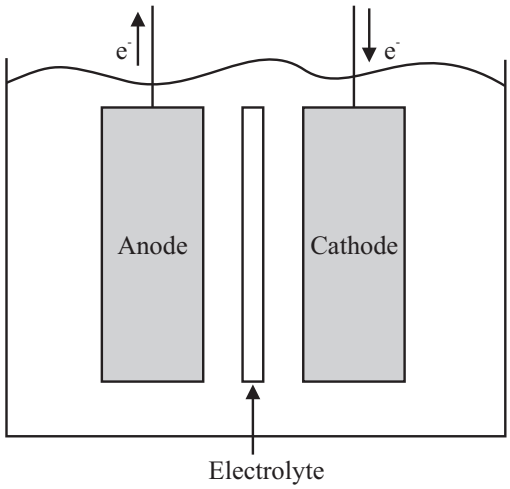

Fig. 1: Schematic picture of an electrochemical cell

is the Kinetic Battery Model. However, some modifications need to be made to adapt the model for the type of batteries used in the wireless devices.

The rest of this report is organized as follows. In Section 2 we give an introduction to the battery physics and the major battery properties we want to model. The different types of battery models are discussed in Section 3 through 6. In Section 7 the discussed models are evaluated, and we give a motivation for our choice to combine the Kinetic Battery Model with workload models. Finally, we end with some plans for future work.

2 Battery basics

A battery consists of one or more electrochemical cells, connected in series or parallel. In these cells chemically stored energy is converted into electrical energy through an electrochemical reaction. Figure 1 shows an schematic picture of a electrochemical cell. A cell consists of an anode, a cathode and the electrolyte, which separates the two electrodes. During the discharge, an oxidation reaction at the anode takes place. In this reaction a reductant (R_1) donates m electrons , which are released into the (connected) circuit. At the cathode a reduction reaction takes place. In this reaction, m electrons are accepted by an oxidant (O_2):

$$\begin{cases} R_1 \rightarrow O_1 + me^-, & \text{at the anode,} \\ O_2 + ne^- \rightarrow R_2, & \text{at the cathode.} \end{cases}$$

Modeling the behavior of batteries is complex, because of non-linear effects during discharge. In the ideal case, the voltage stays constant during discharge, with an instantaneous drop to zero when the battery is empty. The ideal capacity would be constant for all discharge currents, and all energy stored in the battery would be used. However, for a real battery the voltage slowly drops during discharge and the effective capacity is

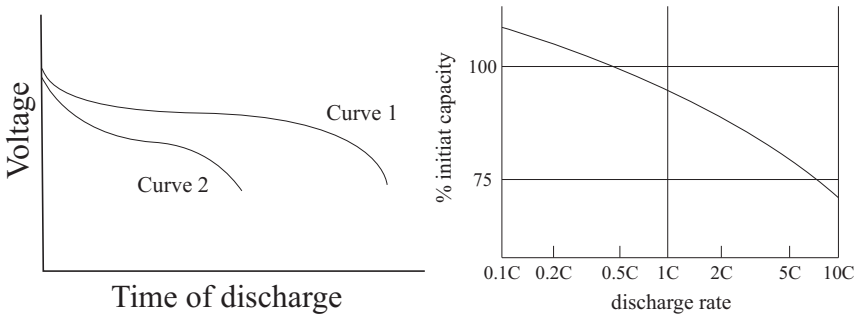

Fig. 2: Rate Capacity effect: The left figure shows the evolution of the voltage over time for a low and high discharge current, curve 1 and 2 respectively. The voltage drops faster for high discharge currents. The right figure shows the capacity as a function of the discharge rate. The discharge rate is given in terms of C rating, a C rating of $2C$ means that the battery is discharged in $\frac{1}{2}$ hours. The measured capacities are given relatively to the capacity at the 2 hour discharge rate, 0.5 C. The figure shows that the effective capacity drops for high discharge rates

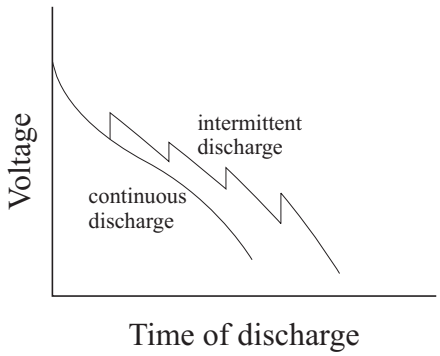

Fig. 3: Recovery effect: for intermittend discharges the battery can recover during idle periods. In the plot of the intermittent discharge the off-time is not shown, so one can clearly see the extension of the battery lifetime.

lower for high discharge currents, cf. Figure 2. This effect is termed the *rate capacity effect*. Besides this, there is the so-called *recovery effect*: during periods of no or very low discharge, the battery can recover the capacity "lost" during periods of high discharge to a certain extent, cf. Figure 3. In this way the effective capacity is increased and the battery lifetime is lengthened. For all types of batteries these effects occur. However, the extent to which they are exhibited depends on the battery type.

For constant loads, we can easily calculate the ideal battery lifetime (L) by dividing its capacity (C) by the discharge current (I): $L = C/I$. However, due to the rate capacity and the recovery effects this relation does not hold for real batteries. Many models have been developed to predict real battery lifetimes under a given load. In the following sections several of these models will be discussed.

3 Electrochemical models

The electrochemical models are based on the chemical processes that take place in the battery. The models describe these battery processes in great detail. This makes these models the most accurate battery models. However, the highly detailed description makes the models complex and difficult to configure.

Doyle, Fuller and Newman developed an electrochemical model for lithium and lithium-ion cells [1, 2, 3]. This model consists of six coupled, non-linear differential equations. Solving these equations gives the voltage and current as functions of time, and the potentials in the electrolyte and electrode phases, salt concentration, reaction rate and current density in the electrolyte as functions of time and position in the cell.

Dualfoil is a Fortran program that uses this model to simulate lithium-ion batteries. The program is freely available on the internet [4]. It computes how all the battery properties change over time for the load profile set by the user. From the output data, it is possible to obtain the battery lifetime. Besides the load profile, the user has to set over 50 battery related parameters, e.g., the thickness of the electrodes, the initial salt concentration in the electrolyte and the overall heat capacity. To be able to set all these parameters one needs a very detailed knowledge of the battery that is to be modelled. On the other hand, the accuracy of the program is very high. The program is often used as a comparison against other models, instead of using experimental results to check the accuracy.

4 Electrical-circuit models

The first electrical-circuit models were proposed by Hageman [5]. He used simple PSpice circuits to simulate nickel-cadmium, lead-acid and alkaline batteries. The core of the models for the different types of batteries is the same:

- a capacitor represents the capacity of the battery,
- a discharge-rate normalizer determines the lost capacity at high discharge currents,

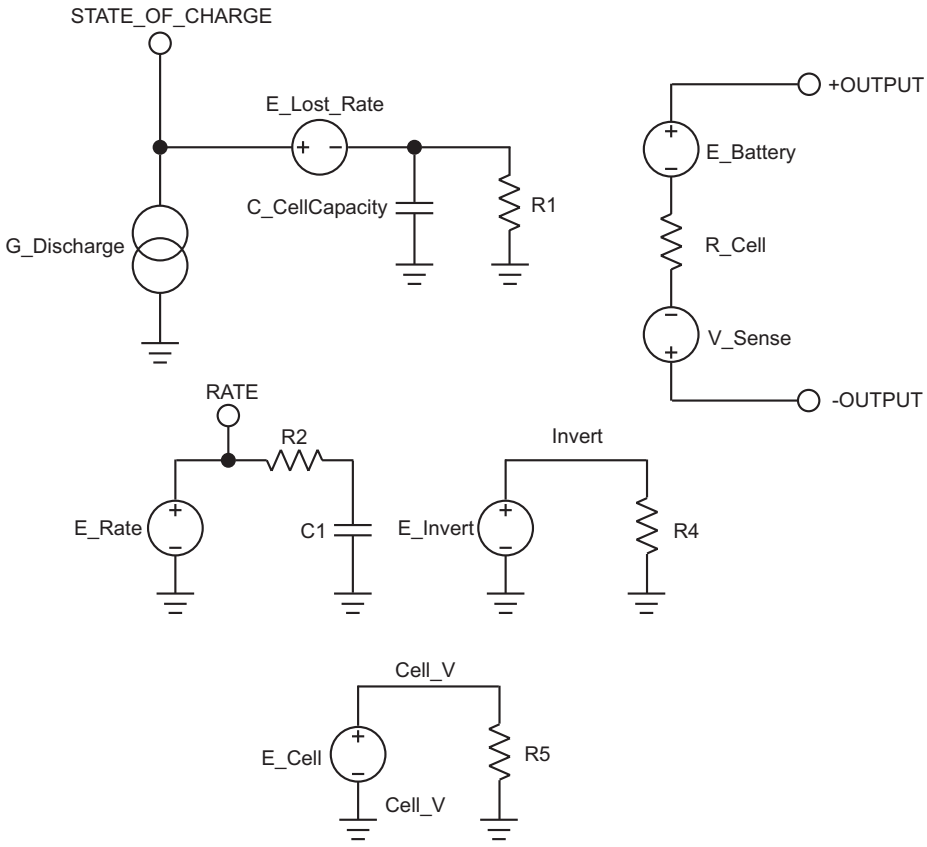

Fig. 4: Basic functional schematic covering all the modeled cell types. This basic schematic requires minor changes to complete the models for each specific cell[5].

- a circuit to discharge the capacity of the battery,
- a voltage versus state-of-charge lookup table,
- a resistor representing the battery's resistance.

Figure 4 shows the basic circuits used to model an arbitrary cell. Minor changes have to be made to complete the model for a specific cell type. Although the models are much simpler than the electrochemical models and therefore computationally less expensive, it still takes some effort to configure the electrical-circuit models. Especially the lookup tables used in the model require much experimental data on the battery's behavior. Furthermore, the models are less accurate, having an error of approximately 10%.

5 Analytical models

Analytical models describe the battery at a higher level of abstraction than the electrochemical and electrical circuit models. The major properties of the battery are modeled

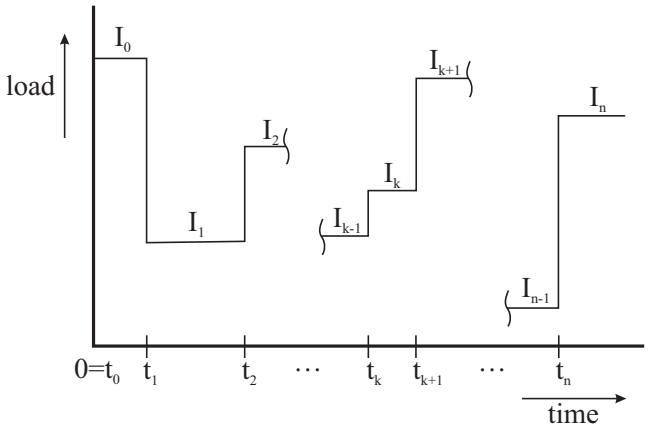

Fig. 5: An example of a piecewise constant discharge profile

using only a few equations. This makes this type of model much easier to use than the electrochemical and electrical circuit models.

5.1 Peukert's law

The simplest model for predicting battery lifetimes that takes into account part of the non-linear properties of the battery is Peukert's law [6]. It captures the non-linear relationship between the lifetime of the battery and the rate of discharge, but without modeling the recovery effect. According to Peukert's law, the battery lifetime (L) can be approximated by:

$$L = \frac{a}{I^b}, \tag{1}$$

where I is the discharge current, and a and b are constants which are obtained from experiments. Ideally, a would be equal to the battery capacity and b would be equal to 1. However, in practice a has a value close to the battery's capacity, and b is a number greater than one. For most batteries the value of b lies between 1.2 and 1.7 [7].

The results obtained by applying Peukert's law for predicting battery lifetimes are reasonably good for constant continuous loads. But the model does not deal well with variable or interrupted loads. In [6], Rakhmatov and Vrudhula give an extended version of Peukert's law for non-constant loads. In (1) I is replaced by the average current up to $t = L$. For a piecewise constant discharge profile, with t_k the points in time of current change, as shown in Figure, 5 this yields:

$$L = \frac{a}{\left[\frac{\sum_{k=1}^{n} I_k(t_k - t_{k-1})}{L}\right]^b}. \tag{2}$$

This equation is not as simple as it looks. It is impossible to easily isolate L in the equation, since L turns up inside the n-term sum as well ($t_n = L$). For $n = 1$ (2) reduces to (1). Although the extended Peukert's law can handle non-constant discharge profiles, it is still

too simple. Only the average discharge current is taken into account, and the recovery effect is still not modelled with it.

5.2 Rakhmatov and Vrudhula

Next to the extended Peukert's law, Rakhmatov and Vrudhula give a new analytical battery model in [6]. The model describes the diffusion process of the active material in the battery. The diffusion is considered to be one-dimensional in a region of length w. $C(x,t)$ is the concentration of the active material at time t and distance $x \in [0,w]$ from the electrode. To determine the battery lifetime one has to compute the time at which the concentration at the electrode surface, $C(0,t)$, drops below the cutoff level C_{cutoff}. The one-dimensional diffusion process is described by Fick's laws [6]:

$$\begin{cases} -J(x,t) &= D\dfrac{\partial C\,(x,t)}{\partial x}, \\ \dfrac{\partial C\,(x,t)}{\partial t} &= D\dfrac{\partial^2 C\,(x,t)}{\partial x^2}, \end{cases}$$

where $J(x,t)$ is the flux of the active matterial at time t and position x, and D is the diffusion constant. According to Faraday's law, the flux at the left boundary of the diffusion region ($x=0$) is proportional to the current $i(t)$. The flux at the right boundary ($x=w$) is zero. This gives the following boundary conditions:

$$\begin{cases} D\left.\dfrac{\partial C\,(x,t)}{\partial x}\right|_{x=0} &= \dfrac{i\,(t)}{\nu F A}, \\ D\left.\dfrac{\partial C\,(x,t)}{\partial x}\right|_{x=w} &= 0, \end{cases}$$

where A is the area of the electrode surface, F is Faraday's constant ($96485.31\,Cmol^{-1}$), and ν is the number of electrons involved in the electrochemical reaction at the electrode surface.

It is possible to obtain an analytical solution from these differential equations and boundary conditions using Laplace transforms. From the solution one can obtain the following expression relating the load, the battery lifetime (L) and the battery parameters:

$$\alpha = \int_0^L \frac{i\,(t)}{\sqrt{L-\tau}} d\tau + 2\sum_{m=1}^{\infty}\int_0^L \frac{i\,(t)}{\sqrt{L-\tau}} e^{-\frac{\beta^2 m^2}{L-\tau}} d\tau,$$

where $\alpha = \nu F A\sqrt{\pi D} C^* \rho\,(L)$, $\beta = \frac{w}{\sqrt{D}}$, C^* is the concentration at $t=0$, and $\rho(L) = 1 - \frac{C(0,L)}{C^*}$.

In the special case of a constant discharge current (I) the expression for α can be simplified to:

$$\alpha = 2I\sqrt{L} + 2\sum_{m=1}^{\infty}\left(\sqrt{L}e^{-\frac{\beta^2 m^2}{L}} - \beta m\sqrt{\pi}\Phi\left(\tfrac{\beta m}{\sqrt{L}}\right)\right),$$

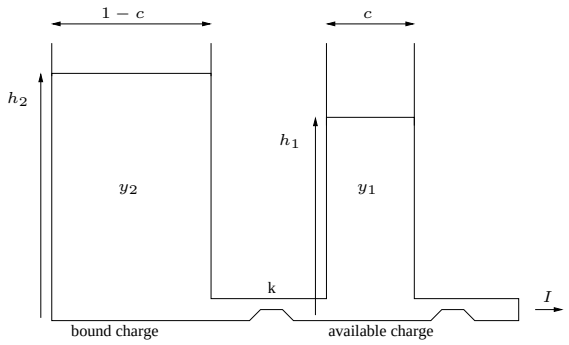

Fig. 6: Two-well-model of the Kinetic Battery Model

where $\Phi(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy$. One can obtain a good approximation for α by taking the first ten terms of the infinite sum. Together with an approximation for Φ this gives:

$$\alpha = 2I\sqrt{L}\left(1 + 2\sum_{m=1}^{10}\left(e^{-\frac{\beta^2 m^2}{L}} - \frac{\pi e^{-\frac{\beta^2 m^2}{L}}}{\pi - 1 + \sqrt{1 + \pi\frac{L}{\beta^2 m^2}}}\right)\right). \tag{3}$$

With (3) one can predict the battery lifetime for a given discharge current. To be able do these predictions one first needs to estimate α and β from experimental data.

In [6, 8, 9] the authors compare their model with the Dualfoil battery simulation program, and the extended version of Peukert's formula which can handle variable loads. The results of the Dualfoil simulation are used as reference values, since these simulations are very precise. For constant continuous loads, the model predicts lifetimes with an average error of 3%, and a maximum error of 6% compared to those obtained using the Dualfoil program. In contrast, Peukert's formula shows an average error of 14% and a maximum error of 43%. Peukert's formula works well for light loads, but the errors will become very large at heavy loads. For interrupted and variable loads in the experiments, the analytical model does even better, with a 2.7% maximum error and an average error of less than 1%. Again Peukert's formula performs a lot worse. This is, of course, clear when we note that Peukert's formula does not take the battery recovery effect into account.

5.3 Kinetic Battery Model

A third analytical model is the Kinetic Battery Model (KiBaM) of Manwell and McGowan [10, 11, 12]. The KiBaM is a very intuitive battery model. It is called kinetic because it uses a chemical kinetics process as its basis. In the model the battery charge is distributed over two wells: the available-charge well and the bound-charge well (cf. Figure 6). The available charge well supplies electrons directly to the load, whereas the bound-charge well supplies electrons only to the available-charge well. The rate at which charge flows between the wells depends on the difference in heights of the two wells, and on a parameter k. The parameter c gives the fraction of the total charge in the battery that is part of

the available-charge well. The change of the charge in both wells is given by the following system of differential equations:

$$\begin{cases} \frac{dy_1}{dt} = -I + k(h_2 - h_1), \\ \frac{dy_2}{dt} = -k(h_2 - h_1), \end{cases} \tag{4}$$

with initial conditions $y_1(0) = c \cdot C$ and $y_2(0) = (1-c) \cdot C$, where C is the total battery capacity. For h_1 and h_2 we have: $h_1 = y_1/c$ and $h_2 = y_2/(1-c)$. When a load I is applied to the battery, the available charge reduces, and the difference in heights between the two wells grows. When the load is removed, charge flows from the bounded-charge well to the available-charge well until h_1 and h_2 are equal again. So, during an idle period, more charge becomes available and the battery lasts longer than when the load is applied continuously. The differential equations can be solved using Laplace transforms, which gives:

$$\begin{cases} y_1 &= y_{1,0}e^{-k't} + \frac{(y_0 k' c - I)(1-e^{-k't})}{k'} - \frac{Ic(k't-1+e^{-k't})}{k'}, \\ y_2 &= y_{2,0}e^{-k't} + y_0(1-c)(1-e^{-k't}) - \frac{I(1-c)(k't-1+e^{-k't})}{k}, \end{cases}$$

where k' is defined as:

$$k' = \frac{k}{c(1-c)},$$

and $y_{1,0}$ and $y_{2,0}$ are the amount of available and bound charge, respectively, at $t = 0$. For y_0 we have: $y_0 = y_{1,0} + y_{2,0}$.

Next to the charge in the battery, the KiBaM models the voltage during discharge. The battery is modelled as a voltage source in series with an internal resistance. The level of the voltage varies with the depth of discharge. The voltage is given by:

$$V = E - IR_0, \tag{5}$$

where I is the discharge current and R_0 is the internal resistance. E is the internal voltage, which is given by:

$$E = E_0 + AX + \frac{CX}{D - X}, \tag{6}$$

where E_0 is the internal battery voltage of the fully charged battery, A is a parameter reflecting the initial linear variation of the internal battery voltage with the state of charge, C and D are parameters reflecting the decrease of the battery voltage when the battery is progressively discharged, and X is the normalized charge removed from the battery. These parameters can be obtained from discharge data. At least 3 sets of constant discharge data are needed for the non-linear least square curve fitting, which is described in detail in [11].

The KiBaM was developed to model large lead-acid storage batteries. These batteries have a flat discharge profile, which is well captured by (5) and (6). These equations do not hold for the modern batteries used in mobile devices, like Li-ion batteries, which have a sloped discharge profile. However, if one is only interested in the battery lifetime, and not so much in its actual voltage during discharge, one can still use the two-well model of

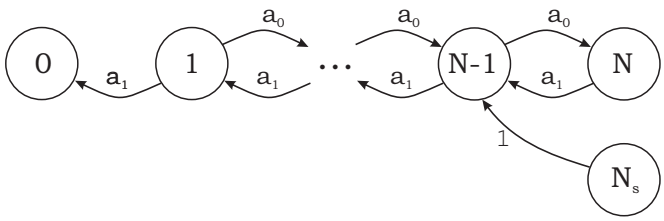

Fig. 7: The basic Markov chain battery model by Chiasserini and Rao [14]

the KiBaM, because the two-well model describes both the rate capacity and the recovery effect. One still may need to adapt the term of the flow charge between the two wells in (4) to the used battery type, which for example is done in [13] for Ni-MH batteries.

6 Stochastic models

Stochastic models aim to describe the battery in an abstract manner, like the analytical models. However, the discharging and the recovery effect are described as stochastic processes.

6.1 Chiasserini and Rao

The first stochastic battery models were developed by Chiasserini and Rao. Between 1999 and 2001 they published a series of papers on battery modeling based on discrete-time Markov chains [14, 15, 16, 17]. In [14] two models of a battery of a mobile communication device for transmitting packets are described. In the first and simplest model, the battery is described by a discrete time Markov chain with $N + 1$ states, numbered from 0 to N (cf. Figure 7). The state number corresponds to the number of charge units available in the battery. One charge unit corresponds to the amount of energy required to transmit a single packet. N is the number of charge units directly available based on continuous use. In this simple model, every time step either a charge unit is consumed with probability $a_1 = q$ or recovery of one unit of charge takes place with probability $a_0 = 1 - q$. The battery is considered empty when the absorbing state 0 is reached or when a maximum of T charge units have been consumed. The number of T charge units is equal to the theoretical capacity of the battery ($T > N$).

The second model described in the paper is an extended version of the first. Again, we have a discrete Markov chain with $N + 1$ states. But, in this second model more than one charge unit can be consumed in any one time step, with a maximum of M charge units ($M \leq N$). In this way, a more bursty consumption of energy can be modeled. Another new aspect is that there is a non-zero probability of staying in the same state. This means no consumption or recovery takes place during a time step.

In the models in [15, 16, 17] further extensions are made. To improve the model, the recovery probability is made state dependent. When less charge units are available the probability to recover a charge unit will become smaller. Next to the state dependence of

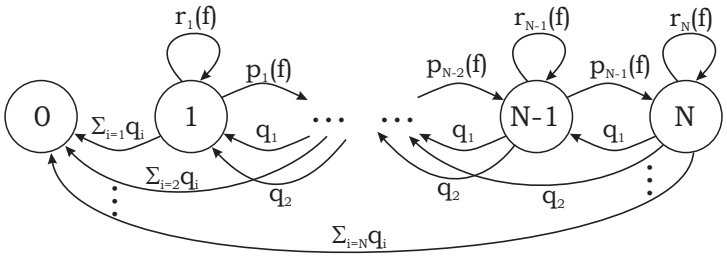

Fig. 8: The extended Markov chain battery model by Chiasserini and Rao [17]

the recovery, there is a phase dependence. The phase number (f) is a function of the number of charge units that has been consumed. When more charge units have been consumed, the phase number increases and this causes the probability of recovery to decrease.

In Figure 8, the state transition diagram of the model with all the extensions is shown. With probability q_i, i charge units are requested in one time slot. During the idle periods, the battery either recovers one charge unit with probability $p_j(f)$, or stays in the same state with probability $r_j(f)$. The recovery probability in state j and phase f is defined as [17]:

$$p_j(f) = q_0 e^{(N-j)g_N - g_C(f)} \tag{7}$$

where g_N and $g_C(f)$ depend on the recovery behavior of the battery. One can model different loads by setting the transition probabilities appropriately. However, one cannot control the order in which the transitions are taken. Thus, it is impossible to model fixed load patterns, and compute their impact on the battery lifetime.

The main property investigated by Chiasserini and Rao is the gain (G) obtained by a pulsed discharge relative to a constant discharge. This gain is defined as: $G = m/N$, with m the mean number of transmitted packages. G is computed as a function of the average number of packets arriving in a given time frame, and for different N varying from 3 through 50. The gain increases when the load is decreased, due to the higher probability to recover.

In [17] the final version of the model is used to model a Li-ion battery. To model the battery, N is set to $\sim 2 \cdot 10^6$, and 3 phases are used. This results in a Markov chain with approximately $6 \cdot 10^6$ states. The model is analyzed by numerical computations, and the results are compared with the electro-chemical model developed by Doyle et al. (cf. Section 3) [1]. With both models, the gain obtained from pulsed discharge compared to constant discharge is calculated for different discharge currents, cf. Figure 9. The gain increases for lower discharge demand rates and higher current densities. The latter is mainly due to the fact that the current densities are close to the specified limits of the battery. When the current density is over this limit the battery's capacity drops excessively fast, and therefore the gain obtained by pulsed discharge increases.

The results of the stochastic model have a maximum deviation of 4% from the electro-chemical model, with an average deviation of 1%. These results show that the stochastic model gives a good qualitative description of battery behavior under pulsed discharge.

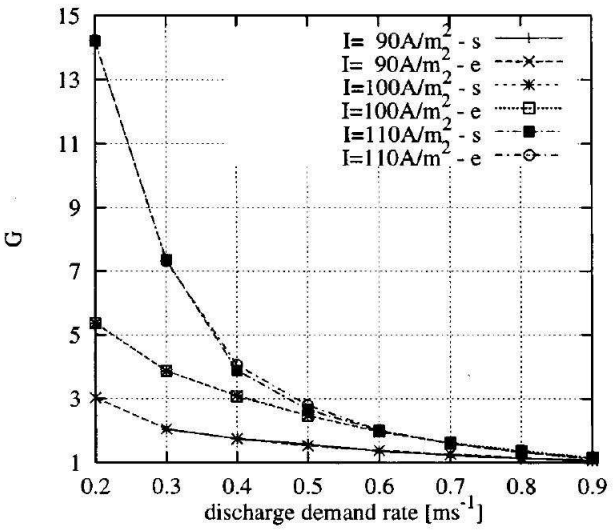

Fig. 9: Gain obtained under pulsed discharge with respect to constant discharge for different current densities. Results of the electrochemical model by Doyle et al.(**e**) and the stochastic model by Chiasserini and Rao (**s**) are compared.[17]

However, it is unclear how well the model performs quantitatively, since only relative numbers of lifetime are compared.

6.2 Stochastic modified KiBaM

Rao et al.[13] proposed a stochastic battery model in 2005, based on the analytical Kinetic Battery Model (KiBaM) proposed by Manwell and McGowan. The stochastic KiBaM is used to model a Ni-MH battery, instead of a lead-acid battery for which original KiBaM was developed. To be able to model this different type of battery, a couple of modifications have been made to the model. First, in the term corresponding to the flow of charge from the bound charge well to the available charge well an extra factor h_2 is added, changing (4) into:

$$\begin{cases} \frac{dy_1}{dt} = -I + k_s h_2 (h_2 - h_1), \\ \frac{dy_2}{dt} = -k_s h_2 (h_2 - h_1). \end{cases} \tag{8}$$

This causes the recovery to be slower when less charge is left in the battery. The second modification is that in the stochastic model the possibility of no recovery during idle periods is added.

The battery behavior is represented by a discrete time transient Markov process. The states of the Markov chain are labeled with three parameters (i, j, t). The parameters i and j are the discretized levels of the available charge well and bounded charge well

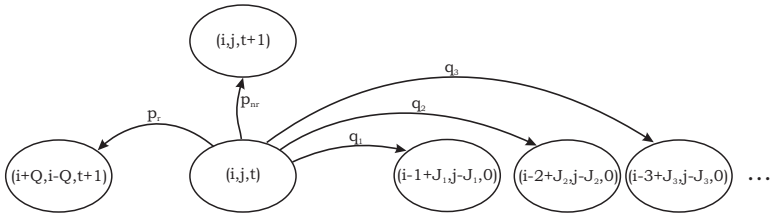

Fig. 10: Part of the state transition diagram of the stochastic KiBaM[13]

respectively, and t is the length of the current idle slot; this is the number of time steps taken since the last time some current was drawn from the battery.

Figure 10 shows a part of the state transition diagram. The transitions are summarized by the following equations:

$$(i,j,t) \longrightarrow \begin{cases} (i+Q, j-Q, t+1) \\ (i,j,t+1) \\ (i-I+J, j-J, 0) \end{cases} \tag{9}$$

The first two equations correspond to the time steps in which the current is zero. With probability p_r, the battery recovers Q charge units, and with probability p_{nr} no recovery occurs. Both p_r and p_{nr} depend on the length of the idle timeslot (t). The third equation corresponds to the time steps in which a current is drawn from the battery. With probability q_I, I charge units are drawn from the available charge well, and at the same time J charge units are transfered from the bound to the available charge well.

The probabilities q_I are defined by the load profile. Since the q_I are equal for all states, it is impossible to control in what sequence the currents are drawn from the battery in this model, and thus to fully model a real usage pattern.

In the model of the Ni-MH battery the charge in the bounded and available charge well is discretized in $27 \cdot 10^7$ and $45 \cdot 10^7$ charge units respectively. This results in a Markov chain too big to handle as a whole. So, no analytical solution to the model can be given. To obtain battery lifetimes several runs of discharging the battery are simulated with the model.

In [13], Rao et al. compare the calculated battery lifetimes with some experimental results. In a simple experimental setup, different periodic loads are applied to an AAA Ni-MH battery, and its lifetime is measured. In the first set of experiments the frequency of the applied load is varied, keeping the ratio of on and off time constant at one. In these experiments the battery lifetime increases as the frequency decreases. In a second set of experiments, the ratio between on and off time is varied by keeping the on-time fixed to 2 seconds and increasing the off-time from 0 to 3.5 seconds. As expected, the lifetime and the delivered charge increase when the off-time increases, since the battery has more time to recover.

The results of the simulations show that the model is quite accurate for predicting battery lifetime and charge drawn from the battery, since a maximum error of 2.65% for the simulations was found.

	battery type	R.C. effect	Recov effect	# pars.	accuracy
Dualfoil [1, 2, 3]	Li-ion	+	+	> 50	very high
Electrical circuit [5]	Ni-Cd, alkaline Lead-acid	+	+	15-30	medium
Peukert [6]	all	+	-	2	medium, 10% error
Rakhmatov [6]	Li-ion	+	+	2	high ,5% error
KiBaM [10]	Lead-acid	+	+	2	high
Chiasserini [14, 15, 16, 17]	Li-ion	-	+	2	high, 1% error
Stochastic KiBaM [13]	Ni-MH	+	+	2	high, 2% error

Tab. 1: Battery models overview

7 Evaluation

We want to use a battery model to combine it with a workload model. With this combination it will be possible to model the energy consumption of battery powered devices, and predict battery lifetimes for different usage patterns. For this purpose, we need a "simple" battery model that still gives a good description of the most important non-linear effects, i.e., the rate capacity effect and recovery effect.

Table 1 gives an overview of the different battery models and their relevant properties. Most battery models are not well suited to be combined with a performance model. Although the electro-chemical model is the most accurate model and the Dualfoil program is often used as "reality" to check the performance of other battery models, the model is too complex for our needs. A very detailed knowledge of the battery is necessary to be able to set all the parameters of the model. Furthermore, the computational complexity of solving the six coupled partial differential equations is very high, which makes the execution of the program slow. Like the electro-chemical model the electrical circuit models are too complex. The modeling of the battery's electrical properties is too detailed for what we want from the battery model. Peukert's formula, on the other hand, is too simple. It could be easily implemented into a performance model. However, it does not take the recovery effect into account. Therefore, it will underestimate battery lifetimes for usage patterns with idle periods. The stochastic model by Chiasserini is also too limited. It focuses on the recovery effect only. The model is designed for pulsed discharge of the battery, but it does not handle arbitrary load profiles with varying discharge currents.

The KiBaM and the model by Rakhmatov et al. do take into account both the rate capacity effect and the recovery effect. Both models use a system of two differential equations to describe the battery and with both models one can compute the battery lifetime for an arbitrary piecewise constant load profile. However, the second-order differential equation in the model by Rakhmatov and Vrudhula makes it harder to combine it with a performance model. Furthermore, the KiBaM describes the battery in a more intuitive manner. The differential equations give a clear insight on how the battery behavior is

modeled. This makes the KiBaM preferable over the model by Rakhmatov.

The first order differential equations of the KiBaM can be fitted nicely into a Markov reward model, creating the needed combination of a workload and battery model. However, the KiBaM is designed to model a lead-acid battery, which is not used in wireless devices and has a more linear discharge behavior than the modern batteries. Experimental results of Rao et al. indicate that the battery lifetime of a Ni-MH battery depends strongly on the frequency of the applied load. In the KiBaM this dependency is not seen on small time scales. Only at large time scales, (> 30 minutes) the non-linear behavior is seen. Rao et al. solved this problem in their stochastic KiBaM by making the probability to recover during idle periods dependend on the length of the idle period.

In the analytical KiBaM one can also do this by making the recovery parameter k dependend on the time the battery is idle. However, this model can not be combined with the Markov reward model describing the usage pattern of a device anymore. Since, in this model one does not know the time one has spend in a specific state, and thus one can not change the parameter k appropriately.

Another approach is needed to combine the workload with the battery model. One possible approach can be using timed automata. In this type of model one does have the notion of local time, the time spend in a state, and one can change the parameter k according to this time.

8 Outlook and future work

The power consumption of portable devices is an important issue, for example Martin has written his PhD-thesis on the trade-offs between power and performance in the case of CPU speed-setting for mobile computing [7]. Much research has been done on decreasing the average power consumption of these devices. One important approach is power-management [18, 19, 20, 21]. In this approach, the power consumption is reduced by selective slowdown or shutdown of components. However, in most of the research the battery is only considered as a limited-power source. The non-linear properties, rate-capacity and recovery effect, are not taken into consideration.

The combination of a workload model with the kinetic battery model will give a powerful model. With this model one can compare the energy consumption of various workload schemes and their influence on the battery lifetime.

To be able to get meaningful information out of the model, we first need to set the parameters in the kinetic battery model for the specific batteries used in the device. The model parameters can be determined by doing some simple discharge measurements. Furthermore, the battery model still has to be validated for more complex discharge profiles. Both for the determination of the parameters and the validation of the battery model an experimental set-up is needed. With this set-up the battery is discharged according to a given discharge profile. During the discharge the battery voltage and discharge current are monitored, and the battery lifetime is measured. In [13] an example of such a set-up is given.

The applications of the model can be various. Our focus will be on the energy consumption of wireless communication devices.

One application is the energy consumption in WLAN-802.11a. Various hardware designs have different energy consumption rates for the MAC states [22, 23]. Using the model one can determine which design gives the longest battery lifetime.

Another application of the new model can be found in a cooperation with Thales Communications (Huizen). Thales has developed a communication and information module (CIM) for the Dutch army. This is a vest in which various battery powered devices, like a radio, GPS and a display, are integrated. It is, of course, very important to use the batteries energy efficiently. A simple version of this vest just has a GPS-device and a radio transmittor. At fixed time intervals the position of the soldier is determined with the GPS and sent to the commander using the radio. Using the battery-workload model we can predict the the battery lifetime for different schemes of measuring and sending the position of the soldier, and in this way optimize the battery usage of this system. When an appropriate workload model is available the model can also be applied on the other devices integrated in the CIM.

A third application is the comparison of polling schemes in bluetooth [24, 25]. The order in which the slaves are polled by the master determines the way the load of the slaves is distributed over time. For example, in exhaustive round robin polling the slaves have big bursts of active periods with long idle periods in between, whereas in pure round robin polling, the active periods are short and more evenly distributed over time.

In most of the discussed models the battery is considered as a singel chemical cell. However, some batteries consist of more cells connected in series or parallel. In the case of a battery with cells connected in parallel, one can exploit the non-linear properties to extend the battery lifetime. By switching between the cells during discharge, the cells can recover during the idle periods. Of course, the way one switches between the cells will determine the gain in lifetime. It is important to find the optimal strategy for balancing the load and the idle periods between the cells.

The same strategy can be applied for balancing the load over multiple batteries that power a device. By switching between the batteries one will be able to use the device for a longer period of time than by using the batteries sequentially.

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