Near coast tsunami waveguiding: simulations for various wave models

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Abstract

Shallow parts in a sloping bottom toward the coast can be expected to act as a waveguide, in partial analogy with optical waveguiding. We will present numerical simulations that convincingly show that large enhanced wave amplification happens for tsunami waves in certain geometries and bathymetries. Since this is even the case for shallow regions that have cross sections of the order of badly resolved numerical scales, this phenomenon may at least partly explain that tsunami heights and coastal effects as observed in reality show such high variability along the coastline.

This report, following [1], supports a more concise publication that will be published soon [2]. In this report we will provide detailed results of extensive simulations using various wave models and different gridsizes. We will show results obtained with the commonly used Shallow Water Equations and with a more accurate dispersive wave model. For the latter simulations we use a recently derived Variational Boussinesq model. We will also show that relatively small gridsizes are needed to capture the transversal flow near the waveguide; on grids that are too coarse, the enhanced amplification will not be observable. This may provide a partial explanation that spatial variability due to relatively shallow bottom variations will not be present in most simulations.

Key words: Tsunami simulations, wave guiding, variational Boussinesq model, shallow water equations, nearcoast tsunami waveguiding

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1. Introduction

Tsunami heights and effects show a high variability along the coast [3,4], and numerical simulations often seem to underestimate the most extreme heights that are observed in the field. This paper aims to increase our understanding by showing that relatively small and shallow structures can produce largely amplified waves by what we will call near-coast waveguiding. Tsunami wave guiding has been studied for deep water running over ridges. Using the relative index as a characteristic number related to the quotient of depths, this deep water case corresponds to weak waveguides. Near the coast we will experience strong waveguides, leading to much higher amplifications. Although the driving mechanism is different, this is somewhat similar to waveguiding as it appears in optics. In optics, material structure with elongated geometries in the direction of propagation which have the property that the speed of light in the material is lower than in the surrounding area (like a glass fiber in air), causes the light to be confined within (a neighborhood of) the waveguide. Monochromatic, so-called guided modes, are traveling waves with intensity strongly confined within the wave guide and with (exponentially) decaying tails in the exterior. The propagation speed of such guided modes is specific for the profile since these are the eigenvalue and eigenfunction respectively of a spatial eigenvalue problem. The speed of the principal mode is then in between the values of the speed inside and outside the waveguide.

It can be expected that in the case of water waves shallower parts in the bathymetry may give rise to a similar phenomenon, since the wave velocity is smaller in shallower areas. In this paper we will investigate this for the case of long tsunami-like waves that approach the coast and encounter shallower regions transversal to the propagation direction. If the shallower region is more narrow than the width of the wave, as will usually be the case for tsunami waves, the shallower region will act as a waveguide, and we have tsunami waveguiding. The extension of the wave far outside the waveguide is an essential difference with optical guided modes. The quasi-guided tsunami wave will have to adjust to the finite amplitude undisturbed external wave; this will influence both the profile and the velocity of guided tsunami waves. Yet the main conclusion of this paper will be that relatively narrow waveguides lead to large enhanced amplification in the run-up region. We will show the appearance of guided waves with largely amplified amplitudes for the case of a specific geometry with synthetic bathymetry. In another paper we will show that bathymetries leading to such phenomena actually appear for specific parts of the Indonesian coast. In a forthcoming paper we will derive a low dimensional dynamical model that can describe the phenomenon theoretically.

The introduction of the phenomenon described above was introduced already in [1] using shallow water simulations. In this consecutive Technical Report we describe in more detail how linear dispersive simulations affect the precise outcome of the enhancement. All wave models and gridsizes will show qualitatively the same result, but quantitative results of the simulations depend somewhat on the wave model and considerably on the gridsize that is used. Concerning the wave model, we will compare results with the often used shallow-water equations (SWE) with the results of a new Variational Boussinesq model (VBM) that includes essential aspects of dispersion. In this report we will restrict to the linear versions of these codes. Our results, summarised in Table 2, show that dispersive effects are clearly present and can influence run-up heights for the geometries and bathymetries used here. The physical process of tsunami waveguiding is largely determined by additional water flow toward and from the waveguiding region. To capture that transversal flow sufficiently accurate, it is expected that the gridsize should be sufficiently small. Somewhat to our surprise, this effect is very substantial. In the cases of the synthetic bathymetry that we will use in this paper, gridsizes as small as 250 m are required in both directions; this is to be compared with the gridsizes that are commonly used and are 6 to 8 times larger. For these larger gridizes, the simulated amplification is less and severely delayed, as is detailed in Table 3. These two effects of wave model and gridsize may be the underlying explanation of the fact that, when using coarse-grid calculations with SWE, tsunami waveguiding has not yet been reported to be a possible (partial) explanation for large near-coast variability of waveheights, and consecutive effects on the coasts.

The organization of this report is as follows. We will comment on the various wave models in Section 2. In section 3 we specify and motivate the synthetic data for the geometry and bathymetry of a coastal area with a waveguiding structure and for the initial tsunami profile. In Section 4 we consider the simple problem of wave propagation over a flat bottom and in Section 5 the propagation into shallow water toward the coast above a transversally uniform sloping bottom. For the last case, in the absence of a waveguide, it is shown that the theoretical WKB results are close to the numerical results. In section 6 we take this uniformly sloping bathymetry and add a ridge of fixed height that will act as waveguide. Various simulations will show large
enhanced amplification of the crest heights, up to a factor of 2 to 3 in realistic situations. The differences from different grid sizes, and some other aspects will be detailed in the final section 7.

2. Wave models and simulations

In this report we present results of simulations that illustrate the phenomenon of tsunami wave guiding. In this section we discuss the various wave models that have been used for the simulations, and describe their peculiarities and differences.

2.1. Shallow Water Models

The linear and nonlinear wave models for shallow water waves, the so-called Shallow Water Equations, will be denoted by LSW and SW respectively in the following. These are the most commonly used models since the numerical implementations can deal well with the large domains that have to be considered and provide fast results for various implementations such as finite differences, finite elements and pseudo-spectral. Characteristic for the LSW-model is that waves of different wavelength travel with the same speed; above a flat bottom, this speed is $\sqrt{gh}$, where $g$ is the gravitational acceleration and $h$ is the depth. As a consequence, any initial profile will translate undisturbed in shape. When running into shallower water, part of the wave will be reflected. For the forth going wave, the speed and the wavelength decrease proportional to $h^{1/2}$, and, as a consequence of energy conservation, the wave height will increase proportional to $h^{-1/4}$. These bathymetric induced changes are remarkably well described by the explicit theoretical expression known as the WKB-approximation (Wentzel-Kramer-Brillouin), as we will see in Section 4.

2.2. Dispersive effects and Variational Boussinesq Model

The LSW and SW model ignore changes in the vertical direction in the fluid layer. Differences in surface elevation cause such variations to an extent that depends on the shallowness, defined as the ratio of depth and wavelength. These variations lead to the effect of dispersion, which means that harmonic waves of smaller wavelength propagate slower than waves of larger wavelength. For the largest waves, the speed approaches that of the shallow water limit, $\sqrt{gh}$. Since any initial surface elevation on a bounded area will contain harmonic waves of different wavelength, the time evolution of the initial disturbance will show the separation of the constituent waves. For a characteristic tsunami bipolar initial profile the result will be that a ‘dispersive tail’ consisting of slower, smaller amplitude waves will become visible. The development of the tail leads to a decrease of the initial wave height. However, as we shall see, entrance in the still water area may compensate this for bipolar waves as considered here.

It could be argued that for tsunami-type of waves the dispersive effects can be neglected since such waves have initial wavelength of at least, say, 50 km and are typically generated in the ocean where the depth is, say, 4000 m. The initial shallowness ratio 4/50 will become only smaller when running into shallower water, proportional to $h^{1/2}$. Nevertheless, as we will see from simulations in this report, dispersion causes differences for tsunamis running above a flat bottom, a uniformly sloping bottom and when a waveguide is present.

The results of simulations to be presented were obtained using a pseudo-spectral implementation of a new variational Boussinesq model [5, 6]; we will refer to the linear version as LVB, and to the full nonlinear one as VB in the following.

3. Initial wave, geometry and bathymetry

In the following we will take a specific quantitative example of a tsunami approaching the coast with a waveguide in the propagation direction. We motivate the choice of quantitative parameters in the simulations by the example of a tsunami of wavelength 40 km generated by tectonic plate motions at a depth of 4000 m.
3.1. Initial waveprofile

We consider propagation in the direction of the coast, taken as the $x$-axis. At the generation region at a depth of 4000m we assume the wave to have a wavelength of $\Lambda_{4000} = 40$km, and that it is uniform in the transversal ($y$-) direction, or at least has a transversal width much larger than the width of the waveguide. However, to reduce the required calculation domain, we start the simulation as if the wave has traveled without dispersion till a depth of 1000m. Neglecting the dispersion in this area, will affect the rest of the calculations, compare the following figures Fig. 2 and 3. We note that, therefore, all dispersive effects that will be visible in the simulations to follow are solely due to dispersion in the part of the numerical domain; a dispersive calculation over the full physical domain would show more dispersive effects.

Calculated approximately with LSW, using the relation that holds for harmonic wavetrains between wavelength, propagation speed (approximated by $c = \sqrt{gh}$) and period, the period of the wave is found to be $T = 200s$, i.e. angular frequency $\omega = \pi/100$.

For the initial wave profile we take a single period (smoothened) bipolar profile of the form $\partial_x (\cos^4 (x))$, extended by zero outside the interval $(-\pi/2, \pi/2)$ and scaled to get the desired amplitude and wavelength. In detail, the expression is given by

$$\eta_0 = -a.3 \cos^2 [\pi/2 \times \text{min}(2 \times |x| / \Lambda, 1)]. \sin [\pi \cdot \text{sign}(x). \text{min}(2 \times |x| / \Lambda, 1)],$$

with $2a$ the amplitude and $\Lambda$ the wavelength. A plot of this initial wave is given in Fig. 2.

To simulate that the initial wave elevation is caused by a fast bottom excitation of that form, we assume that this initial wave profile is released without speed; then this profile falls apart above a flat bottom in a right and left traveling wave of the same amplitude $a$, see Fig. 2.

3.2. Bathymetry

We will be interested in a sloping bottom toward the coast ($x$) which has a waveguiding structure in the transversal ($y$) direction; we will describe the details below, first the uniform sloping bottom, then a ridge structure, and after that the combined bathymetry.

3.2.1. Uniformly sloping bottom

Without the presence of a waveguide, we take a bottom profile that is uniform in the transversal direction described by the function $h_u(x)$.

In the simulations the bottom connects a depth $h_0$ for $x < X_0$ with a flat bottom region of smaller depth $h_1$ downstream for $x > X_1$. The depth is taken constant $h_0$ to the left to prevent possible reflection from bathymetry at the left. A smooth transition with average slope $(h_0 - h_1)/(X_1 - X_0)$ in the interval $x \in [X_0, X_1]$ is taken as follows

$$h_u(x) = h_0 + (h_1 - h_0) \cdot H ((x - X_0)/(X_1 - X_0))$$

where the smoothened transition function $H$ is given by $H(x) = 0$ for $x < 0$ and $H(x) = 1$ for $x > 1$ with transition

$$H(x) = \frac{1}{2} + \frac{1}{2} \tanh \frac{x - \frac{1}{2}}{x - x^2} \text{ for } 0 < x < 1.$$

3.2.2. Waveguide geometry

We introduce a waveguiding structure by a transversal depth variation that describes a shallow part in between deeper areas.

For the numerical calculations we consider a symmetric ridge of height $R_0$ with sloping sides. At half-height, we take the width to be $2W$, with a flat top of width $2W_0$ and sloping sides of width $S$; for all simulations we will take $2W = 4km$, $2W_0 = 2.5km$ and $S = 1.5km$. To prevent artificial reflections of the incoming wave from the appearance of the wave guide, we let the ridge height increase smoothly from 0 at $x = X_0/4$ till $R_0$ at $x = 3X_0/4$. The total waveguide structure is then given by

$$R(x, y) := R_0 \cdot (H ((y + W_0 + S)/S) - H ((y - W_0)/S)) \cdot H ((x - X_0/4)/(X_0/2)).$$
3.2.3. Waveguide on sloping bottom

To illustrate the waveguiding phenomenon we consider as bathymetry the uniform sloping bottom that is combined with the transversal ridge wave guide above:

\[ h(x, y) = h_u(x) - R(x, y) \]  \hspace{1cm} (4)

A schematic 3D plot of the bathymetry is shown in Fig. 1.

4. Evolution above flat bottom

In Fig. 2 at the left we show a 1D plot of the cross-section of the initially plane wave with wavelength \( \Lambda = 20 \text{km} \). This wave is released above a horizontal bottom at depth 1000m. For the LSW-model, the resulting evolution along the \( x \)-axis is shown in the right plot. It illustrates the splitting of the initial wave above a flat bottom into a right and left travelling wave. In the plot we observe the surface elevation at the end of the calculation time interval as the solid line. The dashed lines show some basic elements of the past dynamics; we will use these curves, to be abbreviated by MTC and MTT curve, in similar plots in the following. The MTC and MTT curves (which are variants of the maximal temporal amplitude introduced in [5]) denote the maximal temporal crestheight and minimal temporal throughdepth respectively. That is to say, at a position \( x \), they represent the maximal and minimal waterheight as function of the time, respectively, so

\[ MTC(x) = \max_t \eta(x, t), \quad MTT(x) = \min_t \eta(x, t). \]

We show the usefulness of these MTC and MTT curves to illustrate the effects of dispersion for flow above a flat bottom. In Fig. 3 we plot the evolution above a flat bottom as in Fig. 2 but now with dispersive effects, for the LVB- and the VB-model. The effect of nonlinearity is small, as can be expected. But the effect of dispersion is considerable: the bipolar profile changes, partly as an effect of running into still water. For the
right running wave the negative wave elevation in front is reduced while the consecutive positive elevation is enlarged. We will call this 'dispersive amplitude exchange' caused by running into still water. Besides this, compared to the (L)SW-model, we observe that a dispersive tail develops behind the bipolar wave.

5. Run-up on a uniform slope

We present in this section results for a uniform sloping bottom; in the next section we consider the effect of the presence of a waveguide to deal with waveguided run-up.

5.1. Geometry and numerical parameters

For the numerical calculations we take the geometry described by $h(x)$ with parameter values $X_0 = 25km$, $X_1 - X_0 = 10km$, and $h_0 = 1000m$. We will consider three cases of depth after run-up: $h_1 = 100m$, $h_1 = 60m$ and $h_1 = 20m$; these choices are based on the cases to be considered in the next section about waveguided run-up. As initial wave we took the profile (1) with wavelength $\Lambda_{1000} = 20km$. For the simulations we used a gridsize of 250m in each direction. The unnecessary small gridsize is taken for comparison with the simulations of the waveguided run-up for which the choice of this small grid is essential.

5.2. Simulation results

Fig. 4 shows results of simulations for the LSW-model. In the left plot the wave evolution is shown after 1200s; to the left ($x < 0$) the wave is running over a flat bottom at depth 1000m, and to the right ($x > 0$) the wave is running-up till a depth of 100m. In the plot at the right the wave profile after run-up is shown.

Similar calculations as in Fig. 4 were performed with the SW-model; for the case considered here, the results are almost the same and will not be shown. In Fig. 5 we show plots for the dispersive dynamics with the LVB-model as in Fig. 4.

In the next plots we show density and 3D images for the uniform run-up simulations considered above: Fig. 6 for LSW and Fig. 7 for LVB.

5.3. Discussion of simulation results

We briefly comment here on the results shown in the plots on uniform run-up.

5.3.1. Qualitative observations

All results show clearly the basic physical phenomena that during run-up the crestheight increases and that the wavelength and the propagation speed decrease, very close to predictions by the WKB-method, as we
Fig. 4. LSW-simulation. To the left the wave is running above a flat bottom at depth 1000m. To the right there is a uniform run-up from 1000m to 100m depth between 25km and 35km. Observe the reflected wave, traveling to the left. The right plot shows the waveform after run-up.

Fig. 5. Uniform run-up for LVB at the left, and the wave profile after run-up at the right, for the same bathymetry as Fig.4.

Fig. 6. Uniform run-up for LSW-simulation. At the left the density plot and at the right a 3D impression of the waveprofile after run-up.

shall see below.

All simulations show a substantial reflection from the run-up region, visible in the left travelling wave near position $-50km$.

The difference between the non-dispersive LSW and the dispersive LVB-model is the distortion of the wave form: the LSW-evolution retains almost the skew-symmetric profile, while the LVB evolution shows a substantial exchange of amplitude between the front and consecutive wave, and the development of a dispersive tail.
5.3.2. Quantitative results

A summary of the results of the crestheight amplification for the two cases will now be given. For a uniformly sloping bathymetry, the run-up is essentially one-dimensional and we can use simple expressions for wavelength and amplitude changes. For LSW, the wave speed $c$ is related to the depth $h$ by $c = \sqrt{gh}$. Then the WKB-approximation holds as a consequence of energy conservation. Although this approximation is commonly only used for harmonic waves running over slowly varying bathymetry, the result is more generally true for arbitrary wave forms. Hence, assuming mild slopes, the wavelength $\Lambda$ decreases with the square root and the amplitude increases with the fourth root of the depth, so

$$\Lambda(h) = \Lambda(h_0) \sqrt{h/h_0}$$
$$a(h) = a(h_0) \sqrt{h_0/h}. \tag{5}$$

In the table below, the maximal crestheights (immediately) after run-up are given for the theoretical WKB approximation and for the numerical results for LSW and LVB.

Table 1. Table of amplitude amplification for wave run-up from 1000m depth till depth $h_1$.

<table>
<thead>
<tr>
<th>depth $h_1$</th>
<th>WKB</th>
<th>LSW</th>
<th>LVB</th>
</tr>
</thead>
<tbody>
<tr>
<td>100m</td>
<td>1.8</td>
<td>1.9</td>
<td>1.9</td>
</tr>
<tr>
<td>60m</td>
<td>2.0</td>
<td>2.0</td>
<td>2.2</td>
</tr>
<tr>
<td>20m</td>
<td>2.7</td>
<td>2.8</td>
<td>2.8</td>
</tr>
</tbody>
</table>

All numerical values are close to the theoretical linear WKB-approximation. The good approximation of the WKB-result is actually somewhat remarkable if we take into account that the bottom steepness is not particularly small (slope $\approx 1 : 11$), as was evidenced by the substantial large reflection.

6. Waveguided run-up

In this section we investigate the run-up when a waveguide is present. We consider two cases of ridge height: $R_0 = 40m$ and $R_0 = 80m$, so that for $x > X_1$ the depth above the waveguide is 60m and 20m respectively, with 100m outside the waveguide.

As a measure of the strength of a waveguide we introduce the ‘relative index’ $N$ as the quotient of the velocities outside and inside the waveguide. For the waveguided run-up this index is then given by

$$N = \sqrt{h_u/(h_u - R_0)}. \tag{6}$$

Since the exterior depth decreases during run-up, the strength will increase during run-up; for $R_0 = 40m$ the strength increases from $N = 1.02$ at $x = X_0$ to $N = 1.29$ for $x > X_1$, while for $R_0 = 80m$ the strength increases from $N = 1.04$ at $x = X_0$ to $N = 2.24$ for $x > X_1$.

For ease of comparison with the case above without waveguide, we start to present an overview of the waveguided run-up process by showing the evolution in the $x$-direction at the symmetry line $y = 0$ of the geometry. Then we investigate in detail the transversal aspects.
6.1. Numerical parameters

For the numerical calculations we take the geometry as described before, $h_u(x)$ with parameter values $X_0 = 25\, km$, $X_1 - X_0 = 10\, km$, and $h_0 = 1000\, m$. We will consider two cases of depth after run-up: $h_1 = 60, 20\, m$. In all cases the width of the wave guide is taken as $2W = 4\, km$.

As initial wave we took as before the profile (1) with wavelength $\Lambda_{1000} = 20\, km$.

For these simulations, we used a gridsize of 250 m in each direction. This choice of a relatively small gridsize was made to assure that in the longitudinal and transversal direction all aspects of the waves are covered well. In the longitudinal direction, at the most shallow depth of 20 m that we will encounter in the simulations, the wavelength of nondispersive waves would be approximately $20/7\, km$, so that there are at least ten grid points in the waves of smallest wavelength. In the transversal direction, this small gridsize turned out to be necessary to simulate correctly the flow toward and from the waveguide; see Section 7 for remarks about simulations with coarser grids.

Also care has been taken for a sufficiently wide numerical window in the transversal direction. It had to be taken rather wide (30 km around the waveguide of 4 km) to assure that at the boundary the incoming wave was not affected by the presence of the waveguide during the time interval of evolution. For a window that is too narrow, the periodicity would lead to incorrect, too large, wave amplifications.

6.2. Simulation results for tsunami waveguiding

The symmetry in the profiles that we found causes the maximal heights to appear at the center of the waveguide, at $y = 0$. In fact, for the given width of the waveguide, we only observed the 'fundamental' mode, which is symmetric. For wider waveguides, higher order modes were also found. Therefore, one way to show the process, is to plot the evolution of the wave elevation in the coastal direction by using the MTC and MTT curves at the cross section $y = 0$. We will first show these results for the various models and for two heights of the ridge. Then we will give 3D impressions after which we will show in more detail transversal cross sections near the waves of maximal crestheight.

6.2.1. Evolution in the coastal direction

We present for the two cases $R_0 = 40\, m$ and $R_0 = 80\, m$ the results of simulations with the LSW- and LVB-models in Fig. 8, 9, 10 and 11.

6.2.2. Transversal deformations

The initial wave is uniform in the transversal $y$-direction. The presence of the waveguide will disturb the uniformity: inside and in a neighbourhood of the waveguide the surface elevation will differ from the elevation further out.
We first present density and 3D plots to illustrate the deformation in a global way, and then show transversal cross-sections. We concentrate on the region near the maximal crestheight. We give details for the case of the waveguide with $R_0 = 40m$ for the LSW- and the LVB-model. Fig. 12 and 13 give impressions of the surface elevation at the time of maximal crestheight, and the plots in Fig. 14 and 15 show some transversal crosssections in more detail.

While running further downstream, the wave disintegrates and develops a more pronounced tail while the
Fig. 12. Tsunami waveguiding with LSW-evolution for $R_0 = 40 m$; snapshots at the time of maximal crestheight ($t = 720 s$) of the wave, at the left a density plot and the right a 3D impression.

Fig. 13. Tsunami waveguiding with LVB-evolution for $R_0 = 40 m$ as in Fig. 12.

Fig. 14. For LSW-evolution and $R_0 = 40 m$, transversal crossections ($y$-axis horizontal) show the waveprofile of maximal crestheight (at $t = 720 s, x = 41 m$, denoted by 0) and the profiles at the same instant 1 and 2km behind ($-1, -2$ in left plot) and 1, 2, 3 and 4km ahead ($+1, \ldots, +4$ in right plot) of the extreme position.
Fig. 15. For LVB-evolution and $R_0 = 40m$ as Fig. 14.

Fig. 16. For LSW-evolution and $R_0 = 40m$, the waveprofile in the propagation direction at the centreline (left) and a 3D impression (right) at the final time 1200s.

Fig. 17. For LVB-evolution and $R_0 = 40m$ as Fig. 16.

crestheight decreases; Fig. 16 and 17 show some characteristics at the end of the calculation interval of 1200s.
6.3. Discussion about simulation results

6.3.1. Qualitative observations

The most dominant phenomenon of tsunami waveguiding is visible in all simulations: the substantial enlargement and delay of the wave above the waveguide compared to the transversal surrounding wave. During the waveguiding, the wavefront changes considerably; the wave of highest crestheight is found at the end of the run-up region. The profile has a substantially reduced negative wave in the front, a very much enlarged consecutive positive wave and a consecutive negative wave, which is deeper for LVB than for LSW. The deformation also happens for the LSW-evolution which didn’t show this phenomenon for the uniform run-up. The crestheight and waveheight are very large compared to the depth in case $R_0 = 80m$; the waveheight is more than half the depth. Yet, because the wavelength is still very large, the waves are not steep and vertical motions are still rather small.

The transversal deformations are considerable, caused by the delay of the wave above the waveguide. The sections show that transversal deformations extend till 4 times the width of the waveguide (for the time interval under consideration). Outside that area, the wave adjusts to the surrounding wave that runs-up uniformly undisturbed till the depth $100m$. This implies that there it propagates with the speed determined by the exterior depth, and hence that the wave as a whole during run-up travels with that speed. This is confirmed by the numerical simulations: with or without the presence of the waveguide, the progress of the front of the wave is almost the same. The transversal variations are the main reason why the discretization in the transversal direction has to be sufficiently small and has to be extended sufficiently far.

Having arrived at the flat waveguided area, the distorted wave seems to adapt to the horizontal waveguide. In the cases shown, initially the crestheight decreases, after which stronger deformations (shown in the plots of the profile at the endtime) determine the further details.

6.3.2. Summary of quantitative results

In the following Table 2 we summarise the quantitative results for the maximal crestheight. In the first column the height of the waveguide is given; the second column describes the wave model. The total amplification is given in column 3, where we take the maximal crestheight (compared to the initial crest height) as measure. For comparison with the crestheight increase for uniform run-up, the quotient of the maximal amplitudes with and without the presence of the waveguide, the waveguide-enhancement, is denoted by $Q$. Depending on whether we take the exterior depth or the depth above the waveguide as reference, using the results listed in Table 1, we get the values of the quotients $Q_{ext}$ and $Q_{wg}$ as given in columns 4 and 5.

<table>
<thead>
<tr>
<th>$R_0$</th>
<th>wave model</th>
<th>TWG</th>
<th>$Q_{ext}$</th>
<th>$Q_{wg}$</th>
</tr>
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<tbody>
<tr>
<td>40m</td>
<td>LSW</td>
<td>4.3</td>
<td>2.4</td>
<td>2.1</td>
</tr>
<tr>
<td>40m</td>
<td>LVB</td>
<td>4.2</td>
<td>2.2</td>
<td>1.9</td>
</tr>
<tr>
<td>80m</td>
<td>LSW</td>
<td>8.5</td>
<td>4.7</td>
<td>3.1</td>
</tr>
<tr>
<td>80m</td>
<td>LVB</td>
<td>7.8</td>
<td>4.1</td>
<td>2.8</td>
</tr>
</tbody>
</table>

7. Discussion

In this paper we presented results of numerical simulations that convincingly show the phenomenon of tsunami waveguiding. To our best knowledge, this phenomenon did not receive attention yet in the literature.

In this final section we comment on various different aspects.

7.1. Choice of the synthetic data

The synthetic data that we use are subject to discussion.
A first remark in this respect concerns the observation that the invariance of the shallow wave equations imply that the results are robust and characteristic for a class of geometries and incoming waves. In fact, for any scaling factor $\alpha$ the invariance is given by $(x, y, \omega) \rightarrow (x/\alpha, y/\alpha, \alpha\omega)$. If we take the depth of the generation of the tsunami fixed, the scaling in the frequency corresponds to a scaling of the initial wavelength $\Lambda \rightarrow \alpha\Lambda$. Hence, if the width of the waveguide is scaled similarly, $W \rightarrow W/\alpha$, the same wave amplification factors are found.

The choice of the specific waveguide is more problematic.

As waveguiding structure we took here a simple ridge of fixed width and height on top of the sloping bottom. The assumption on the horizontal structure seems less severe than the one on the height. This can be seen already by the different results for $R_0 = 40m$ and $R_0 = 80m$ ridge height. In both cases the entrance region doesn’t seem to have a noticeable effect since the strength of the waveguide at the deep bottom is small. The waveguided run-up, however, is very different. For $R_0 = 80m$ the total amplification from 1m to approximately 8m takes place in the run-up region of 10km length, and 1km behind it. For $R_0 = 40m$ the increase in the run-up region is approximately from 1m to approximately 2.5m, but a substantial further increase is observed while running over an additional distance of 6km over the horizontal part $x > X_1$. In both cases, after the increase a slow ($R_0 = 40m$) or fast ($R_0 = 80m$) decrease sets in, related to the development of a more profound tail.

7.2. Reversed wave

The choice of the initial wave is motivated by tsunami waves that are generated by tectonic plate motions relatively close to the coast of interest. Then, in case of rupture, the subduction of the oceanic plate under the land plate causes a downward motion of the bottom, and hence a negative surface elevation at the landside, and an upward motion with positive surface elevation at the ocean side. Investigating this so-called reversed wave, travelling in the direction with positive wavepart in front, the symmetry in the (linear) equations, it can be expected that a similar phenomenon happens. The following results of calculations for a reversed initial wave show that this is indeed the case: now the negative wavepart shows largest amplification, and the plots are almost the same for the corresponding case with opposite sign of elevation. For LSW- and LVB-evolution, compare the plots for the reversed wave Fig. 18, 19 and 20 with the previous results Fig. 8, 12 and 9.

As a final remark we mention that we are investigating data for regions where such waveguiding bathymetry may occur in the Indonesian coastal area; the results will be published in a forthcoming paper.

7.3. Gridsize dependence

The reduced speed of the wave above the waveguide causes transversal flows which lead to the large amplification. This is a subtle process that may be influenced by the accuracy of calculation. To investigate this, we performed calculations on gridsizes that are larger than used in the rest of this paper. Assuming that
the waveguide at the shallow flat part after run-up continues, we found the results as listed in the Table 3; there the maximal crestheight is given together with the position and time of maximal crestheight. It shows that the maximal crestheight is lower for coarser grids, but in particular that the process of amplification seems to be delayed considerably.

These results indicate that the grids that are often used in tsunami calculations, $1' \times 1' \approx 1850m \times 1850m$, are too coarse to capture the effect of tsunami waveguiding phenomenon.
Table 3: *Maximal crestheight with position and time of occurrence.*

<table>
<thead>
<tr>
<th>Grid (m x m)</th>
<th>Max Crestheight (m)</th>
<th>Position (km)</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>250x250</td>
<td>4.3</td>
<td>41</td>
<td>720</td>
</tr>
<tr>
<td>500x500</td>
<td>3.8</td>
<td>54</td>
<td>1275</td>
</tr>
<tr>
<td>1000x100</td>
<td>3.7</td>
<td>61</td>
<td>1395</td>
</tr>
</tbody>
</table>

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**References**


