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A multiple-choice knapsack based algorithm for CDMA downlink rate differentiation under uplink coverage restrictions

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Abstract

This paper presents an analytical model for downlink rate allocation in Code Division Multiple Access (CDMA) mobile networks. By discretizing the coverage area into small segments, the transmit power requirements are characterized via a matrix representation that separates user and system characteristics. We obtain a closed-form analytical expression for the so-called Perron-Frobenius eigenvalue of that matrix, which provides a quick assessment of the feasibility of the power assignment for a given downlink rate allocation. Based on the Perron-Frobenius eigenvalue, we reduce the downlink rate allocation problem to a set of multiple-choice knapsack problems. The solution of these problems provides an approximation of the optimal downlink rate allocation and cell borders for which the system throughput, expressed in terms of downlink rates, is maximized.
Keyword: CDMA, transmit power feasibility, Perron-Frobenius eigenvalue, border optimization, downlink rate allocation.
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1 Introduction

In Code Division Multiple Access (CDMA) systems, transmissions of different terminals are separated using (pseudo) orthogonal codes. The impact of multiple simultaneous calls is an increase in the interference level, that limits the capacity of the system. The assignment of transmission powers to calls is an important problem for network operation, since the interference caused by a call is directly related to the power. In the CDMA downlink, the transmission power is related to the downlink rate. Hence, for an efficient system utilization, it is necessary to adopt a rate allocation scheme in the transmission powers assignment.

The rate assignment problem has been extensively studied in the literature [4, 8, 14, 18, 20]. In [8], a joint power and multiclass rate allocation algorithm for the uplink is considered. The problem is formulated as a non-linear optimization problem that is solved in two steps. In the first step, one decides for each user a SIR level that guarantees a maximum system utility. In the second, the SIR levels obtained are used to determine an optimal rate allocation. In [14] several rate assignments are analyzed in the context of the trade-off between fairness and over-all throughput. The algorithms presented in [8] and [14] find a solution to the rate optimization problem via the Lagrangian dual. Another approach for joint optimal rates and powers allocation, based on Perron-Frobenius theory, is presented in [4] and [18]. [4] presents a distributed algorithm for assigning base station transmitter (BTSs) powers such that the common rate of the users is maximized, while in [18] multiple rates are considered. [10] develops a model for characterizing downlink and uplink power assignment feasibility based on the Perron-Frobenius theory. It is shown that the transmission power assignment problem can be translated into the border location optimization problem, where these optimization problems for the downlink rate and optimal cell border take into account both uplink and downlink feasibility.

Based on the result in [10], in this paper we extend the model towards a downlink multi-rate allocation scheme.

We study transmission power feasibility and data rate allocation problem for CDMA networks that supports variable transmission rate. The objective is to derive an analytical model and to develop an algorithm for downlink rate allocation with variable transmission rates. Moreover the model is used to determine an optimal cell border by taking into account both uplink and downlink feasibility. Our approach for the downlink rate allocation algorithm is based on the Perron-Frobenius eigenvalue of a power assignment related matrix that allows us to determine first the cell border location and then a near to optimal rate allocation. We call a rate allocation optimal if it maximizes the system utility. We show that the rate allocation problem can be reduced to a set of multiple choice knapsack problems for which efficient algorithms are known. Based on Fully Polynomial Time Approximation Scheme (FPTAS) for the multiple choice knapsack problems, we are able to design a FPTAS for the rate allocation problem. Hence, for an $\epsilon > 0$, our algorithm returns an allocation of value at least $(1-\epsilon)*$optimum in polynomial time in the size of the input data and $(1/\epsilon)$.

The remainder of this paper is organized as follows. We present the downlink interference and feasibility model in Section 2. The downlink rate allocation algorithm and a combined down- and uplink capacity allocation model are presented in Section 3. In Section 4 we present the numerical results and we conclude our work in Section 5.

2 Model

This paper focuses on the modeling of downlink rate allocation in a CDMA system consisting of BTSs located along a highway. Specifically, we focus on a two cell model, where only the area in between two base stations is taken into account. The description can readily be generalized to larger networks.

2.1 Cell Model

Consider a cell model as presented in [10]. We focus on modeling BTSs located along a highway to include both non-homogeneity of the call distribution, and mobility of calls. Users are located in cars passing through the cells. Due to e.g. traffic jams ("hot spots") the load of the cells will not be distributed evenly along the road. To characterize the distribution of a single type of calls in the cells, we propose a discretized-cell model. Each cell is divided into small segments. Then, the nonhomogeneous load can be characterized by the mean number of calls and fresh call arrival rates in the segments. Taking into account interference between segments in neighboring cells and between segments within the cells, we express the generated downlink interference per segment towards the other segments.

We consider a linear network model consisting of two BTSs, X and Y, say. Let the area between these BTSs be divided into segments of length δ. For the description below, we fix the radii of the cells. Let cell X contains I segments, labelled as $i = 1, ..., I$, and let cell Y contains J segments, labelled as $j = 1, ..., J$. Then $L_1 = I\delta$ is the radius of cell X, see Figure 1. Let $D = \delta(I + J)$, the distance between the BTSs.

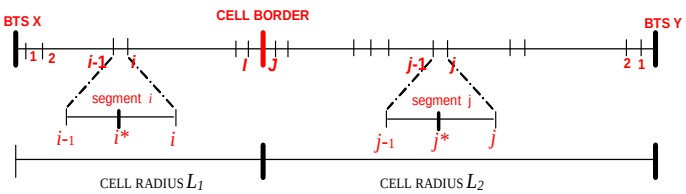

Fig.1. Discretized Cell Model

We assume that the segments are small, so that we may approximate the location of terminals in a segment to be in the middle of that segment, i.e. for segment i of cell X, users are located at distance $i^* = \delta\left[(i-1)+i\right]/2$ from X. Furthermore, we consider a deterministic Okumura-Hata path loss propagation model between a transmitter and a receiver, which performs reasonably in flat service areas (see [1, 12]):

$$P_i = P_0 \left(d_i\right)^{-\gamma}, \tag{1}$$

where P_i is the received power, P_0 is the transmitted power, d_i is the distance between transmitter and receiver and γ the path loss exponent.

2.2 Downlink Interference Model

For the discretized cell model, we consider the downlink interference model proposed in [10]. Assume that the number of terminals in each segment in both BTSs is known. Let $\mathbf{U}_X = (n_1, n_2, \cdots, n_I)$ be a vector representing the number of terminals connected to BTS X, where n_i is the number of terminals in segment $i = 1, 2, \cdots, I$; and let $\mathbf{U}_Y = (m_1, m_2, \cdots, m_J)$ be a vector representing the number of terminals connected to BTS Y, where m_j is the number of terminals in segment $j = 1, 2, \cdots, J$. Furthermore, let $\mathbf{X} = (X_1, X_2, \cdots, X_I)$ and $\mathbf{Y} = (Y_1, Y_2, \cdots, Y_J)$ be the transmit power vectors of BTS X and BTS Y to terminals in the segments. Then, the energy per bit to interference ratio, $\left(\frac{E_b}{I_0}\right)$, for a terminal $i \in \{1, 2, \cdots, I\}$ is defined as (see e.g. [13])

$$\left(\frac{E_b}{I_0}\right)_i = \frac{W}{r_i} \frac{X_i \left(i^*\right)^{-\gamma}}{I_{\text{intracell}} + I_{\text{intercell}} + N_0}, \tag{2}$$

where W is the system chip rate, r_i is the data rate, $I_{\text{intracell}}$ is the interferences from users within the cell, i.e. $I_{\text{intracell}} = \alpha \left(i^*\right)^{-\gamma} \left(\sum_{l=1}^{I} n_l X_l - X_i\right)$ with α is non-orthogonality factor, $I_{\text{intercell}}$ is the interferences from users in neighboring cell, i.e. $I_{\text{intercell}} = \left(D - i^*\right)^{-\gamma} \sum_{j=1}^{J} m_j Y_j$, and N_0 is the thermal noise. A similar expression to (2) holds for $j \in \{1, 2, \cdots, J\}$.

For a terminal $i \in \{1, 2, \cdots, I\}$, downlink transmission at sufficient quality requires the energy per bit to interference ratio to exceed a certain threshold ϵ^*_{iD}, i.e., a terminal in segment i requires BTS X to transmit enough power such that

$$\left(\frac{E_b}{I_0}\right)_i \geqslant \epsilon^*_{iD}, \text{ for } i \in \{1, 2, \cdots, I\}, \tag{3}$$

and similarly for a terminal segment j requires BTS Y to transmit enough power such that

$$\left(\frac{E_b}{I_0}\right)_j \geqslant \epsilon^*_{jD}, \text{ for } j \in \{1, 2, \cdots, J\}. \tag{4}$$

We assume that the $(\frac{E_b}{I_0})-$target is the same, i.e., $\epsilon^*_{iD} = \epsilon^*_{jD} = \epsilon^*_D$ for all $i = 1, 2, \cdots, I$ and $j = 1, 2, \cdots, J$. The model can be easily extended to different $(\frac{E_b}{I_0})-$target.

Under the quality of service (QoS) constraints, we are interested in finding a downlink rate assignment such that there exist non negative transmit power vectors $\mathbf{X}$ and $\mathbf{Y}$ satisfying (3). Without loss of generality, in this paper we assume that all users in the same segment have the same rate r_i chosen from a finite set of possible transmission rates $R_i \subseteq \{r_1, r_2, \cdots, r_K\}$. Note that, if in a segment the maximum rate r_K is not requested, then $R_i \subset \{r_1, r_2, \cdots, r_K\}$. This assumption leads to a better use of the resources. Let $\mathbf{R}_X = (r_1, r_2, \cdots, r_I)$, respectively $\mathbf{R}_Y = (r_1, r_2, \cdots, r_J)$, be the rates assigned to terminals in cell X, respectively cell Y.

Under the assumption of perfect power control, (3) becomes:

$$X_i = \alpha V_i \sum_{i=1}^{I} n_i X_i + V_i p_i \sum_{j=1}^{J} m_j Y_j + V_i N_0 d_i, \quad \text{for } i = 1, 2, \cdots, I, \tag{5}$$

where

$$V_i = \frac{\epsilon_D^* r_i}{W + \alpha \epsilon_D^* r_i}; \quad p_i = \left(\frac{D - i^*}{i^*}\right)^{-\gamma} \text{ and } d_i = \left(\frac{1}{i^*}\right)^{-\gamma}, \quad \text{for } i = 1, 2, \cdots, I. \tag{6}$$

(5) express the required BTS X transmit powers to all segment i, $i = 1, 2, \cdots, I$. Similarly, we can write the required BTS Y transmit powers to all segment j, $j = 1, 2, \cdots, J$, in the cell Y. Thus system (3) and (4) can be rewritten in matrix form as follows

$$(\mathbf{I} - \mathbf{T})\mathbf{Z}^D = \mathbf{c}. \tag{7}$$

where $\mathbf{Z}^D = \left((X_i)_{i=1,2,\cdots,I}, (Y_j)_{j=1,2,\cdots,J}\right)$ is the transmit powers vector; $\mathbf{c} = N_0 \begin{pmatrix} \mathbf{V}_X \mathbf{D}_X \\ \mathbf{V}_Y \mathbf{D}_Y \end{pmatrix}$ with diagonal matrices $\mathbf{V}_X = diag(V_i)$ and $\mathbf{V}_Y = diag(\bar{V_j})$, and vectors $\mathbf{D}_X = (d_i)_{i=1,2,\cdots,I}$ and $\mathbf{D}_X = (d_j)_{j=1,2,\cdots,J}$ given by (6); and the square matrix

$$\mathbf{T} = \begin{pmatrix} \mathbf{V}_X & 0 \\ 0 & \mathbf{V}_Y \end{pmatrix} \begin{pmatrix} \alpha \mathbf{1}_X & \mathbf{P}_X \\ \mathbf{P}_Y & \alpha \mathbf{1}_Y \end{pmatrix} \begin{pmatrix} \mathbf{N} & 0 \\ 0 & \mathbf{M} \end{pmatrix}. \tag{8}$$

where $\mathbf{P}_X = (p_{ij}^X)$, $p_{ij}^X = \left(\frac{D - i^*}{i^*}\right)^{-\gamma}$ is a matrix of size $I \times J$ that represents the inter-cell path loss from segment j in cell Y and segment i in cell X, and $\mathbf{P}_Y = (p_{ji}^Y)$, where $p_{ji}^Y = \left(\frac{D - j^*}{j^*}\right)^{-\gamma}$; the square matrices $\mathbf{N} = \mathbf{U}_X \mathbf{I}$ and $\mathbf{M} = \mathbf{U}_Y \mathbf{I}$ give the distribution of users.

System (7) is the downlink power control equation. Since matrix $\mathbf{T}$ is a non-negative matrix, we can use the Perron-Frobenius theory to characterize the feasibility of (7). Thus, given the number of terminals in each cell $\mathbf{U}_X$ and $\mathbf{U}_Y$ and the current rate allocation $\mathbf{R}_X$ and $\mathbf{R}_Y$, we can verify whether a feasible transmit power solution exists. This will be described in the next section.

2.3 Downlink Feasibility

According to the Perron-Frobenius theorem (see [19]), the feasibility of (7) is determined by the Perron- Frobenius (PF) eigenvalue $\lambda(\mathbf{T})$ of the matrix $\mathbf{T}$, i.e.,

$$\mathbf{Z}^D \geq \mathbf{0} \text{ exist} \quad \text{and} \quad \mathbf{Z}^D = (\mathbf{I} - \mathbf{T})^{-1}\mathbf{c} \iff \lambda(\mathbf{T}) < 1. \tag{9}$$

Hence, downlink transmit power feasibility is completely characterized by the matrix $\mathbf{T}$ given in (8). Characterization (9) provides a clear motivation for discretization. By discretizing the cell into segments, we obtain an analytical model for characterizing the transmit power feasibility for a certain rate allocation and a certain users distribution. We obtain a downlink interference model that is very similar to uplink models such as studied in [3, 9, 11] where feasibility of the uplink power control algorithm is characterized via the Perron-Frobenius (PF) eigenvalue of a matrix containing the number of calls in the cell (not in segments).

The explicit expression of the PF eigenvalue has been derived for the case $r_i = r_j = R_D$ for all i and j (see [10]). The result can be extended to the case of different rates. For this purpose, we do a dimension reduction of the power control matrix, as was done for the uplink (see [11, 16, 22]). The following lemma provides a proof for the dimension reduction of matrix (8).

Lemma 1 *System (7) is feasible if and only if the following system is feasible*

$$(\mathbf{I} - \mathbf{T}')\mathbf{Z}' = \mathbf{c}', \tag{10}$$

where

$$\mathbf{T}' = \begin{pmatrix} \alpha \sum_{i=1}^{I} V_i n_i & \sum_{i=1}^{I} V_i\, n_i p_i \\ \sum_{j=1}^{J} \overline{V}_j m_j p_j & \alpha \sum_{j=1}^{J} \overline{V}_j m_j \end{pmatrix}, \tag{11}$$

$\mathbf{Z}' = \left(\sum_{i=1}^{I} X_i n_i,\ \sum_{j=1}^{J} Y_j m_j \right)^T$ *is the total transmit power vector of BTS X and BTS Y,*

and $\mathbf{c}' = N_0 \left(\sum_{i=1}^{I} V_i n_i d_i,\ \sum_{j=1}^{J} \overline{V}_j m_j d_j \right)^T$.

Proof. First, we prove that if system (7) has a positive solution then system (10) has one. Let $(X_i)_{i=1,2,\cdots,I}$ and $(Y_j)_{j=1,2,\cdots,J}$ be positive solutions of (7). If we multiply each equation in (5) with the number of users in segment i, n_i, and sum them up for all i, $i = 1, 2, \cdots, I$, we obtain

$$\sum_{i=1}^{I} X_i n_i = \alpha \left(\sum_{i=1}^{I} V_i n_i \right) \sum_{i=1}^{I} X_i n_i + \left(\sum_{i=1}^{I} V_i n_i p_i \right) \sum_{j=1}^{J} Y_j m_j + \sum_{i=1}^{I} V_i n_i N_0 d_i. \tag{12}$$

Similarly, we obtain an expression for BTS Y. It follows immediately that $(\sum_{i=1}^{I} X_i n_i, \sum_{j=1}^{J} Y_j m_j)$ is a solution of (10).

Next, we prove that if (10) has a positive solution, then system (7) has one. Let $\mathbf{Z}' = (X', Y')$ be a solution of (10). Define:

$$X_i = \frac{1}{1 + \alpha \Gamma_i} [\Gamma_i p_i Y' + \alpha \Gamma_i X' + \Gamma_i N_0 d_i], \tag{13}$$

$$Y_j = \frac{1}{1 + \alpha \overline{\Gamma_j}} [\overline{\Gamma_j} p_j X' + \alpha \overline{\Gamma_j} Y' + \overline{\Gamma_j} N_0 d_j]. \tag{14}$$

It can be seen that $(X_i)_{i=1,2,\cdots,I}$ and $(Y_j)_{j=1,2,\cdots,J}$ is a solution of system (7).
This completes the proof. ■

Remark *From Lemma 1 follows that the feasibility of system (7) can be characterized by the PF eigenvalue of matrix* T'.

The matrix $\mathbf{T}'$ represents the BTSs total transmit power due to intra-cell and intercell interferences. The explicit expression of the PF eigenvalue of $\mathbf{T}'$ can be calculated easily and is equal to

$$\lambda(\mathbf{T}') = \frac{1}{2} \left\{ \alpha \left(\sum_{i=1}^{I} V_i n_i + \sum_{j=1}^{J} \overline{V}_j m_j \right) + \sqrt{\alpha^2 \left(\sum_{i=1}^{I} V_i n_i - \sum_{j=1}^{J} \overline{V}_j m_j \right)^2 + 4 \left(\sum_{i=1}^{I} V_i n_i p_i \right) \left(\sum_{j=1}^{J} \overline{V}_j m_j p_j \right)} \right\}, \tag{15}$$

Based on the explicit formulation of the PF eigenvalue of matrix $\mathbf{T}'$ in (15), we develop an optimization model for rate allocation for a given distribution of users over segments. This is done in the next section.

3 Analysis

We model the problem of finding a maximum system utility as a discrete optimization problem. We choose the system utility as the total sum of rates allocated to users. If the rates used are assigned

to a certain price, i.e., euro per bit used, then this optimization model can be interpreted as the total revenue of the system. Note that the algorithm we present also works for the other definition of system utility, such as in [2, 7, 14, 18, 21]. In particular, we develop a joint uplink and downlink optimization model with downlink rate differentiation. There are two objectives of our model. The first objective is to find a set of possible border location that maximizes the total number of uplink users. Then, given the set of border locations, we find an approximation of downlink rates allocation that maximizes the total sum of downlink rates allocated.

3.1 Border Optimization

We extend the result of [10], where the allocation of borders was analyzed using the downlink and uplink PF eigenvalues of the transmit powers matrix. In [10], an optimization for a single downlink rate was formulated. Here, we formulate the joint uplink and downlink optimization with downlink rate differentiation. We are interested in deciding the position of the cell borders such that the maximum number of users is covered in the uplink and for these users, a maximum downlink total rate is insured. Note that the coverage of a cell is equal to the number of segments covered by the cell. Thus, the border of cell X is defined as the point located after segment I and the border of cell Y is defined as the point located after segment J . The combined optimization problem is formulated as follows:

Find the set of border locations, I and J, for which the number of carried calls under the uplink feasibility is maximized and then find a downlink rate allocation such that the system utility is maximized while downlink feasibility is maintained.

Let $R = \{r_1, r_2, ..., r_K\}$ be the set of admissible rates. For each segment i, let x_{is} (respectively y_{js}) be $0-1$ variables indicating whether users in segment i in cell X , respectively segment j in cell Y, receive rate $r_s \in R$. The problem can be formulated as follows

$$\max_{\mathbf{x},\mathbf{y}} \sum_{i=1}^{I} \sum_{r_s \in R_i} r_s n_i x_{is} + \sum_{j=1}^{J} \sum_{r_s \in R_j} r_s m_j y_{js} \tag{16}$$

$$\text{s.t.} \qquad \lambda(I, J, \mathbf{x}, \mathbf{y}) < 1,$$

$$\sum_{r_s \in R_i} x_{is} = 1 \text{ for each } 1 \le i \le I, \tag{17}$$

$$\sum_{r_s \in R_j} y_{js} = 1 \text{ for each } 1 \le j \le J, \tag{18}$$

$$x_{is}, y_{js} \in \{0, 1\},$$

$$I, J \in \arg\max \left(\sum_{i=1}^{I} n_i + \sum_{j=1}^{J} m_j \right)$$

$$\text{s.t.} \quad \rho(I, J) < 1$$

$$i = 1, 2, \cdots, I; \; j = 1, 2, \cdots, J,$$

where $\lambda(I, J, \mathbf{x}, \mathbf{y})$ is obtained by expressing (15) with the help of the indicator vectors $\mathbf{x}$ and $\mathbf{y}$; $\rho\,(I, J)$ is the uplink PF eigenvalue as given in [10]

$$\rho\,(I, J) = \frac{\Gamma}{2} \left(\sum_{i=1}^{I} n_i - 1 + \sum_{i=1}^{J} m_j - 1 \right) + \sqrt{\left(\sum_{i=1}^{I} n_i - \sum_{i=1}^{J} m_j \right)^2 + 4 \sum_{i=1}^{I} p_i n_i \sum_{j=1}^{J} p_j m_j}, \tag{19}$$

where $\Gamma = \epsilon_U^* \frac{R_U}{2W}$, ϵ_U^* is energy per bit to interference ratio threshold for uplink and R_U is the uplink

rate; $N_I = \sum_{i=1}^{I} n_i$, respectively $M_J = \sum_{i=1}^{J} m_j$, is the total number of users in cell X, respectively cell Y. Note that in the uplink we do not take into account rate differentiation. Constraints (17) and (18) ensure that a single rate from the set of possible rates is assigned to each segment. We solve the optimization problem (16) in two stages. In the first stage, we find by enumeration the set of optimal border locations. In the second stage, we find a rate allocation that ensures a system utility close to the optimum. We propose a rate allocation algorithm based on the multiple-choice knapsack problem [17].

3.2 Downlink Rate Allocation Algorithm

Let us consider the second stage of the optimization problem. The goal is to allocate rates to users in all segments such that downlink feasibility is maintained and the total sum of allocated rates is maximized. We will show that the downlink rate allocation problem can be reduced into a set of multiple choice knapsack problems, which are NP-hard. Then, we propose an algorithm for finding a rate assignment that, for a specific ϵ, gives a solution of value at least $(1-\epsilon)$ times the optimum, in polynomial time in the size of the instance and in $\frac{1}{\epsilon}$. Such an algorithm is called a fully polynomial time approximation scheme (FPTAS).

For given border locations, I and J, the downlink rate allocation problem is formulated as follows

$$
\begin{aligned}
(P) \quad & \max_{\mathbf{x},\mathbf{y}} \sum_{i=1}^{I} \sum_{r_s \in R_i} r_s n_i x_{is} + \sum_{j=1}^{J} \sum_{r_s \in R_j} r_s m_j y_{js} \\
& \text{s.t.} \quad \lambda(\mathbf{U}, \mathbf{x}, \mathbf{y}) < 1, \\
& \sum_{r_s \in R_i} x_{is} = 1 \text{ for each} 1 \le i \le I, \\
& \sum_{r_s \in R_j} y_{js} = 1 \text{ for each} 1 \le j \le J, \\
& x_{is}, y_{js} \in \{0,1\},
\end{aligned}
\tag{20}
$$

It can be proven that $\lambda(\mathbf{U}, \mathbf{x}, \mathbf{y}) < 1$ is equivalent with the following three conditions:

$$
\begin{aligned}
& \frac{1 - \alpha \sum_{i=1}^{I} \sum_{r_s \in R_i} V_s n_i x_{is}}{\sum_{i=1}^{I} \sum_{r_s \in R_i} V_s n_i p_i x_{is}} > \frac{\sum_{j=1}^{J} \sum_{r_s \in R_j} \overline{V}_s m_j p_j y_{js}}{1 - \alpha \sum_{j=1}^{J} \sum_{r_s \in R_j} \overline{V}_s m_j y_{js}}, \\
& \alpha \sum_{i=1}^{I} \sum_{r_s \in R_i} V_s n_i x_{is} \le 1, \\
& \alpha \sum_{j=1}^{J} \sum_{r_s \in R_j} \overline{V}_s m_j y_{js} \le 1.
\end{aligned}
\tag{21}
$$

Next, based on the special form of the conditions (21), we show how the problem (P) can be decomposed in a set of multiple-choice knapsack problems.

Let **OPT** be the optimal value of the optimization problem (P). Denote

$$t_{\min} = \min_{\mathbf{y}\in\{0,1\}^J} \left\{ \frac{\sum_{j=1}^{J} \sum_{r_s\in R_j} \overline{V}_s m_j p_j y_{js}}{1-\alpha \sum_{j=1}^{J} \sum_{r_s\in R_j} \overline{V}_s m_j y_{js}} \right\}, \tag{22}$$

$$\text{and} \quad t_{\max} = \max_{\mathbf{x}\in\{0,1\}^I} \left\{ \frac{1-\alpha \sum_{i=1}^{I} \sum_{r_s\in R_i} V_s n_i x_{is}}{\sum_{i=1}^{I} \sum_{r_s\in R_i} V_s n_i p_i x_{is}} \right\}. \tag{23}$$

For any $t \in [t_{\min}, t_{\max}]$, consider the following problems:

$$P_1(t): \quad \max_{\mathbf{x}} \sum_{i=1}^{I} \sum_{r_s\in R_i} r_s n_i x_{is} \tag{24}$$

$$\begin{aligned} \text{s.t.} \quad & \frac{1-\alpha \sum_{i=1}^{I} \sum_{r_s\in R_i} V_s n_i x_{is}}{\sum_{i=1}^{I} \sum_{r_s\in R_i} V_s n_i p_i x_{is}} \geq t, \\ & \sum_{r_s\in R_i} x_{is} = 1 \text{ for each } 1 \leq i \leq I, \\ & x_{is} \in \{0,1\}, \end{aligned}$$

and

$$P_2(t): \quad \max_{\mathbf{y}} \sum_{j=1}^{J} \sum_{r_s\in R_j} r_s m_j y_{js} \tag{25}$$

$$\begin{aligned} \text{s.t.} \quad & \frac{\sum_{j=1}^{J} \sum_{r_s\in R_j} \overline{V}_s m_j p_j y_{js}}{1-\alpha \sum_{j=1}^{J} \sum_{r_s\in R_j} \overline{V}_s m_j y_{js}} < t, \\ & \sum_{r_s\in R_j} y_{js} = 1 \text{ for each } 1 \leq j \leq J, \\ & y_{js} \in \{0,1\}. \end{aligned}$$

Let $\mathbf{OPT}_1(t)$, respectively $\mathbf{OPT}_2(t)$, be the optimal values of $P_1(t)$, respectively of $P_2(t)$. Then, the optimal value of (P) can be found by solving $P_1(t)$ and $P_2(t)$ for all $t \in [t_{\min}, t_{\max}]$. This is formalized in the following lemma

Lemma 2 $\mathbf{OPT} = \max_{t\in[t_{\min},t_{\max}]} \mathbf{OPT}_1(t) + \mathbf{OPT}_2(t)$.

Proof. Consider a $t \in [t_{min}, t_{max}]$. Let x, respectively y, be optimal solutions of $P_1(t)$, respectively $P_2(t)$. Clearly, (x, y) is a feasible solution of (P), and therefore $OPT_1(t) + OPT_2(t) \leq OPT$. Hence, $\max_{t\in[t_{min},t_{max}]} OPT_1(t) + OPT_2(t) \leq OPT$.

In order to prove the reverse inequality, consider an optimal solution (x^*, y^*) of (P). Let

$$t = \frac{1-\alpha \sum_{i=1}^{I} \sum_{r_s\in R_i} V_s n_i x^*_{is}}{\sum_{i=1}^{I} \sum_{r_s\in R_i} V_s n_i p_i x^*_{is}}.$$

Since x^* is feasible for $P_1(t)$ and y^* is feasible for $P_2(t)$, $OPT \leq OPT_1(t) + OPT_2(t)$. ■

Lemma 2 implies that the optimum rate allocation can be found by solving independently the set of optimizations problems, $\{P_1(t) \mid t \in [t_{\min}, t_{\max}]\}$and $\{P_2(t) \mid t \in [t_{\min}, t_{\max}]\}$, where each set characterizes only one cell, the cells interactingly only through the parameter t. Next, we show that $P_1(t)$ and $P_2(t)$ are multiple choice knapsack problems. For this, we rewrite $P_1(t)$ and $P_2(t)$ as:

$$P_1(t): \begin{array}{ll} \max\limits_{\mathbf{x}} & \sum\limits_{i=1}^{I} \sum\limits_{r_s \in R_i} r_s n_i x_{is} \\ s.t. & \sum\limits_{i=1}^{I} \sum\limits_{r_s \in R_i} V_s n_i (\alpha + p_i t) x_{is} \leq 1, \\ & \sum\limits_{r_s \in R_i} x_{is} = 1 \text{ for each } 1 \leq i \leq I, \\ & x_{is} \in \{0,1\} \end{array} \tag{26}$$

and

$$P_2(t): \begin{array}{ll} \max\limits_{\mathbf{y}} & \sum\limits_{j=1}^{J} \sum\limits_{r_s \in R_j} r_s m_j y_{js} \\ s.t. & \sum\limits_{j=1}^{J} \sum\limits_{r_s \in R_j} \overline{V}_s m_j (\alpha + \frac{p_j}{t}) y_{js} < 1, \\ & \sum\limits_{r_s \in R_j} y_{js} = 1 \text{ for each } 1 \leq j \leq J, \\ & y_{js} \in \{0,1\}. \end{array} \tag{27}$$

The input to the multiple choice knapsack problems $P_1(t)$, respectively $P_2(t)$ is: the objects are the pairs $\{(i, r_s), i \in \{1, ..., I\}, r_s \in \{r_1, ..., r_K\}\}$, respectively $\{(j, r_s), j \in \{1, ..., J\}, r_s \in \{r_1, ..., r_K\}\}$; a class consists of the objects corresponding to the same segment; the profit of an object (i, r_s) is $n_i r_s$ and its size is $V_s n_i$ for $i \in \{1, ..., I\}$, respectively $m_j r_s$ and $V_s m_j$ for $j \in \{1, ..., J\}$. The volumes of the knapsacks are 1, respectively t. There are two categories of algorithms for tackling these problems: exact (see [17]) but with no polynomial running time, or approximation algorithm (see [6]) running in polynomial time in the size of the input data and $1/\epsilon$, where ϵ is the error of the solution. Depending on the data, one can choose to solve the multiple choice knapsack problems, $P_1(t)$ and $P_2(t)$, by one of these type of algorithms. Next, we will show that even if one OPTs for using a FPTAS, a solution of value at least $(1-\epsilon)*$OPT can be obtained.

For an $\epsilon > 0$ and $t \in [t_{\min}, t_{\max}]$, let $K_1(t, \epsilon)$ and $K_2(t, \epsilon)$, be the value of the solution given by a FPTAS for $P_1(t)$, respectively $P_2(t)$. The following Lemma shows that by approximating $\max\limits_t (OPT_1(t) + OPT_2(t))$ with $\max\limits_t \{K_1(t, \epsilon) + K_2(t, \epsilon)\}$, we obtain a solution of the value at least $(1-\epsilon)*$OPT.

Lemma 3 *For each $\epsilon > 0$,*

$$\max_{t \in [t_{\min}, t_{\max}]} \{K_1(t, \epsilon) + K_2(t, \epsilon)\} \geq (1-\epsilon) OPT.$$

Proof. Let $t^* \in [t_{\min}, t_{\max}]$ such that $OPT(t^*) = OPT_1(t^*) + OPT_2(t^*)$. From Lemma 2 follows

$$\begin{aligned} & \max_{t \in [t_{\min}, t_{\max}]} \{K_1(t, \epsilon) + K_2(t, \epsilon)\} \\ & \geq K_1(t^*, \epsilon) + K_2(t^*, \epsilon) \\ & \geq (1-\epsilon) OPT_1(t^*) + (1-\epsilon) OPT_2(t^*) \\ & \geq (1-\epsilon) OPT. \end{aligned}$$

■

The only bottleneck in designing a FPTAS for (P) is to find $\max\limits_{t \in [t_{\min}, t_{\max}]} \{K_1(t, \epsilon) + K_2(t, \epsilon)\}$, in polynomial time in the size of the instance and in $\frac{1}{\epsilon}$.

However, if $\epsilon \geq 1/2$, a feasible solution of (P) of value at least $(1-\epsilon)OPT$ can be found just by calculating $K_1(t_{\min}, 2\epsilon - 1)$ and $K_2(t_{\max}, 2\epsilon - 1)$. Based on the monotonicity of $OPT_1(t)$ and $OPT_2(t)$, the following Lemma can be easily derived.

Lemma 4 *If $\epsilon \geq \frac{1}{2}$, then*

$$\max\{K_1(t_{\min}, 2\epsilon-1) + K_2(t_{\min}, 2\epsilon-1), \quad K_1(t_{\max}, 2\epsilon-1) + K_2(t_{\max}, 2\epsilon-1)\} \geq (1-\epsilon)OPT.$$

The proof of Lemma 4 can be found in [5]. For the case of $\epsilon < \frac{1}{2}$, we proceed as follows. Consider the sets A_l and $\overline{A_l}$, for $l \in \{0, 1, ..., \lfloor \frac{1}{\epsilon} \ln \frac{1}{\epsilon} \rfloor + 1\}$ defined as

$$\begin{aligned} A_0 &= \{t|\ K_1(t_{\min}, \epsilon) < K_1(t, \epsilon)\}, \\ \overline{A_0} &= \{t|\ K_2(t_{\max}, \epsilon) < K_2(t, \epsilon)\}, \\ A_l &= \{t|LB_l < K_1(t, \epsilon) < UB_l\},\ \text{for } l \geq 1, \\ \overline{A_l} &= \{t|\overline{LB}_l < K_2(t, \epsilon) < \overline{UB}_l\},\ \text{for } l \geq 1, \end{aligned}$$

where $LB_l = (1-\epsilon)^l K_1(t_{\min}, \epsilon)$, $\quad UB_l = (1-\epsilon)^{l-1} K_1(t_{\min}, \epsilon)$, $\overline{LB}_l = (1-\epsilon)^l K_2(t_{\max}, \epsilon)$ and $\overline{UB}_l = (1-\epsilon)^{l-1} K_2(t_{\max}, \epsilon)$.

Let J_1 be a set containing the highest t in each non empty set A_l and J_2 a set containing the smallest t in each non empty set $\overline{A_l}$. The following lemma shows that by calculating $K_1(t, \epsilon')$ and $K_2(t, \epsilon')$ only for $t \in J_1(\epsilon') \cup J_2(\epsilon')$ and for a well chosen ϵ, we find a feasible solution for (P) of value at least $(1-\epsilon)OPT$.

Lemma 5 *For $\epsilon' = 1 - \sqrt[3]{1-\epsilon}$ the following relation holds*

$$\max_{t \in J_1(\epsilon) \cup J_2(\epsilon)} \{K_1(t, \epsilon') + K_2(t, \epsilon')\} \geq (1-\epsilon)OPT.$$

The proof of Lemma 5 can be found in [5].

Hence, the number of points we are looking at in order to find a solution close to the optimum is reduced to $|J_1(\epsilon')| + |J_2(\epsilon')| = \frac{2}{\epsilon'} ln \frac{1}{\epsilon'} + 2 = O(\frac{1}{\epsilon} ln \frac{1}{\epsilon})$. Note that the points in $J_1(\epsilon') \cup J_2(\epsilon')$ can be found in polynomial time by the search procedure described in [15], where at every query, a FPTAS for the multiple choice knapsack problem is performed. This implies that the procedure of finding $\max_{t \in J_1(\epsilon') \cup J_2(\epsilon')} \{K_1(t, \epsilon') + K_2(t, \epsilon')\}$ runs in polynomial time in the size of the instance and in $\frac{1}{\epsilon}$ and that the following procedure is a FPTAS for problem (P):

Algorithm 1

1. Let $\epsilon' = 1 - \sqrt[3]{1-\epsilon}$.
2. Find the sets $J_1(\epsilon')$ and $J_2(\epsilon')$.
3. For all $t \in J_1(\epsilon') \cup J_2(\epsilon')$, calculate $K_1(t, \epsilon')$ and $K_2(t, \epsilon')$, by using a FPTAS for the multiple choice knapsack problem.
4. Choose the $t \in J_1(\epsilon') \cup J_2(\epsilon')$ for which $\max_{t \in J_1 \cup J_2} \{K_1(t, \epsilon') + K_2(t, \epsilon')\}$ is attained.

4 Numerical Examples

In this numerical examples, we use the following system parameters of the Wideband-CDMA system [13]: the system chip rate $W = 3.84$ MHz, thermal noise $N_0 = -169$ dBm/Hz, path loss exponent $\gamma = 4$, downlink non-orthogonality $\alpha = 0.3$, QoS required $\frac{E_b}{I_0}$, $\epsilon^* = 5$ dB, uplink transmission rate $r_U = 14$ kbps, downlink transmission rate $r_i \in \{14, 32, 64, 144\}$ kbps.

We first consider two BTSs with a non-homogeneous traffic load as shown in Figure 2, i.e., there is a block-shaped traffic jam located at 650m from BTS X. The distance between the two BTSs X and

Y is 2000 meter. We divide the cells into small segments such that the load of a segment is at most $\rho_s = 1$ Erlang. In this case, we obtain 400 segments of width 5m. The system is overloaded, i.e., not all calls can be assigned a positive rate. For this typical traffic load, we investigate the optimal border location and downlink rate allocation obtained from (16) using Algorithm 1.

We solve the first stage problem using the algorithm for uplink optimal border location presented in [10]. Figure 3 depicts the optimal border locations. From the figure we conclude that the optimal uplink cell borders are obtained if the uplink cell borders are located between the vertical lines in Figure 3. Thus, the optimal border for cell X is at 850m from BTS X and for cell Y is at 1000m from BTS Y. Figure 4 depicts the optimal number of uplink users in cell X and cell Y. Notice that there is a coverage gap between BTS X and Y, which means that in order to maintain uplink feasibility some users have to be dropped.

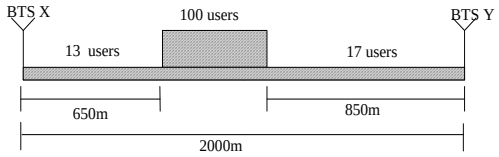

Fig.2. Rectangular hot spot

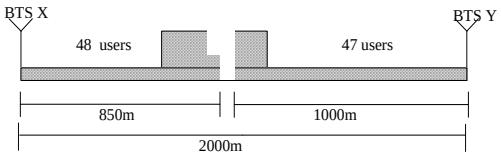

Fig. 3. Optimal border location

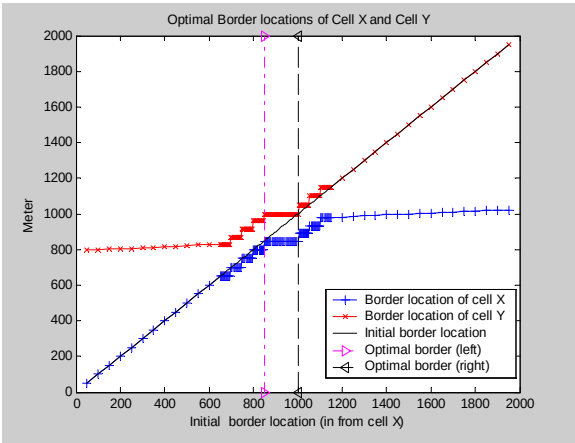

Fig. 3. Optimal border location

Next, we investigate the FPTAS for the downlink rate allocation problem. Given the optimal uplink border from the first stage, we determine a downlink rate allocation which is close to the optimum. For finding a feasible solution of (26) and (27), we use Algorithm 1. For finding a feasible solution of the multiple-choice knapsack problems involved, we use a FPTAS based on dynamic programming as described in [6, 17]. We choose $\epsilon = 0.1$, i.e., we are interested in obtaining a solution of value at least $90\% * OPT$.

We consider two cases of the rate allocation according to the required transmission rates per segment. Firstly, we will consider the case where all segment in the cells can choose rate r_i from the same set of possible transmission rates $R_i \subseteq \{14 \text{ kbps}, 32 \text{ kbps}, 64 \text{ kbps}, 144 \text{ kbps}\}$. Secondly, we will consider the case where each segment only request some rates, i.e., each segment has different set of possible transmission rates $R_i \subset \{14 \text{ kbps}, 32 \text{ kbps}, 64 \text{ kbps}, 144 \text{ kbps}\}$.

Case I:

First, we find the sets $J_1(\epsilon)$ and $J_2(\epsilon)$. Figure 5 and Figure 6 depict the numerical results if the border of cell X is at 850m from BTS X and border of cell Y is at 1000m from BTS Y.

The next step is to find the maximum value of the total system utility, i.e., $\max_{t \in J_1(\epsilon') \cup J_2(\epsilon')} \{K_1(t, \epsilon') + K_2(t, \epsilon')\}$. The maximum utility is attained at $t = 0.0628$ with value of 4784 units. The related rate allocation is shown in Figure 7, i.e., for cell X: 1 user with rate 32 kbps, 2 users with rate 64 kbps and 29 users with rate 144 kbps, and 16 users are dropped (receive 0 rate); and for cell Y: 2 users with 144 kbps, 5 users with rate 32 kbps and 40 users are dropped (receive 0 rate).

It can be seen that the maximum utility is attained by allocating maximum rate to most of the users in cell X which are close to BTS X and only few users in cell Y have a non zero rate. This

confirms the intuitive rate allocation in the interference limited system, i.e., as the main interference sources are users from the other cell, it is optimal to allocate rate only to one of the cells at a time. Note that this numerical example describes an extreme situation, when cell X is heavily loaded. Therefore, after allocating rates to cell X, few resources remain available for cell Y, resulting in a small number of users in cell Y with a non-zero rate. In the case of less loaded cells, the number of users with non-zero rate in cell Y increases.

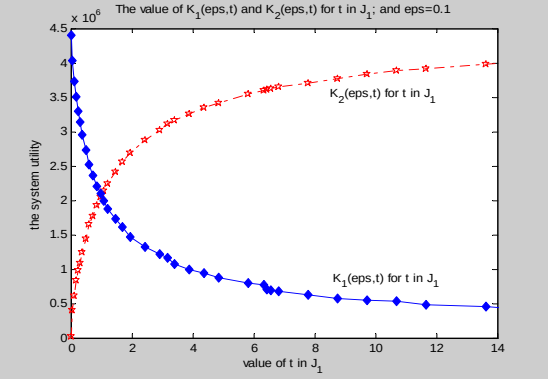

Fig. 5. $K_1(t,\epsilon)$and $K_2(t,\epsilon)$for $\epsilon = 0.1$and $t \in J_1$

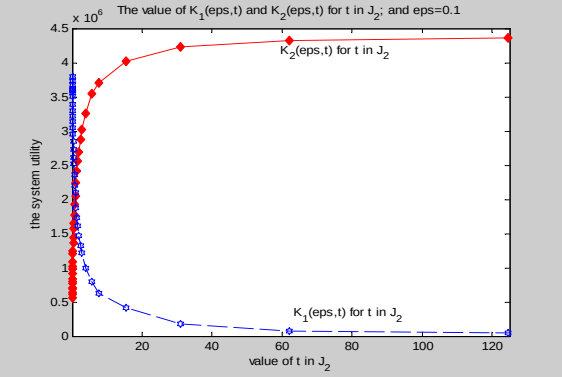

Fig. 6. $K_2(t,\epsilon)$and $K_1(t,\epsilon)$for $\epsilon = 0.1$and $t \in J_2$.

Case II:

In the second case, we consider the case where each segment has different set of possible transmission rates $R_i \subset \{14 \text{ kbps}, 32 \text{ kbps}, 64 \text{ kbps}, 144 \text{ kbps}\}$.

Suppose users in cell X require a service with the following rates: users in segment $i = 1, \cdots, 13$ require service with rate either 14 kbps or 32 kbps; users in segment $i = 14, \cdots, 33$ require a service with rate either 32 kbps or 64 kbps; users in segment $i = 34, \cdots, 43$ require a service with 144 kbps and users in segment $i = 44, \cdots, 48$ require a service with either 64 kbps or 144 kbps. In cell Y all users require a service with rate either 32 kbps or 64 kbps. The maximum system utility is again approximated by $\max_{t \in J_1(\epsilon') \cup J_2(\epsilon')} \{K_1(t,\epsilon') + K_2(t,\epsilon')\}$. The maximum utility is attained at $t = 0.0020$ with value of 7424 units. The related rate allocation is shown in Figure 8, i.e., for cell X: 13 users with rate 32 kbps, 49 users with rate 64 kbps and 26 users with rate 144 kbps; and for cell Y: 2 users with 64 kbps, 6 users with rate 32 kbps and 39 users are dropped (receive 0 rate).

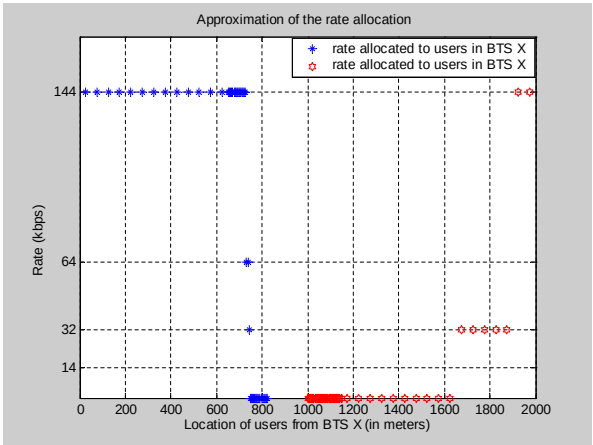

Fig. 7. Rate allocation for traffic load in Figure 4

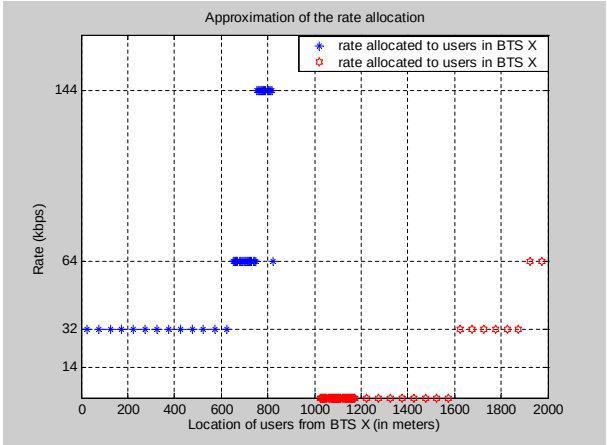

Fig. 8. Rate allocation for traffic load in Figure 4

Notice that by restricting the set of available rates in a segment to the set of requested rates, a higher system utility is obtained (4784 in case I versus 7424 in case II) and less users are dropped (56 users in case I versus 39 users in case II).

5 Conclusion

This paper has provided a model for determining an optimal cell border in CDMA networks. We have formulated a joint uplink and downlink optimization problem for the downlink and uplink power assignment feasibility. Based on the Perron-Frobenius eigenvalue of the power assignment matrix, we have reduced the downlink rate allocation problem to a set of multiple-choice knapsack problems, yielding an approximation of the downlink rate allocation. We used our combined downlink and uplink feasibility model to determine cell borders for which the system throughput, expressed in terms of downlink rates, is maximized.

This approach proves to have several advantages. First, the discrete optimization approach has eliminated the rounding errors due to continuity assumptions of the downlink rates. Using our model, the exact rate that should be allocated to each user can be indicated. Second, the rate allocation approximation we proposed guarantees that the solution obtained is close to the optimum. Moreover, we have control on the error of the approximation and the running time of the algorithm. Last, the result of our method confirms the intuitive rate allocation in CDMA systems, i.e., users with lower interference obtain maximum rate. The numerical results have shown that the system utility is maximized when other-cell interferences are minimized. Therefore, users close to the border may receive 0 rate. Such a rate allocation may seem unfair. It is among our aims for further research to develop a downlink rate allocation for which the fairness over time and limited transmit powers are included in the model.

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