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The decompression of basaltic magma into a sub-surface repository

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Abstract

We examine the ascent of volatile-rich basaltic magma through a vertical dike that intersects a horizontal tunnel of comparable cross-sectional area to the dike and located 300 m below the surface and initially filled with air at atmospheric pressure. This process is a simplified representation of some aspects of the possible interaction of a basaltic fissure eruption with a man-made tunnel, as part of a risk assessment for the proposed high level waste repository at Yucca Mountain, Nevada, U.S.A. We study the decompression and flow that develops following breakthrough into the tunnel using a one-dimensional model averaged over the prescribed dike and tunnel geometry. The main volatile phase in the basaltic magma is water and this is exsolved from the melt as the mixture decompresses. We neglect any motion of the vapor bubbles relative to the mixture and use a parameterization of the bulk viscous resistance. The model predicts that for 2 wt% water, the magma-gas mixture decompresses rapidly into the tunnel, and generates a pressure jump in the air, which travels at a speed of order 500 m/s. Two end-members references simulations are investigated: one in which the dike-drift nozzle (about 20 m²) opens instantly and is relatively smooth, and another one in which the dike-drift nozzle is opened from a small area (< 1 m²) to its steady opening (of about 20 m²) in a minute. In either case the tunnel is eventually filled with high-pressure magma at about its initial dike-tip pressure within a few minutes. In the faster case the pressure jump is reflected and amplified by a factor of 20 – 45 thereby producing a high pressure region at the end of the tunnel away from the dike. In the slower case, the tunnel fills more gradually in about two minutes. Further flow behavior is investigated in a parameter study. The results suggest that this pressurization of the tunnel could lead to rock fracture and magma breakthrough to the Earth’s surface.

Keywords: magma-repository interactions, decompressing magma flows, hyperbolic equations

AMS Subject Classification: 93A30, 35L65, 35L15, 74A50
1. Introduction

There has been considerable interest in quantifying eruption recurrence and long term eruption forecasting in order to warn communities near volcanoes, guarantee aviation safety, and to assess risks involved in storing waste in the proposed nuclear waste repository at Yucca Mountain, Nevada. This repository site is located in an area that has experienced relatively recent volcanic activity in the geological record. Indeed, the Lathrop Wells cinder cone is the product of the most recent eruptions in the vicinity of Yucca Mountain about (0.1 ± 0.05) Ma ago [Connor and Hill, 1995; Connor et al., 1997; and Heizler et al., 1999]. New methodology for predicting volcanic recurrence rates, based on the historical record and geological constraints, have been developed by Condit and Connor [1996], Connor and Hill [1993,1995], and Connor et al. [1997, 2000]. This work has identified that in the immediate area of Yucca Mountain, there is a probability of volcanic activity in the range $10^{-3} - 10^{-4}$ over the next 104 years. Such estimates of volcanic activity are significant and lead to the interesting scientific question of the magma flow that may ensue if relatively volatile-rich basaltic magma, ascending in a dike, were to intersect the repository. It is proposed that the repository consists of a series of tunnels or drifts at a depth of order 300 m, and initially filled with air at atmospheric pressure (Fig. 1). In this study we examine a simplified picture of such an event as part of a risk assessment of the Yucca Mountain repository site. As well as the new insights into explosive magma flow, the problem is of great interest because of the possible associated release of nuclear waste into the environment, which may arise in the waste repository context.

![Figure 1. Schematic of a magma dike ascending through the Yucca Mountain repository site.](image-url)
parameterization to examine the effect of the opening or closing of the rock at the dike-drift intersection.

iv) The basaltic magma and exsolved volatiles form a multiphase mixture. Following Wilson and Head [1991], we parameterize this as a pseudo-one-phase fluid with a monotonic relationship between pressure $p$ and density $\rho$ using Henry's law [Sparks, 1978]. Hence the magma is effectively compressible from a macroscopic view point. Although there has been some analysis of the effects of the relative motion between the bubbles in a conduit, the bulk phenomena are comparable [Vergniolle and Jault, 1986]. For simplicity we therefore neglect these effects herein.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2.png}
\caption{(a) A magma dike is moving upward towards a drift filled with air at atmospheric pressure. (b) In an idealized and experimental configuration of a volcanic dike and drift, a vertical diaphragm (depicted at the beginning of the drift as a dashed light area) separates magma from air. When the membrane is removed, the dike-drift interaction begins. (c) The flow in a flow-tube model depends only on a smooth coordinate $\xi_1$ which follows the dike, and turns via an arc into the drift; variations of the cross-sectional area $A(\xi_1, t)$ of dike, connecting arc, and drift are captured in the model. By symmetry we consider only explicitly the fluid flowing into one side of the tunnel.

v) The complex three-dimensional fluid dynamics of magma and air is simplified into a one-dimensional flow-tube model in an idealized geometry. The one-dimensional flow-tube model sketched in Fig. 2c describes the flow from one sector of the dike into the tunnel on one side of the dike following the left-right symmetry of the problem (see also Fig. 3).

In the flow-tube model, all fluid variables such as velocity, density, pressure and energy depend only on time $t$ and on a curvilinear spatial coordinate $\xi_1$ along the average flow path. The flow-tube configuration in Fig. 2c displays the connecting nozzle between dike and drift. The dashed-dotted line $\xi_1$ indicates the central line of flow. Typically, the area of the nozzle is $\pi d w/2$, which corresponds to half the area of a circular band as wide as the dike with $d$ the diameter of the drift and $w$ the width of the dike. The dynamics in the flow-tube model consists of compressible dynamics for basaltic magma using Henry’s law, and gas dynamics for air using the ideal gas law. Alternatively, the flow-tube model can be derived by averaging the three-dimensional compressible flow equations over the cross-sectional areas of the flow tube. To obtain closure, the resulting Reynolds stress terms are parameterized to leading order by simple frictional parameterizations. In its inviscid limit, the one-dimensional flow-tube model is equivalent to flow in a circular tube with varying cross-section and gravitational influence. The introduction of frictional terms introduces geometrical effects which lead to a greater friction in the narrow dike than in the drift.

The ensuing flow in the flow tube model is therefore expected to resemble dynamics in a classical shock tube problem. The latter consists of initially quiescent flow with different pressures at either side of a diaphragm that is removed instantly [Whitham, 1974, 10.12]. The shock-tube analogy arises because the decompressing magma is compressible owing to the presence of the water-vapor bubbles; the original speed of the magma in the dike is low compared with the ensuing speed of the rapidly decompressing mixture in the air-filled drift; and the flow inertia dominates the frictional stresses. Although we will consider magma-air interactions in a more complex geometry where gravity plays a role, many essential aspects of the flow evolution of magma and air are captured by classical shock tube and idealized shock reflection problems. We therefore explore simplified models to develop some basic understanding before moving on to numerical simulations.

The advantage of an averaged one-dimensional flow-tube model in contrast to higher-dimensional flow models is its simplicity. Neglecting a detailed study of the transition from vertical magma flow in the dike to a horizontal one in the drift may seem severe, but flow-tube models allow study of transient volcanic fluid dynamics for a substantially larger part
of parameter space. It is therefore more straightforward to assess the sensitivity of numerical solutions to variations in volatile content, overpressure, and the frictional parameterizations. We also examine the effects of variations in the cross-sectional area between dike and drift, both as a function of position and time, including a simplified model in which the dike has a fixed length in order to examine the effects of dike closure following the drop in pressure after breakthrough. In this case only a finite amount of magma is released into the drift, although we may expect the dike to open again at a later stage as the dike-drift intersection is repressurized.

The results presented herein provide fundamental insights into magma-air interactions in a dike and drift system. They represent bounding calculations on the effects of explosive magma-air interactions in man-made repository tunnels by using a simplified geometry and flow model which allows for a detailed study of parameter space. Our premise is that reality will lie somewhere within the parameter range studied.

The mathematics of a flow tube model are introduced in section 2. The dike-drift geometry and the basic processes involved are explained in 3. The fluid dynamics of the shock tube and the shock reflection flows are explored in a series of idealized problems in section 3.1. Two reference simulations of magma-air flow in the model dike-drift geometry are then interpreted in terms of these idealized processes in section 3.2. The sensitivity of these interactions is studied in a parameter study in section 4 as a function of various parameters and geometries and we draw some conclusions in section 5.

2. Flow-tube model

The fluid equations of the compressible dynamics of magma and air are introduced next. These equations form the basis for our analytical and numerical investigations of the magma-repository interactions.

The multiphase basaltic fluid of melt and volatiles is modeled as an isothermal compressible fluid with a parameterized equation of state. The density \( \rho \) of this fluid equals the reciprocal of the volume occupied by a unit mass of the mixture of dissolved volatiles (gas), dissolved volatiles (liquid) and melt. The mass fraction of dissolved volatiles is \( n(p) \) with \( p \) the pressure. Dissolved volatiles and melt are lumped together as an incompressible mixture of mass fraction \( 1 - n(p) \) and density \( \sigma \), giving

\[
\rho(p) = \left( \frac{n(p)}{\sigma} + \frac{1 - n(p)}{\sigma} \right)^{-1}.
\]

Here \( R_v \approx 462 \text{ J kg}^{-1} \text{K}^{-1} \) is the gas constant for \( \text{H}_2\text{O} \) in the basaltic mixture, the mixture temperature \( T \approx 1350 \text{K} \), and lumped melt-liquid density \( \sigma \approx 2500 \text{ kg m}^{-3} \). Volatiles, bubbles and melt are assumed to be in chemical equilibrium modeled using Henry’s law [Sparks, 1978]

\[
n(p) = n_0 - n_{sh} \equiv n_0 - s_H p^\beta.
\]

Here the total volatile content \( n_0 = 1.0 - 2.5 \text{ wt\%} \) (weight percent), the solubility constant \( s_H \approx 3 \times 10^{-6} \text{ Pa}^{1/2} \) and \( \beta \approx 1/2 \) for basaltic magmas. The model is a leading order approximation to the complicated physics of the multi-phase melt-volatile mixture. Supersaturation effects are neglected [Sparks, 1978; Woods, 1995], and we assume there is negligible slip between the phases.

Eventually all the volatiles become dissolved when the pressure reaches a critical pressure \( p_c \) where \( n(p = p_c) = 0 \) [equation (2)]. For example, for gas mass fractions \( n_0 = 0.01, 0.025 \text{ wt\%} \), the critical pressure \( p_c = (n_0/s_H)^{1/3} \approx 11.69 \text{ MPa} \). For \( p > p_c \), the purely liquid flow is “nearly” incompressible with a typical sound speed of about \( 1400 - 2000 \text{ m/s} \). In the present study we only consider pressures \( p < p_c \), for which there is a highly compressible dissolved volatile phase, although one could in principle couple the nearly incompressible purely liquid region with \( \rho \approx \sigma \) for \( p > p_c \) and the compressible region of the pseudo one-phase fluid modeled by (1) and (2) for \( p < p_c \) as has been done in models with stationary flow [Wilson et al., 1980; Woods, 1995].

In the dike-drift configuration a representative \( \xi(x, y, z) \)-coordinate may be identified with corresponding cross-section \( A(\xi, t) \), normal to \( \xi \)-isolines, as is illustrated in Fig. 2c by the dashed-dotted line. Any time-dependence of the cross-sectional area \( A \) aims to include the effects of a prescribed opening or closing of the dike on the fluid dynamics, or to the movement of large objects in the drift as a consequence of pressure gradients and viscous forces. A one-dimensional system, obtained by averaging over cross-sections and by neglecting flow in cross-sectional planes, is derived by considering mass and momentum density conservation in a control volume \( A(\xi, t) \, d\xi \). Momentum density is defined as \( \rho A u_1 \). Pressure in the \( \xi \)-direction acts both on the slices at \( \xi \) and
\[ \xi_1 + \Delta \xi_1, \text{ giving a contribution } \sim -\partial(p A)/\partial \xi_1, \text{ and on the flow tube walls between these slices, giving a contribution } \sim p \partial A/\partial \xi_1. \] The gravitational force is a volume force. For the basaltic fluid we then find the following equations of motion (following Whitham, 1974, §8.1):

\[
\frac{\partial(p A u)}{\partial t} + \frac{\partial}{\partial \xi_1} \left( p A u^2 + p A \right) = -A F_1 + p \frac{\partial A}{\partial \xi_1} - \rho g A \frac{\partial z(\xi_1)}{\partial \xi_1},
\]

\[
\frac{\partial(p A)}{\partial t} + \frac{\partial}{\partial \xi_1} \left( p A u \right) = 0, \tag{3}
\]

where \( u = u_1 \) is the \( \xi_1 \)-component of the velocity, and \( -F_1 \) the parameterized forcing and/or dissipation. In a vertical dike or conduit, we have \( \xi_1 = z \) and in a horizontal tunnel \( \xi_1 = x \). We can regard (3) as an average of the three-dimensional compressible flow equations over cross sections in which the frictional stress terms are replaced by dissipation \( F_1 \).

It is not well understood how to model the frictional forces of the bubbly magma liquid owing to its complex rheology. The effective viscosity has been shown to increase with volatile exsolution and also with the pressure of bubbles [Jaupart and Allègre, 1991]. These authors proposed an empirical parameterization of frictional dissipation in a high-viscosity magmatic foam, averaged over the cross-sectional area, and proportional to the velocity:

\[
F_1 = 3 \mu_0 e^{(6-100 \mu_0 \alpha) (1 - \phi)^{-5/2}} \frac{2 L_c^2}{u}, \tag{4}
\]

where \( L_c \) is a typical length scale, for example the width of the dike or the radius of drift or conduit, and \( \mu_0 = 10 - 100 Pa s \). Relation (4) has validity when the void fraction of the mixture, \( \phi \), given by

\[
\phi = \frac{1}{1 + (1 - n) p/(n R_T T \sigma)} \tag{5}
\]

remains below the fragmentation threshold \( \phi < \alpha \), where \( \alpha = 0.7 \sim 0.9 \) [Woods, 1995]. As \( \phi \) evolves through this regime, the gas becomes the continuous phase and frictional forces diminish. When \( \phi > \alpha \), a simple parameterization for turbulent flow in a pipe was proposed by Wilson et al. [1980]:

\[
F_1 = 0.0025 \frac{\rho p u^3}{L_c}, \tag{6}
\]

We can, of course, view these frictional parameterizations as a very crude, leading order (turbulent) closure for the unknown Reynolds stress terms. Both parameterizations refer through the length scale \( L_c \) to the geometry of the cross section. In the inviscid limit when \( F_1 = 0 \) the shape of the cross sections is arbitrary while their area is specified, but when the frictional terms described above are added the area shape is specified through a characteristic cross-sectional width \( L_c \). In our dike-drift system \( L_c \) is five times smaller in the dike than in the drift; friction in the dike is thus five to 25 times larger than in the drift.

The one-dimensional, averaged compressible equations of motion for air (following Whitham, 1974, §8.1) are:

\[
\frac{\partial(p A u_a)}{\partial t} + \frac{\partial}{\partial \xi_1} \left( p A u_a^2 + p A \right) = -A F_a + p \frac{\partial A}{\partial \xi_1} - \rho g A \frac{\partial z(\xi_1)}{\partial \xi_1},
\]

\[
\frac{\partial(p A)}{\partial t} + \frac{\partial}{\partial \xi_1} \left( p A u_a \right) = 0, \tag{7}
\]

in which subscripts “a” distinguish variables in air from ones in the basaltic fluid, and in which

\[
\Omega = p_A \left( \frac{1}{2} u_a^2 + e + gz \right) \tag{8}
\]

is the energy density of air with internal energy \( e \). Air is modeled as an ideal gas with \( p_A = \kappa(s) \rho_A^\gamma = (\gamma - 1) \rho_A e \), where \( \kappa = \kappa(s) \) is a function of entropy \( s \) and \( \gamma = c_p/c_v = 1.4 \) is the ratio of specific heats at constant pressure and volume, respectively. Viscous forces in air will be ignored, \( F_a = 0 \).

The basaltic fluid and air are separated by an interface. In the one-dimensional flow-tube model, the interface between the basaltic fluid on one side and air on the other is marked by a fluid parcel at position \( \xi_1 = \xi_i(t) \). The dynamics of this parcel is governed by

\[
\frac{d \xi_i}{dt} = u_i, \quad \frac{du_i}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial \xi_i} \bigg|_{\xi_i(t)} = -\frac{1}{\rho_A} \frac{\partial p_A}{\partial \xi_1} \bigg|_{\xi_i(t)}, \tag{9}
\]

where velocity \( u_i(t) = u(\xi = \xi_i(t), t) \) and pressure are continuous across the interface, while density is generally not. Since we focus on high-speed inertial effects, diffusion of heat across the interface is neglected. The boundary conditions for the one-dimensional flow tube model are in- or outflow at the beginning of the
flow tube or dike at depth in the Earth, and \( u = 0 \)
at the end wall in the tunnel. In- or outflow conditions are either implemented by simply extrapolating
interior values, or by using an approximate approach based on the Riemann invariants for the frictionless
situation with constant cross section \( A \).

Except for the forcing, geometric and gravitational
terms, the remaining hyperbolic part of the equa-
tions of motion is written in conservative form. The
dynamics of shocks in basaltic flows is governed by
momentum and mass conservation across disconti-
nuities. Energy is additionally conserved in air be-
cause entropy increases across a shock in accordance
with the second law of thermodynamics [Courant and
Friedrichs, 1948, \textsection50]. Conservative formulations of
(3) and (7) form the basis of numerical discretizations
in which shocks are properly simulated.

2.1. Scaling and numerics

Numerical integration is performed using dimen-
sionless equations. Numerical values are chosen to
lie close to unity to improve numerical stability. By
considering the equations of motion and state for the
magma, the following scalings for various parameters
have been adopted:

\[
\begin{align*}
p &= P_0 \rho', & \rho &= \rho_0 \rho', & t = (L_s / U) t', & x = L_s x', \\
u &= U u', & n(p) &= n_0 n'(p'), & \sigma = \rho_0 \rho'
\end{align*}
\]

with \( R_h = n_0 R_{h} T / \rho_0 \) and \( U^2 = P_0 / \rho_0 \). A Froude
number \( F_r = P_0 / (\rho_0 g L_s) \), frictional number \( v = \rho_0 / (L_s \sqrt{\rho_0}) \), and volatile saturation \( \epsilon = \epsilon P_0 \beta / n_0 \)
will appear in the scaled equations of motion and
equations of state for basaltic magmas and air. We
used \( \rho_0 = 100 \text{ kg/m}^3 \) and \( L_s = 5 \text{ m} \).

In order to solve the partial differential equations
(3) and (7) for such high-speed flows, in which frictional
effects play a minor role, the major task is to
discretize the inviscid, conservative and gravita-
tional components of the dynamics properly. Shock-
capturing second- and third-order Local Lax Friedrich
and Essentially Non-Oscillatory numerical schemes
have been used [Shu and Osher, 1988, 1989; Liu and
Osher, 1998]. Code validation and development have
been reported in detail in Bokhove [2001a]. In particu-
lar, code validation consisted of comparing numeri-
cal solutions with exact inviscid solutions of station-
ary, moving and reflected shocks, and shock tubes for
pure magma, pure air, and magma and air combined.

3. Basic processes in a dike-drift
system

A typical cross section of the magma dike and
repository drift (Fig. 2) is given in Fig. 3. (As a
simplification, the drift is assumed to be empty since
the cross-sectional area of the canisters is small rela-
tive to the cross section of the drift.) In the one-
dimensional flow-tube model the connection between
dike and drift is for simplicity represented as a verti-
cal segment and circular arc, and based on symmetry
we only consider the flow going into one side of the
drift.

![Figure 3. A vertical cross section of the magma dike and drift system defines the various length scales involved in typical flow-tube model simulations. Using a symmetric configuration, we only consider half of the flow into the drift.](image)

Characteristic scales in the configuration are: the
length of the magma dike \( L_{dike} \), the distance from the
origin \( O \) to the end of the drift \( L_{drift} \), the connection
during diketip and the center of the tunnel via a cir-
cular arc of length \( L_{arc} = \pi w / 4 \) plus vertical segment
d/2 – w/2. For dikes of greater extent, the pressure
rises above the critical pressure \( p_c = (\rho_0 / s_f)^{1/3} \)
for which all volatiles are dissolved in the magma.
Although the dike continues to the magma chamber, we
restrict the dike lengths in our computational domain
so that the pressure remains below this critical pres-
sure \( p_c \). Given the geometry, the effect of gravity in
the flow-tube model is present in the dike, but dimin-
ishes in the arc and ultimately has no influence in the
drift [cf., equation (3)].

The cross-sectional area \( A(\xi_1, t) \) of the flow tube
is constant in most of the dike, \( A_{dike} \), for \( \xi_1 < L_{end} - L_{drift} - (1/2) L - d \). However it narrows in a
nozzle or transition zone between dike and drift with
minimum area \( A = \pi d w \) before it enlarges to the
constant drift area, \( A_{drift} \). The cross-sectional area
in the nozzle is chosen to be the area of a circular strip of width \( w \) and diameter \( d \), e.g., \( \pi d w \), or a large fraction thereof and is thus smaller than the drift area. The characteristic spacing between drifts determines \( L \). This smooth transition from the dike to the narrower nozzle area over a length \( L/2 \) or \( L \) has some similarity with the bulbous dike-tip model solutions of Lister [1990]. This transition zone model is, however, a much simplified picture of the characteristic three-dimensional flow path and volume between dike and drift. In the flow-tube model, pressure, density and velocity depend only on one spatial coordinate. Most spatial resolution for the numerical solution is needed in the region of prime interest around the dike-drift intersection and in the drift. Instead of \( \xi_1 \), a smooth coordinate transformation to another coordinate is made for which the numerical grid is regular. This yields a coarser \( \xi_1 \)-grid near the bottom of the magma dike.

![Diagram of magma dike and drift with pressure and time profiles](image)

**Figure 4.** This schematic explains the three characteristic phases in time during magma-air interactions in a dike-drift system. Scales are exaggerated.

We consider a basaltic magma with the following reference parameter values \( T = 1350 K, \sigma = 2500 \, kg \, m^{-3}, n_0 = 2 \, wt\%, \eta_H = 3 \times 10^{-6} \, Pa^{-1/2}, R_v = 462 \, J \, kg^{-1} \, K^{-1}, \) and \( \beta = 0.5 \). Given the proposed dimensions of the repository site at Yucca mountain and its geology, we also use the following reference parameters: \( L = 80 \, m, w = 1.5 \, m, d = 5 \, m, A_{dike} \approx 19.64 \, m^2, A_{drift} = 120 \, m^2, L_{drift} = 200 \, m, L_{dike} = 800 \, m; \) friction \( \mu_0 = 10, 100 \, kg \, m^{-1} \, s^{-1} \). The lithostatic pressure \( \mu_L = \sigma g D = 7.5 \, MPa \), at a depth of \( D = 300 \, m \), and overpressure \( P_o = 5, 10 \, MPa \) give a total pressure at the dike tip of about \( P_t = 12.5, 17.5 \, MPa \); and the critical void fraction at the point of fragmentation is taken as \( \alpha = 0.7 \).

The initial condition for the reference simulations is zero flow on either side of the drift entrance, and the air in the tunnel has atmospheric pressure and room temperature. The initial magma pressure and density can be determined by combining the equation of state and the hydrostatic balance condition in one relation, whose root can be found numerically to yield the pressure. Except for the hydrostatic balance, the initial condition is reminiscent of one for a classical shock tube.

![Pressure and time profiles for magma dike and drift](image)

**Figure 5.** Initial condition and the first few profiles of pressure in the dike and drift, and in the dike only, at times \( t = 0, 0.044, 0.088, ..., 0.492 \, s \).

We can distinguish three phases in the fluid dynamical processes that occur after the dike encounters the tunnel. These phases are found in the schematic Fig. 4 of the dike-tunnel system with its curvilinear coordinate \( \xi_1 \) and varying cross section. After the magma breaks through into the tunnel, the
The initial flow closely resembles a classical, idealized shock tube problem in a uniform, horizontal pipe with magma on the left and air on the right. The corresponding idealized problem can be solved analytically. The first 12 consecutive pressure profiles versus spatial coordinate $\xi_1$ of a simulation of the flow, in Fig. 5, show that as time advances a rarefaction wave travels into the high pressure magma to the left, while a shock wave develops in the air which is displaced by the magma. The initial shock wave in air is visible as the small jump of the pressure in air. Since pressure and velocity are continuous across the magma-air interface, this interface is only revealed explicitly in the density profiles (see e.g. Fig. 10). The influence of gravity is negligible due to the high inertia of the flow.

In the second phase, 2, the incoming shock wave in air reflects against the end wall of the tunnel. After a series of shock reflections back and forth between the drift end wall and the magma-air interface, a strongly amplified shock develops in the magma and travels back into the tunnel against the flow of incoming magma. The shock amplification process against drift wall and magma-air interface is clearly visible in the last four profiles in Fig. 5.

In the final phase, 3, the rarefaction wave and reflected shock waves travel down into the dike and diminish in magnitude due to hydrostatic and frictional effects. After these waves leave the domain, the flow comes to rest, in hydrostatic balance, while the air in the drift is compressed to a small volume against the drift end wall.

Table 1. Summary of presented simulations in the three-dimensional pressure/density/velocity-space-time plots.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Figure</th>
<th>Configuration</th>
<th>Friction</th>
<th>Dike tip $P_t$ (MPa)</th>
<th>Duration (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference 1a</td>
<td>5</td>
<td>dike &amp; drift</td>
<td>$\rho_0 = 10kgm^{-1}s^{-1}, \alpha = 0.7$</td>
<td>17.5</td>
<td>0.492</td>
</tr>
<tr>
<td>Resonance</td>
<td>9</td>
<td>horizontal</td>
<td>none</td>
<td>17.5</td>
<td>0.6712</td>
</tr>
<tr>
<td>Reference 1a</td>
<td>10</td>
<td>dike &amp; drift</td>
<td>$\rho_0 = 10kgm^{-1}s^{-1}, \alpha = 0.7$</td>
<td>17.5</td>
<td>1.774</td>
</tr>
<tr>
<td>Reference 1b</td>
<td>11</td>
<td>dike &amp; drift</td>
<td>$\rho_0 = 10kgm^{-1}s^{-1}, \alpha = 0.7$</td>
<td>17.5</td>
<td>8.868</td>
</tr>
<tr>
<td>Time-varying</td>
<td>12</td>
<td>dike &amp; drift</td>
<td>$\rho_0 = 100kgm^{-1}s^{-1}, \alpha = 0.7$</td>
<td>12.5</td>
<td>70.940</td>
</tr>
</tbody>
</table>

To gain understanding of the shock reflection phase, two idealized problems will be considered next. We initially deal with a horizontal shock tube, and neglect the effects of gravity and the geometry of the dike-tunnel system. Subsequently we examine how the simple results are modified in a reference simulation. Note that the initial conditions in these idealized simulations and the reference simulation are different. We have summarized the various simulations in Table 1.

3.1. Idealized solutions of rarefaction and shock waves

It is not possible to derive an exact solution for the reflection of the shock wave arising from a shock tube because of the evolving rarefaction wave. In order to help develop insight, we separately examine an idealized shock tube problem and the reflection of a shock in air between the tunnel wall and the magma-air interface. For the reflecting shock problem, we assume constant but generally different values of pressure, phase or density on either side of shocks and interfaces in order to understand the magma and air interaction. Parameter values for the magma are taken to equal those we used earlier but the initial conditions are varied.

Figure 6. Four regions appear in this space-time sketch for a shock tube in magma, left of the interface, and air, right of the interface.
3.1.1. A shock tube in magma and air. The space-time diagram in Fig. 6 explains the dynamics in a horizontal shock tube, in which gravity and viscosity are ignored. Initially, there is zero flow with the dense, high-pressure magma to the left and the air at atmospheric pressure and room temperature to the right of the interface. Subsequent flow consists of a high-speed shock in air, which propagates to the right ahead of the interface and a backward propagating rarefaction wave. The exact shock relations in air are well known [Courant and Friedrichs, 1948; Whitham, 1974, §6.10] and can be connected to the exact expressions for the rarefaction wave in magma by requiring continuity of velocity and pressure across the magma-air interface. In Fig. 7a, we plot the speed of the shock in air as a function of the initial magmatic overpressure $P_i$ in the dike (solid line). In Fig. 7b, we plot the time required for this shock wave to reach the end of a 200 m tunnel, as a function of the overpressure (solid line). The shock speed in air is high, of the order of 400 – 600 m/s, and reaches the end of a 200 m tunnel, in a fraction of a second. The speed of the incoming magma and air behind the shock wave is lower, since the shock wave propagates quickly ahead of the interface. The speed of the interface between magma and air is given by the thin solid line (Fig. 7a). An increased overpressure is seen to lead to higher sound speeds and shorter travel times. In addition to the magma-air shock tube problem, we have also plotted the shock speeds and travel time for shock tube problems with only a magmatic fluid and only air, respectively. In these latter two cases, the high-pressured magma or air lies to the left, while the magmatic or air pressure to the right is atmospheric. These solutions for pure magma and pure air are denoted by dashed and dotted lines respectively (Fig. 7). With a thin dashed-dotted line we also indicate the speed of the internal interface in pure air. (There is no such interface in a pure magmatic fluid since the density, or equivalently pressure, and velocity govern the dynamics as the magma is isothermal). By comparing the solutions for pure magma, pure air, and magma and air side by side, we note that shock and interface speeds are highest in pure air and lowest in pure magma, with speeds in the magma and air combination lying between these two pure cases. More importantly, we note that the shock speeds do not drop significantly until $P_i < 2 MPa$. In other words, even for dike-tip pressures significantly lower than the 7.5 MPa lithostatic pressure at a depth of 300 m, shocks remain high speed. All the above solutions of shock tube problems are solved by standard methods [Whitham, 1974; Bokhove, 2001a] and are exact except for the use of a root-finding routine.

![Figure 7](image_url)

**Figure 7.** (a) The speed of the shock (denoted by “S”) versus the dike-tip pressure $P_i$ in i) air for the fluid with air only (dashed-dotted line), in ii) magma for a pure magma fluid (dashed line), and in iii) air for the separated magma-air fluid (solid line). Speeds of pure air and magma-air fluid interfaces (denoted by “In”) are thin versions of the shock lines. (b) The travel time of these shocks in a 200 m long tunnel as function of $P_i$ for the i) air, ii) magma, and iii) magma-air fluids, respectively.

3.1.2. Shock reflection in magma and air. When states with constant values are considered in the absence of gravity and explicit friction and in a horizontal pipe as a model of the end of the drift, the dynamics may be solved exactly (at least within some time interval) by tracking each shock-shock, shock-
wall and shock-interface interaction in space and time. The space-time diagram in Fig. 8 explains the evolution.

![Space-time diagram of resonating shock interactions](image)

**Figure 8.** Space-time diagram of the resonating shock interactions between tunnel wall and melt-air interface. The vertical axis is space, the horizontal axis is time. The interface is denoted by a dashed-dotted line and shocks by a solid line. Initially, there is an upper state 1 in the basaltic fluid, an intermediate air state 2, and a lower air state 3 at rest. Effects of gravity and friction are absent.

Initially, dense, high-pressure basaltic fluid in constant state 1 flows towards the wall; across the interface in state 2, velocity and pressure in air are continuous and hence the same as in state 1, but the air density has a lower value; the shock in air approaches the wall faster than the interface approaches the wall, and demarcates air state 2 from air state 3 while the latter is at rest. At time \( T_0 \), the shock in air reflects and amplifies against the wall. The reflected shock then reflects against the interface at time \( T_1 \), and so on. The interface-shock reflection yields a reflected shock and a transmitted shock, the latter propagates in the basaltic fluid. The first couple of transmitted shocks are still swept towards the wall because they are unable to overcome the incoming fluid speed \( u_1 \) in state 1. They are, however, traveling away from the interface. Several transmitted shocks are thus generated in the basaltic fluid, the later ones overtake and annihilate the earlier ones and increase the speed of the shock front in the magma. Finally, a strongly-amplified shock in the magma propagates away from the wall with great speed. The initial shock in the tunnel propagates in the less dense medium: air. Each subsequent shock-interface interaction yields two shocks until finally the air has become dense enough to support the propagation of the shock through the interface while a rarefaction wave unfolds in the air. At this stage the speed of sound in air has become larger than the speed of sound in magma [Courant and Friedrichs 1948, §79.]

![Pressure, velocity and density profiles](image)

**Figure 9.** Pressure, velocity and density profiles are shown for a simulation of resonating shocks between a tunnel wall and magma-air interface for 0.6715 s and \( T = 1350 \text{K} \). The wall of the horizontal tunnel is at the right; the boundary on the left is open and allows inflow (and outflow). Initial values of the pressures are \( p_1 = 0.2 \text{MPa (magma)} \), \( p_2 = 0.2 \text{MPa (air)} \) and \( p_3 = 0.1 \text{MPa (air)} \). The magma-air interface is denoted by the dashed-dotted line in the \( \xi_1 = t \)-plane. Observe the strong compression of air as time advances. Gravity plays no role.

Numerical evidence in Fig. 9 supports this picture. Pressure profiles have been plotted after regular time intervals. Note that each consecutive profile has been shifted upward by a constant amount, relative to the previous profile. That is, zero pressure for the initial pressure profile corresponds to the zero on the vertical scale, zero pressure for the first profile corresponds to \( \Delta P \approx 0.2 \text{MPa} \) on the vertical scale, and so on. The shock reflection in Fig. 8 can, in principle, be solved exactly until the pressure becomes high enough for a
rarefaction wave to appear in solving the matching conditions across the interface, although it requires root finding routines.

As the magma-air interface slows down and finally arrests, the dynamics is no longer presented accurately by the flow-tube model. Upon slowing down, a gravity current develops as the air pocket in the upper corner of the tunnel slowly starts to spread along the top of the tunnel. The dynamics of gravity currents is not captured by the flow-tube model. Speeds of these gravity currents, of the order of \( \sqrt{g' d} \approx 7 \text{ m/s} \) with reduced gravity \( g' \) [Simpson, 1997], are much slower than those of the acoustic shock and rarefaction waves.

3.2. Reference simulations

![Reference simulations](image_url)

**Figure 10.** Pressure, velocity and density profiles are shown for magma-air interactions in the total dike-drift system and in the dike only. The simulation encompasses \( 1.7735 \text{ s} \) and each of the 41 profiles is spaced \( 0.044 \text{ s} \) apart. Short-time reference simulation 210.

In the following, a comprehensive account is given of a fast and slow end-member simulation with parameter values typical for relatively volatile-rich basaltic magmas. Remaining simulations that comprise the parameter study are straightforward variations of the basic geometry and parameter set used therein. The initial conditions and geometries of simulations presented onwards differ from those presented in figures Fig. 7 and Fig. 9. Nearly all simulations have been verified against double resolution runs.

**Figure 10.** Continued.

First, we consider the fast reference simulation with \( P_i = 17.5 \text{ Ma} \) and \( \mu_0 = 10 \text{ Pa s} \). Flow profiles during the first \( 1.7735 \text{ s} \) of the reference simulation are shown in Fig. 10, which is a continuation of Fig. 5. The typical shock-tube profiles of phase 1 (see Fig. 4) govern the dynamics till the shock in air reflects against the end wall of the tunnel at \( \xi_1 = 1002.9 \text{ m} \). A rarefaction wave is then seen to travel into the magma dike but it is modified by the presence of gravity in the arc and dike, because the pressure in the dike is observed to be close to hydrostatic, and at later times by the high viscosity below the fragmentation level. Both pressure and velocity profiles reveal the interplay of reflections between...
drift wall and magma-air interface during the first tens of seconds. The density profile again marks the movement of the interface and the transmitted shock. Near the end of the simulation, at \( t = 1.7735 \text{s} \), a strongly amplified pressure jump is seen to emerge in the magma. The pressure drop across the jump is about 42 times larger than the pressure jump across the initial shock wave in air. Inertial flows dominate during the first couple of seconds and friction and fragmentation are of minor importance. Remember that fragmentation is solely modeled through the dependence of the frictional parameterization on void fraction. The shock reflection process in the reference simulation (Fig. 10) therefore closely resembles the inviscid shock reflection process (Fig. 9) described before.

\[ \begin{align*}
\text{density (kg/m}^3) \\
0 & 1000 \\
1000 & 1500 \\
1500 & 2000 \\
2000 & 2500 \\
2500 & 3000 \\
3000 & 3500 \\
\end{align*} \]

\[ \begin{align*}
\text{pressure (MPa)} \\
0 & 5 \\
5 & 10 \\
10 & 15 \\
15 & 20 \\
20 & 25 \\
25 & 30 \\
30 & 35 \\
\end{align*} \]

\[ \begin{align*}
\xi_1 (m) \\
0 & 200 \\
200 & 400 \\
400 & 600 \\
600 & 800 \\
800 & 1000 \\
\end{align*} \]

\[ \begin{align*}
\text{time (s)} \\
0 & 1 \\
1 & 2 \\
2 & 3 \\
3 & 4 \\
4 & 5 \\
\end{align*} \]

**Figure 10.** Continued.

When the simulation is followed for \( 8.8676 \text{s} \) several new phenomena become apparent (Fig. 11). The remarkable pressure amplification considered in the previous simulation occurs within the first two seconds. Note that in this figure, the first few frames summarize phase 2. While the rarefaction wave travels back into the dike, the large reflected shock wave propagates away from the wall. Before it reaches the dike-drift transition (at \( \xi_1 = 802.9 \text{m} \)), it encounters an anomaly associated with a choking or hydraulic condition at the nozzle [Courant and Friedrichs, 1948, §147]. This phenomenon appears at an early stage in the simulation (Fig. 10). After the first reflected shock has reached the transition, a transmitted shock of diminished size travels into the dike, dissipates and leaves the domain, while a reflected shock travels back into the drift. This shock then reflects against the wall, and after it reaches the transition it divides into a further set of transmitted and reflected shocks which decay (phase 3). Final profiles are close to hydrostatic equilibrium with a strongly compressed air pocket at the end of the tunnel.

**Figure 11.** Pressure, velocity and density profiles are shown for magma-air interactions in the total dike-drift system and in the dike only. The simulation encompasses \( 8.8676 \text{s} \) and each of the 41 profiles is spaced \( 0.2217 \text{s} \) apart. Long-time reference simulation 212.

Second, we consider a slow reference simulation
with $P_1 = 12.5 \text{ Ma}$ and $\mu_0 = 100 \text{ Pa s}$. The quiescent initial condition used so far mimics the sudden breakthrough of magma from the dike into drift, and the fixed nozzle geometry is designed to represent a typical flow path. Viable alternatives exist that deserve equal attention. The connection between dike and drift could, for example, open gradually. We therefore model flow through a dike-drift nozzle that gradually opens over a period of a minute, with the area increasing from $0.389 \text{ m}^2$ to $23.56 \text{ m}^2$ after the simulation begins. Thereafter, the cross-sectional area remains constant. A small but finite initial opening is used to avoid numerical instabilities. The remaining set-up, including the initial condition, is the same as in the basis simulation in Fig. 11. The marked difference with the fast reference simulation is that the pressure amplification due to reflection is negligible and we estimate the tunnel to be filled in about two minutes instead of 10 seconds. Moreover, the pressure pulse at the end of the tunnel increases more gradually.

Alternative geometries of the dike-drift transition can speed up the dynamics. For example when the cross-sectional area $A(\xi)$ is fixed in time but allowed to vary linearly in $\xi_1$ in the transition from dike to drift, the basic results remain similar, but both shock and rarefaction wave propagate slightly faster. Since the nozzle is expanding smoothly from drift to dike, the reflected shock wave in magma is mainly transmitted at the nozzle, in contrast to the situation for the simulation in Fig. 11 where an approaching shock is partly transmitted and partly reflected because the cross-sectional area first contracts at the nozzle before it expands to its large dike value.

![Figure 11](image_url)

**Figure 11.** Continued.
function of the parameter under study. The pressure amplification $S$ is defined as the pressure drop across the reflected pressure jump in magma over the initial pressure drop across the shock in air. When the maximum pressure coincides, numerically, with the initial pressure at the dike-tunnel intersection, we use the maximum pressure in the part of the tunnel that lies at least 50 m away from this intersection.

Figure 12. Pressure profiles are shown for magma-air interactions in a dike-drift system with a time-dependent cross section $A(\xi, t)$ increasing to its “reference” value from 0.589 m$^2$ to 23.56 m$^2$ during 60 seconds. Simulation 299 ends at $t = 70.940$ s. 41 profiles are shown each set 1.774 s apart.

We begin to consider changes in initial dike-tip pressure $P_1$ at the start of the simulation, while leaving all other parameters fixed. The maximum pressure observed in the tunnel lies between 10 MPa to 33 MPa (Fig. 13a) and the shock amplification lies between 28 and 51 (Fig. 13b) for dike-tip pressures in the range 12 – 25 MPa. For increasing values of $P_1$, we find that the rarefaction wave is larger and propagates faster into the dike; yet the position of the magma-air interface hardly changes over the range of dike-tip pressures investigated (not shown).

A decrease in volatile content can result in the pressure in the magma dike being larger than the critical pressure $p_c$ above which all volatiles are dissolved so that the magma is nearly incompressible. To avoid these incompressible regions, the dike-tip pressure and the length of the magma dike have been reduced so that the pressure in the computational domain remains smaller than $p_c$. The predictions of the maximum pressure in the tunnel and the shock amplification are shown in Fig. 14. Note that for the lower volatile content simulations the dike-tip pressure has changed. By interpreting Fig. 13 and Fig. 14 in tandem, we see that an increase in volatile content leads to a small decrease of the maximum pressure in the tunnel and of the shock amplification, for fixed dike-tip pressure.

Increased values of the viscosity $\mu_0$ [equation (4)] lead to reduced pressures in the drift and hence smaller pressure changes across reflected magma shocks, while the magnitude of the shock in air remains similar because friction in air is negligible (not shown). Increased friction drastically slows down the speed of the rarefaction wave in the magma dike (not shown).
When the void fraction (5) of the magma is small, the mixture resembles a high-viscosity foam and the viscosity is parameterized by (4). For flows with void fraction in excess of the critical value for fragmentation, \(\alpha\), the frictional resistance decreases, as parameterized by the turbulent flow law (6). Therefore, the explosive behavior of magma and air is expected to reduce if the critical void fraction increases. Simulations with \(\alpha = 0.7, 0.8\) and 0.9, respectively, reveal a sharp decrease in the reflected shock wave amplitude (not shown), while the maximum pressure observed in the tunnel and the rarefaction wave remains the same. As the critical void fraction for fragmentation increases part of the flow is subjected to the large viscous frictional dissipation [equation (4)]. The rarefaction wave propagates in the highly viscous magma in the dike and is unchanged, but the greater friction of the magma flowing in the drift implies lower and slower shock magnitude and speed. The increased frictional resistance essentially leads to a reduced volume flux, which then yields a smaller amplification of the shock reflected in the magma.

![Graph](image1.png)

**Figure 14.** (a) Maximum pressure \(P_{max}\) in the tunnel, and (b) shock amplification \(S\) are shown versus volatile content \(n_0\) for simulations (with different initial dike-tip pressures): 211) \(n_0 = 2.5\text{ wt}\%, P_t = 17.5\text{ MPa};\) 212) \(n_0 = 2\text{ wt}\%, P_t = 17.5\text{ MPa};\) 220) \(n_0 = 1.5\text{ wt}\%, P_t = 15\text{ MPa};\) and 222) \(n_0 = 1\text{ wt}\%, P_t = 8\text{ MPa}.

### 4.1. Alternate dike-drift transitions

Some aspects of the dynamics in a dike that closes up as magma is withdrawn can be modeled in a simple fashion by assuming that the dike has only a short and finite depth. We consider therefore the ensuing magma flows after breakthrough for four dikes of finite depth 50, 100, 300, and 500\(m\), respectively. In all these four cases, the maximum pressure observed in the drift remains above 10 \(MPa\) due to the shock amplification process (Fig. 15), even though the final pressure in the system after about 15 \(s\) is smaller (between 6 \(MPa\) and 15 \(MPa\)). For dike depths in excess of about 100\(m\), the shock is still amplified by a factor of about 34 (Fig. 15).

![Graph](image2.png)

**Figure 15.** (a) Maximum pressure \(P_{max}\) in the tunnel, and (b) shock amplification \(S\) are shown versus the length of a finite-depth dike for runs: 216) 50\(m\), 217) 100\(m\), 218) 300\(m\), and 219) 500\(m\).

### 5. Discussion and conclusions

In this paper, we have analyzed a flow-tube model of magma-air interactions in an idealized dike-drift geometry (Fig. 2). A dike of constant width and characteristic length is smoothly connected with a uniform and cylindrical horizontal drift. Although in practice the tip of the dike would slowly ascend from a magma chamber and advance towards the subsurface repository drifts, the magma-air interactions studied here start from rest after a diaphragm between dike and drift is broken (Fig. 2b,c).
In accordance with idealized shock tube, hydraulic control, shock-interface and shock-wall reflection problems, the simulations presented herein show a rarefaction wave traveling into the magma dike, and a complex interaction of rarefaction and reflected shock wave interactions in the drift. The initial shock wave in the compressed air travels to the end of the tunnel with speeds of order $500 \text{ m/s}$. When the opening between the dike and drift is immediately relative large strong pressure amplification, between 20 and 45 times, results as a consequence of a “resonating” process of the initial shock in air between the magma-air interface and the drift end wall. That resonance process is consistent with analysis and simulations in idealized interface-wall shock reflection problems (section 3.1). Typically, the resonating process in a drift with an end wall 200 m from the dike-drift intersection is finished in about two to five seconds. After about 10 seconds to a minute, the reflected pressure jump in magma has propagated far back into the dike. Alternatively, when the dike-drift intersection opens more slowly, say in a minute, the tunnels are filled more gradually, on the order of a few minutes. In either case, the tunnels are eventually filled with high-pressure magma at about its initial dike-tip pressure.

The sensitivity of our reference simulation, presented in section 3.2, has been assessed as a function of the initial dike-tip pressure, the volatile content of the magma, friction, fragmentation level, and geometry of the dike-drift interface (section 4). This geometry models the characteristic flow-tube area around the dike-drift transition, where the upward going magma turns around and flows into the horizontal tunnel. In accordance with our expectations, increasing the dike-tip pressure leads to larger rarefaction and amplified reflected shock waves. While the rarefaction and shock waves propagate somewhat faster, the interface movement is basically independent of changes in the absolute dike-tip pressure, for values in the range 7.5 – 17.5 $\text{MPa}$. Changes of volatile content are hard to implement without modifying dike-tip pressure or without a model that can handle both nearly incompressible and compressible magma together (see point (ii) below). Along with the volatile content, the dike-tip pressure and dike depth have also been changed in order to avoid incompressible regions, in which all volatiles are dissolved, in our computational domain. However, we found that for decreasing volatile content, from 2.5 wt% to 1 wt%, the amplitude of the shock wave diminishes. An increase of the viscous frictional parameter $\mu_0$ shows a slow-down in the speed of the rarefaction wave in the dike and a reduction in amplitude of the reflected shock. Another reduction in the reflected shock amplitude occurs upon increasing the critical void fraction for fragmentation level from 0.7 through 0.8 – 0.9. Rarefaction wave propagation speeds and interface positions are not affected by this change because the position of the fragmentation surface only affects the low-pressure regions of magma in which there are a large volume of exsolved volatiles (i.e., in the drift).

Finally, as a simple model of some aspects of the effect of the closing of the dike when magma is withdrawn (section 4.1), we considered cases in which the dike depth is finite. The maximum pressures observed in the tunnel after breakthrough remain high, above 10 $\text{MPa}$ for a 50 m deep finite dike and this rapidly increases for deeper dikes of finite length.

In conclusion, our bounding calculations show that by reducing volatile content and dike-tip pressure to reasonable lower limits and by increasing frictional values and fragmentation levels to reasonable upper limit, the tunnel is rapidly filled between $10 \text{s}$ and a few minutes, whereafter a high-pressure quiescent end state results. The nature of the pressure evolution depends on the initial conditions. In the worst case, for large initial dike tip pressures and large enough initial mass fluxes, a dominant pressure pulse develops at the tunnel end. In the mildest case considered, the tunnel is filled gradually in about three minutes.

In natural or artificial dike-tunnel systems a new dike or conduit may develop if absolute pressures and pressure gradients are sufficiently high, and our study suggests that a breakthrough may arise anywhere along the tunnel. Our one-dimensional model is not able to detect the increase in pressure due to the impact of initial flow after breakthrough against the tunnel roof above the dike-drift intersection. However, preliminary two-dimensional simulations seem to indicate that this initial pressure increase is much lower than the one at the closed end of the tunnel [Bokhove, 2001b, private communication]. Where breakthrough to the surface will occur in the tunnel depends on the complicated and poorly understood interaction between fluid dynamics of the magma-air system and the rock mechanics associated with the dike. Further work is required to study this interaction.

Although the model and parameter study described herein grasp the leading order behavior of explosive magma-air interactions in a dike-drift system, details of several phenomena remain poorly understood. A number of important aspects which merit further re-
search include: (i) coupling the flow-tube model to a simplified model of rock mechanics [Lister, 1990; Rubin, 1993; Mériaux et al., 1999] in order to assess how the dike walls react to pressure fluctuations in the magma after breakthrough; (ii) the stationary and transient flow in a one-dimensional flow-tube model in which a magma dike or conduit has formed at the end of a drift, and has reached the Earth's surface [Woods et al., 2001, subjudice]; (iii) better characterization of the viscosity and bulk rheology of the magma-gas mixture, of the volatile exsolution rate and kinetics, and of the effects of phase separation; and finally (iv) more refined modeling of transient flow profiles at the dike-drift transition and of gravity currents near the interface in two-dimensional laterally averaged or three-dimensional models.

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