Faculty of Mathematical Sciences

University of Twente

University for Technical and Social Sciences

P.O. Box 217
7500 AE Enschede
The Netherlands
Phone: +31-53-4893400
Fax: +31-53-4893114
Email: memo@math.utwente.nl

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Word graphs: The third set

C. Hoede and L. Zhang[1]

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[1]Northwestern University, Xi'an, P.R. China

Word Graphs: The Third Set

C. Hoede and L. Zhang*

Faculty of Mathematical Sciences
University of Twente
P.O. Box 217
7500 AE Enschede, The Netherlands

Abstract

This is the third paper in a series of natural language processing in term of knowledge graphs. A word is a basic unit in natural language processing. This is why we study word graphs. Word graphs were already built for prepositions and adwords (including adjectives, adverbs and Chinese quantity words) in two other papers [2], [3]. In this paper, we propose the concept of the logic word and classify logic words into groups in terms of semantics and the way they are used in describing reasoning processes. A start is made with the building of the lexicon of logic words in terms of knowledge graphs.

Key words: Knowledge graphs, word graphs, logic word, classification, ontology.

AMS Subject Classifications: 05C99, 68F99.

1 Introduction

Natural language is a kind of special symbol system that is used to express human ideas and pass on information. Each natural language has evolved into a kind of traditional symbol system which has its own word types and word structures, in which different components have some relationship and these related components play a role as a whole that has some meaning. In science a particularly important part of the special symbol system is the set of words used in logic. It is therefore necessary to study the logic phenomena in natural language and the common rules in different languages in order to reveal and explain pure logical forms, other logical structures and logical rules in sentences of a natural language text.
In particular in knowledge graph theory the meaning, which is considered identical with the graph structure, of a sentence is a function of the meanings, graph structures, of its various parts. Therefore, to understand the meaning of a sentence one first needs to understand the meaning of each word that occurs in the sentence thus gaining the meaning of the whole sentence from these words and their order, one of the syntactical aspects of the sentence. Finally, combining the sentence graphs we can obtain the meaning of the whole text.

The paper will study the set of words called *logic words*, which have a logical function in a sentence or even a paragraph. In Section 2 we first discuss some ontological questions and take a stand from the point of view of knowledge graph theory. Section 3 deals with classification aspects of Chinese words. Logic words are classified in Section 4. Combining the analysis of

*On leave from Northwestern University, Xi'an, P.R. China

linguistics with the analysis of logic, we try to reveal common properties of logic words, that are independent of the particular language. In Section 5 we discuss word graphs for logic words.

2 Ontological aspects

Before starting our particular study, we should say something about the ontology we are using. We refer to the papers of Hoede and Li [2] and of Hoede and Liu [3] for general information on the ontology of knowledge graphs. We recall only the most essential parts for our discussion. The word graph ontology consists, up till now, of 8 types of binary relationships and 4 types of n-ary relationships, also called *frame* relationships.
The eight binary types describe

Equality	:	EQU
Subset relationship	:	SUB
Similarity of sets, alikeness	:	ALI
Disparateness	:	DIS
Causality	:	CAU
Ordering	:	ORD
Attribution	:	PAR
Informational dependency	:	SKO.

They are seen as means, available to the mind, to structure the impressions from the outer world, in terms of awarenesses of somethings. This structure, a labeled directed graph in mathematical terms, is called *mind graph*. Any part of this graph, can be *framed and named*. Note that here WORDS come into play, the relationships were considered to be on the sub-language level so to say, on the level of processing of impressions by the brain, using different types of neural networks.
Once a subgraph of the mind graph has been framed and named another type of relationship comes in, that between the frame as a unit and its constituent parts. The four n-ary frame-relationships are describing:

Focussing on a situation	:	FPAR
Negation of a situation	:	NEGPAR
Possibility of a situation	:	POSPAR
Necessity of a situation	:	NECPAR.

The situation is always to be seen as some subgraph of the mind graph. It will already be clear that word graphs for logic words will mainly be constructed using the second set of four n-ary relationships.

Let us compare our ontology with two of the many ontologies proposed in history. The first one is of course that of Aristotle. He distinguished:

• Quantity	• Relation	• Time	• Substance	• Activity
• Quality	• Location	• Position	• State	• Passivity.

These ten basic notions clearly focus on the physical aspects of the impressions, as do the first eight notions of word graph ontology. The focus there is on the way the world is built.
The second ontology to consider is that of Kant, who distinguished twelve basic notions:

QUANTITY	QUALITY	RELATION	MODALITY
• Unity	• Reality	• Inherence	• Possibility
• Plurality	• Negation	• Causality	• Existence
• Totality	• Limitation	• Commonness	• Necessity.

Note that Kant clearly focuses on the logical aspects, including modal logic concepts like *possibility* and *necessity.* Of course *negation* is included as well. Together with the *and* concept, which is simply two tokens framed together in knowledge graph theory, the negation gives a *functionally complete* set of logical operators, in terms of which first order propositional logic formulae can be expressed. The possibility of describing all known systems of modal logics by means of knowledge graphs was shown by van den Berg [1].

Here some remarks are due concerning the work of C.S. Peirce [5]. Describing logic by graphs, called *existential graphs* by him, was introduced by him before 1900, starting with the idea of simply indicating *and* ($\wedge$) and *negation* ($\neg$) by two different types of frames. The work of van den Berg can be seen as a direct continuation of this set up. It has often been said that Peirce was guided by the ontology of Kant, who presented the twelve basic notions in four triples, see above, when he introduced the notions of *firstness*, *secondness* and *thirdness* of a concept. Peirce's definitions are not very easy to understand. We quote from Sowa [6]

Firstness:	The conception of being or existing independent of anything else.
Secondness:	The conception of being relative to, the conception of reaction with, something else.
Thirdness:	The conception of mediation, whereby a first and a second are brought into relation.

From the point of view of knowledge graph theory the following stand is taken.

For any concept, token or (vertex) of a mind graph, we can distinguish:

- The token itself, which usually has an inner structure, the definition of the concept.
- The token together with its neighbours, including a subgraph of the mind graph, that we call the *foreground knowledge* about the concept.
- The whole mind graph, considered in relation to the concept, also including what we call the *background knowledge* about the concept.

In our view Kant's triples do not correspond precisely to Peirce's notions and we have the idea that the triple of knowledge graph theory: concept, foreground knowledge, background knowledge, is all that Peirce's notions are about. What is extra in our theory is the fact that the mind graph is not a fixed entity but depends on the particular mind (human) and for one human even on the particular circumstances in which the concept word is to be interpreted. Also the *intention* of the concept, its definition, is often not uniquely determined, although it is one of the major goals in science to get at least the definitions straight. The variation in meaning, possible for a word, is an intrinsic and essential part of knowledge graph theory.

3 Classification of Chinese words and criteria used in classifying

It is necessary to classify the logic words before we study them. This section is inserted because there are more ways to classify than the one used in traditional Chinese linguistics.

How to classify a word depends on the specific purposes that classifiers have. Different purposes will result in different classifications. In order to study the structure of a set of single words it is usual to classify the words into monosyllabic, bi-syllabic and polysyllabic words according to the number of syllables. In order to study the formation of words, we classify words into simple words and compound words according to the number of morphemes. However, this is not the classification of grammar. The classification of grammar is according to the grammatical function of a word. Its purpose lies in explaining the structure of language and the usage of a word. For example, in traditional Chinese linguistics, two categories in the classification according to grammar are the notational word and the function word.

In order to make a distinction between the purposes of traditional linguistic and general purposes we ought to divide the classification into two parts. One is the word classification in the narrow sense, which is according to traditional linguistics, another is the word classification in the general sense, which is according to some general purpose.

Definition 1 A *narrow* word classification is a classification according to the grammar of traditional linguistics.

Definition 2 A *general* word classification is a classification according to the purpose of classifiers.

There are several criteria in a narrow word classification. In Chinese we have three main criteria. Words are considered:

- in terms of the shape feature
- in terms of the syntax feature
- in terms of the meaning of the word.

Of these criteria the second one is considered to be the key one because Chinese has no distinct changes in the shape feature and the criterion of a word's meaning may generate ambiguities. In line with the knowledge graph idea of meaning as a word graph, particularly in Chinese the meaning of a word strongly depends on the context. The specific meaning attached to a word is affected by the philosophy, the social aspects, the ethical aspects, etc., involved in the discussion. Nevertheless, the main goal of knowledge graph theory is to construct a lexicon of word graphs.

For the purpose of natural language processing, we investigate logic words and classify them, but not according to the narrow classification. The classification will be general, the purpose being to make distinction according to the ontology of knowledge graph theory.

4 Logic words and their classification

The words mentioned in traditional logic should be logic words. For example, "and", "or", "not", "if ..... then ....." and "if and only if", which are five words describing connections in proposition logic, are typical logic words. Words like "possible", "necessary", "ought" and "permitted" which are used in modal logic and deontic logic are of course also logic words. But, many other words used in natural language, such as "therefore", "since", "while", "but", "before", etc. are related to logical aspects of utterances as well. So the classification of logic words might include two parts, which are *pure* logic words and *other* logic words. The pure logic words are then, by definition, those words that are mentioned in traditional logic (including proposition logic, predicate logic, tense logic, modal logic, deontic logic and fuzzy logic). This class of words seems quite easy to recognize. The other logic words are words that are somehow related to logical aspects as well. This class turns out not to be as easy to define and structure. We are therefore in need of more precise classification criteria.

4.1 Classification criteria

Before discussing possible classification criteria we should mention the objects that we wanted to classify. A corpus of 2000 English words with the property that they occurred most frequently in a set of 15 texts was established by Hofland and Johansson [4]. Our main goal is to develop a system of *structural parsing*, by means of which from a lexicon of word graphs the sentence graph of a given sentence can be constructed. So first the sentence is taken apart, and then from a representation of the parts, the words, a representation of the sentence is constructed.

a) **Subjective classification**
As a start of the general lexicon these 2000 words should be included. For that reason, and to have a natural restriction for the set of words, we considered these 2000 words and tried to classify them on a five point scale:

1. definitely a logic word
2. clearly related to logic, but not basic
3. related to some form of logical reasoning
4. having a vague logical flavor
5. no relation to logic.

As a result the two authors classified the words into classes $C11$, $C12$, $C13$, $C14$, $C15$, $C22$, $C23$, $C24$, $C25$, $C33$, $C34$, $C35$, $C44$, $C45$ and $C55$. The two indices indicate the scale values mentioned by the authors. We give only the first class resulting from this subjective coding process, in order of frequency of the words.

C_{11}: and, if, no, then, must, might, right, however, every, possible, difference, cannot, necessary, therefore, probably, true, thus, nor, everything, else, unless, truth, impossible, neither, doesn't, wouldn't, everyone, ought, isn't, possibly, nevertheless, possibility, existence, maybe, equal, equivalent, necessarily, hence.

b) *Classification by using Kant's ontology*
Another way of classifying would be to use Kant's ontology and decide whether a word belongs to one of his twelve categories, i.e. expresses something the main feature of which is one of these twelve concepts. As an example, let us consider those words in the class C_{11} that we determined, that would fall in the category of "possibility". We would choose "might", "possible", "cannot", "probably", "impossible", "possibly", "possibility" and "maybe" as elements of this class.

c) *Classification by using knowledge graph theory ontology*
Looking at these words in b) from the knowledge graph point of view we discover that people, students in our case, making a word graph for those words, use the POSPAR-frame in all cases. This prompts another way of classifying, namely according to the occurrence of FPAR, NEGPAR, POSPAR and NECPAR-frames in the word graphs of the word. Note that these frames correspond to Kant's categories *existence, negation* (seen as a quality by Kant), *possibility* and *necessity.*

Definition 3 A *logic word of the first kind* is a word, the word graph of which contains one of the four types of frames in the knowledge graph ontology.

The existence of two somethings, seen as two components of a frame, puts them in an FPAR-relationship with the frame. That frame can be named "and". Similarly, something in a NEGPAR-frame is put in a NEGPAR-relationship with that frame, that now can be named "not". By functional completeness the other connectives from propositional logic follow from equivalences like $p \vee q \Leftrightarrow \neg(\neg p \wedge \neg q)$, for the "or"connective.
When we consider something, say a situation S (German: Sachverhalt) in the form of a graph a POSPAR-frame may be considered around it. We may describe this by saying "S is a possibility", "It is possible that S" or "Possibly we have S". In Chinese these three utterance are translated as
"ke 3 neng 2 S", literally "possible S"
"S shi 4 ke 3 neng 2 de", literally "S is possible"
"you 3 ke 3 neng 2 S", literally "have possible S",
respectively.

We have chosen to give the Chinese sentences and words in this paper in spelling form, Pin 1 Yin 1, followed by a number 1, 2, 3 or 4, indicating the four intonation forms.

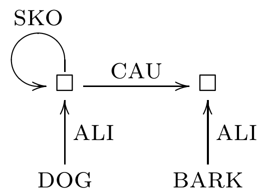
Subtle differences come forward due to the choice of the POSPAR-frame. Suppose S is given by the following graph, in socalled total graph form in which the arc is also described by a vertex,

S : [A] → (CAU) → [B].

A POSPAR-frame around the whole of S would describe "(There is) possibility (that) A causes B". A frame around [A] → (CAU), which by itself reads "cause A", would lead to a description of "A (is a) possible cause (of) B". A frame around (CAU) → [B] would describe "A possibly causes B". In English "possible" is an adjective and "possibly" an adverb. The decision which word to use depends here on "cause" being a noun and "causes" a form of the verb "cause". In Chinese, the three sentences are translated

"A yin 3 qi 3 B shi 4 ke 3 neng 2 de", literally "A cause B is possible of",
"B ke 3 neng 2 you 2 A yin 3 qi 3", literally "B possible have A cause"
"A ke 3 neng 2 yin 3 qi 3 B", literally "A possible cause B".

The reader should remark that the existential and universal quantifiers, “there exists” and “for all”, are not falling under Definition 3. In fact, Peirce already pointed out that making the statement S on the paper of assertion (in whatever form), is equivalent to existential quantification (for closed formulae). That is why we put “There exists” between brackets in our example sentences. The universal quantifier in knowledge graph theory is expressed by the SKO-loop on a token, that should be read as “for all”.



is to be read as “all dogs bark” or “for all dogs (holds) dog bark(s)”. So “all” is not a logic word of the first kind according to Definition 3. We should note here that “all” is falling in the category “totality” of Kant’s ontology. “Exist” is a logic word of the first kind as its word graph is the “be”-frame, the empty frame, filled with “something SUB world”. In the framing and naming process, a subgraph of the mind graph is framed and in that way the definition of a concept C is given. The description is “C is a ....”. Here “is” is a logic word of the first kind too as it describes the FPAR-relationship between C and its frame content. The famous “ISA”-relationship, like in “A dog is a(n) animal”, expresses the FPAR-relationship between “animal” (part of the definition of “dog”) and the “dog”-frame. So here IS in ISA is a logic word of the first kind too.

For the other logic words the classifying feature in their word graphs is chosen to be one of the other eight types of, binary, relationships. An important example is “causality”, basic in words like “cause” or “because”, which also is a category of Kant’s ontology. In knowledge graph theory we would classify according to the occurrence of a CAU-arc. The ALI-link, for “alike” concepts, corresponds to concepts in the “commonness”-category of Kant’s ontology and determines a set of words like “like”, “as ..... as”, etc. Other logic words, so words with one of the eight binary relationships as dominant link in the word graph, are not really expressing aspects of pure logic, but then pure logic is not the whole of thinking, that we like to describe as “linking of somethings”. In expert systems the question “why C?” can be answered if causations are known. If B is a possible cause for C and A a possible cause for B, then the answer may be “Possibly because A”. If we analyse this thinking process, we see that we start with C, and then notice that we know that $B \xrightarrow{\text{CAU}} C$ and $A \xrightarrow{\text{CAU}} B$. By linking these data we obtain $A \xrightarrow{\text{CAU}} B \xrightarrow{\text{CAU}} C$, and have found A as possible cause. This process of linking is particularly interesting when the concepts are *expanded*, i.e. they are replaced by the content, of their frame, that is embedded in the rest of the graph considered. This replacement poses problems of its own, but we are not interested in them here. The expansion process plays an important role in thinking. Given some statements, say in mathematics, and the problem to prove that some goal-statement G is true if the given statements are true, then the way to find the proof may be the following. Expand all given statements as far as possible, with available further knowledge, till a graph A is obtained that has the graph G of the goal statement as a subgraph. The basic process of reasoning is namely that whenever a graph is considered to be true, each of its subgraphs must be true (under specific conditions on the structure of these subgraphs).

In trying to find the (answer) graph A one meets the difficulty of expanding the graph in the right direction. From the given statements expansion, combining of "true" graphs, may lead to many answer-graphs A that are all true, but none of which contains the goal graph G as a subgraph.

Both in this general process of reasoning and in the case of expert systems, a "rulebased" version can be given. "If a graph A is true *then* its subgraph G is true" is the rule in the first case, but in natural language we would use the word "so" (which by the way turned up in class C_{12}): "A so G". "If A then B" and "If B then C" are rule-versions for natural language descriptions like "B because of A" and "C because of B". It is for this reason of almost equivalence in description that it makes sense to speak about other logic words. The rule-version has the pure logical setting in which the pure logic words are used. The statements have to be well-formed closed formulae. In natural language the thinking process is often described by not-well formed statements that nevertheless correspond to certain mind graphs that can be used in the description of the expansion process. We will see that this deviation from pure logic allows for dealing with some other linguistic aspects as well.

Definition 4 *Logic words of the second kind* are words the word graph of which contain one of the eight types of binary relationships of the knowledge graph ontology as dominant link.

We have given a restriction here by demanding that the relationship, e.g. the CAU-relationship, is a dominant link as otherwise all words would be logic words. Meant are those words that describe the linking process in its basic form. Word graphs with more than one type of binary relationship are to be excluded, unless one link is clearly dominant. In the first paper in the series the 15 different word graphs for the Chinese word for "in" were given. In them a SUB-link or a PAR-link was clearly dominant. A preposition like "in" can therefore also be seen as a logic word of the second kind, used often in thinking about structuring the world. To determine which link is determinant we need some measure for dominance. In graph theory measures have been developed for concepts like centrality. Similar measures can be chosen for the concept of dominance and thus used to decide whether a word can be called a logic word of the second kind or not.

Finally, some discussion is due on the words "truth" and "true", definitely words often used in logic. There are two ways of looking at these words. First there is the comparison of a statement or proposition p with a model of the situation expressed by p. The outcome of the comparison determines the truth value of p, which in two-valued logic is "true" or "false". For our knowledge graph view, what is happening is comparison of two mind graphs, one for p and one for the model. "Truth" can then be seen as equality of certain frames, one for p and one for (a part of) the model, and hence is a logic word of the first kind according to Definition 3. The truth values "true" and "false" are nothing but instantiations of the outcome of the comparison that may be replaced by, for example, the numbers 1 and 0, as is often done. These are not logic words.

A second way of looking at "truth" is as an attribute of a framed part of the mind graph. That part may represent the content of a contemplation, or, more close to what was said before, the model of a situation, as perceived. Such a perceived situation may be held to be true or false, i.e. truth is attributed. No comparison is made, there is only one frame, that may of course be described by a statement p. One of the arguments in favor of this second way of looking at truth is that both statement and model are parts of the mind graph. The model is also held to be a correct description of the state of affairs, whether this state of affairs is due to a presupposed "outer world", as in physics, or due to ideas in an "inner world", as in mathematics, where e.g. axioms are simply considered to be true. The statements describing axioms are not considered to be in need of comparison.

The knowledge graph theory slogan "the structure is the meaning" is in line with the second way of looking at truth. The statement "it is raining outside", a standard example, does not need comparison with a model. The structure of the part of the mind graph associated by the listener with the statement is all that matters, as far as meaning attribution is concerned. The truth of the statement is depending on the comparison, with the outer world. In socalled *truth conditional* semantics this comparison is stressed. In our *structural* semantics, the outcome of such a comparison is irrelevant. As a major consequence of our stand even statements that are not well-formed also have a well-defined semantics as far as the corresponding mind graph frames are well-defined. A statement like "$x < 5$" is considered to have no well-defined truth conditional semantics even when a model is given, with proper domain and interpretations, because x is free. Any knowledge graph constructed by a mind as corresponding to the statement is the meaning of the statement in structural semantics.

4.2 Classifying logic words of the first kind

Due to our chosen Definition 3 we would have to construct the word graph to decide whether a word is a logic word of the first kind. We already discussed the operators from proposition logic and the quantification operators. Also modal logic operators were discussed. Of all these only the universal quantification did not involve a frame, but was expressed by a SKO-loop and was therefore a logic word of the second kind. The first set of logic words of the first kind corresponds to the four frames themselves and is given in Table I.

WORD GRAPH	WORD	PARAPHRASE	CHINESE WORD	LITERALLY
Frame	Inherence	be with	Gu 4 you 3 xing 4	primitive have gender
Negframe	Negation	be not	Fei 1	not
Posframe	Possibility	be possible	Ke 3 neng 2 xing 4	possible gender
Necframe	Necessity	be necessary	Bi 4 ran 2 xing 4	necessary gender

Table I

Remark first the use of the word xing 4, "gender", which is used to describe the occurrence of an alternative. Literally possibility is circumscribed by "possible male/female", where male/female only functions to express the two values for possibility, possible/impossible. Secondly, the word Fei 1, for "negation", used in the context of logic, literally must be translated as "not".

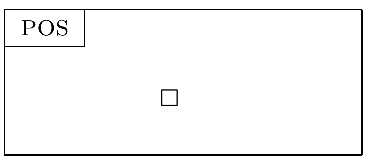
We have chosen the words inherence and negation as these are two of Kant's categories. Note that the word "negation" has a subjective undertone. Similarly, any subgraph that is framed and named gives a concept with the subgraph as *inherent* property set. The word "inherence" clearly expresses more than just "being".

The second set of logic words of the first kind has graphs containing one of the four frames next to other parts. Let a graph P, corresponding to a proposition p, be contained in a frame, which may be described by "it is so that p", or simply by "being p" or even just "p". In the knowledge graph formalism the graph

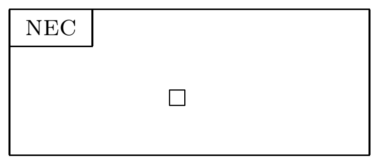
$$P: \qquad p \xrightarrow{\text{EQU}} \square \xleftarrow{\text{ALI}} \text{PROPOSITION}$$

would be given, or, simpler, $\boxed{P}$. A frame containing the frames $\boxed{P}$ and $\boxed{Q}$ is the representation of $p \wedge q$, or p AND q in natural language. The word graph for "and" , She 2, respectively "∧, "yu 3", is

without specifying the contents of the two inner frames, or tokens as they are called. By functional completeness other connections in proposition logic are expressable by the "and"-frame and the NEG-frame. Consider a NEG-frame containing a proposition graph P, then is can be described by "it is not so that p", or simply by "negation p" or "not p". Omitting p from the graph the word graph for the word "not" results. In Chinese this is described by, "bu 1 shi 4 p", literally "not be p". Likewise the POS-frame and the NEC-frame allow expressing "possible p" respectively "necessary p". Omitting p again the word graphs for "possible", ke 3 neng 2 de, and "necessary", bi 4 ran 2 de, result as



and



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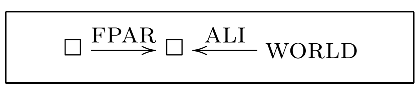
Note that in Chinese the word de is used to express the fact that we are dealing with an adjective.

The frame corresponding to "being" may contain the graph of something considered to exist in the world, so essentially the graph



describes the word "existence", cun 2 zai 4 xing 4, literally "existence gender". Further information on the "world " considered, which may e.g. be a set of numbers, may be added. We now have word graphs for the logic words of the first kind describing logical operators, see Table II

LOGICAL OPERATOR	WORDS	CHINESE WORDS	LITERALLY
Proposition logic	And	Yu 3	and
	Not	Fei 1	not
	Or	Huo 4	or
	If .... then	Ru 2 guo 3 ... ze 2	if ... then
	If and only if	Dang 1 gie 3 jin 3 dang 1	when and only when
Predicate logic	Existence (of)	Cun 2 zai 4	exist
Modal logic	Possible	Ke 3 neng 2 de	possible of
	Necessary	Bi 4 ran 2 de	necessary of

Table II

Note that for “if”, the word “ru 2 guo 3”, is used in “if ... then”, whereas in “if and only if”, the word “dang”, is used, literally meaning “when”.

We should take a stand with respect to other logics like tense logic, deontic logic, or fuzzy logic and the words used therein. Consider for example the word “obligation” in deontic logic. It may be argued that it is a word like “possibility” or “necessity”, but also that obligation is just an attribute of something, say an attribute of an act, attached to the act by the speaker, according to his norm system. If we follow the first argument, we might introduce an OBL (obligation)-frame, analogous to the POS-frame and the word would have to be considered a logic word of the first kind. In that case we consider “obligation” to refer to a given set of rules. We follow the second argument and do not consider “obligation” to be a logic word of the first kind or even an logic word of the second kind, but to be similar to a word like “beauty”. Words used in tense logic are words like “present”, “past”, “future”. They are expressible in terms of the ORD-relationship concerning time values, so at most fall under logic words of the second kind in case the ORD-arc is really dominant in the word graph. “Fuzziness” basically involves unprecise information about values of something. A fuzzy word from natural language is for example “youth”. No specific frame or basic relationship seems present here. The word graph for “youth” will clearly be quite complex.

Concluding, the logic words of the first kind are those mentioned in Table I and II. For those words that have word graphs very close to those for these pure logic words we take the stand that although frames are used, the essential meaning is not expressed by the frame. One example should suffice here. The word “both” is used in expressions like “both a and b are numbers”, meaning “a is a number and b is a number”. “Number $(a) \wedge$ Number (b) may be the formulation in predicate logic. The essential meaning of “both” is that the predicate holding for a and b is the same, the “and”-connective has the, different, meaning of combining two propositions that may have no further commonness at all. “Both” is therefore not considered to be a logic word of the first kind. Whether it should be seen as a logic word of the second kind, because the EQU-relationship is clearly present, will be discussed in the next section.

4.3 Classification of logic words of the second kind

For the logic words of the first kind the four frames in the ontology of knowledge graph theory were chosen as defining entities. For the logic words of the second kind we choose the eight binary relationships.
The representation we choose is that of the total graph in which the arc is also represented by a, labeled, vertex. The reason for this is that we can then consider subgraphs of the graph

$$\square \longrightarrow (\text{TYPE}) \longrightarrow \square\ ,$$

where the label TYPE may be one of the eight types we distinguish. Table III gives the words corresponding to the whole graph of three vertices and the graph in which an encompassing be-frame is considered as well. For the type EQU the graph is considered to represent the word “equal”, whereas “being equal” is considered to be a synonym of “equality”.

TYPE	WORD	WORD (+ BE)	CHINESE WORD	CHINESE WORD (+ BE)
EQU	Equal	Equality	Xiang 1 deng 3	Xiang 1 deng 3 xing 4
SUB	In	Containment	Li 3	Bao 1 han 2 xing 4
ALI	Alike	Community	Xiang 1 tiang 4	Gong 4 xing 4
DIS	Distinct	Disparateness	Bu 4 tong 2	Cha 1 yi 4 xing 4
ORD		Ordering	Shun 4 xu 4	Pai 2 xu 4
CAU	Causation	Causality	Qi 3 ying 1	Yin 1 guo 3 xing 4
PAR		Attribution		Shu 3 xing 4
SKO		Dependency	Ying 3 she 4	Yi 3 lai 4 xing 4

Table III

Some remarks are due here. First we could not determine an English or Chinese word for the PAR-link. Secondly we notice that for the ORD-link, within a BE-frame, in Chinese the alternative indicating word xing 4, “gender”, is not used. Thirdly the Chinese words tend to extend the description considerably, i.e. a more complicated word graph is expressed. Bao 1 han 2, literally means “around inside”, hence “containment”. This hints at the fact that also the English word “containment” expresses more than just a SUB-link in a BE-frame. This explains why the two columns for Chinese words given are so different apart from the words for “equal” and “equality”. It is simply so that not every knowledge graph has a precisely describing word. We might even have left more places open in the table for “lack of words”.

Now let us consider the subgraphs of the form $\square \longrightarrow$ (TYPE) and the corresponding words, given in Table IV, and those of the form (TYPE) $\longrightarrow \square$, given in Table V.

TYPE	WORD	CHINESE WORD	LITERALLY
EQU			
SUB	Of ...	... de yi 1 bu 4 fen	... of one part
ALI			
DIS			
ORD	From	Cong 2	from
CAU	By	You 2 ... ying 3 qi 3	have .... cause
PAR	Of	... de xing 4 zhi 4	... of attribute
SKO	Dependent (on)	Yi 3 lai 4	depend on

Table IV

TYPE	WORD	CHINESE WORD	LITERALLY
EQU	Equal (to)	Deng 3 yu 2	equal (to)
SUB	With	Ju 4 you 3 ...	have ...
ALI	Similar (to)	Xiang 1 si 4 ya 2	similar to
DIS	Distinct (from)	Bu 4 tong 2 yu 4	not equal to
ORD	To	Dao 4	to
CAU	With	Yu 4 guo 3 ... xing 4 zhi 4	have ... attribute
PAR			
SKO			

Table V

Note, as discussed in [2], that the mereological stand is that SUB, PAR and FPAR are the three mereological relationships, mixed up in the English language by the fact that the words "of" and "with" are used in all three cases, like in the examples:

Part A of B, respectively B with part A,
Attribute A of B, respectively B with attribute A,
Property A of B, respectively B with property A,

or, more concretely:

The tail of the dog,
The beauty of the dog,
The barking of the dog;

if barking is part of the definition of "dog". Some people define a dog as "something that barks and sniffs".

The graphs $\square \longrightarrow (\text{FPAR})$ and $(\text{FPAR}) \longrightarrow \square$ are worded "of" and "with" too, but in Chinese the two graphs are described by "... de te 4 zheng 1" respectively "yu 4 you 3 ... te 4 zheng". We see that in Chinese strict distinction is made for the three mereological relationships. Literally we have

.... de yi 1 bu 4 fen	:	.... of one part of
... de xing 4 zhi 4	:	... of attribute
... de te 4 zheng 1	:	... of property,

where in English only the word "of" is used, and

ju 4 you 3 ...	:	with have ...
ju 4 you 3 ... xing 4 zhi 4	:	with have ... attribute
ju 4 you 3 ... te 4 zheng 1	:	with have ... property,

where in English only the word "with" is used.
It is quite remarkable that no fourth mereological relationship is used in Chinese, which fact supports our choice of only three mereological relationships. This was not apparent from English.

The EQU, ALI and DIS-relationships are symmetric. This is probably the reason why in Table IV no words are given, which is due to the fact that they do not seem to be present in language.
If y is a function of x,
$x \xrightarrow{\text{ALI}} \square \longrightarrow (\text{SKO}) \longrightarrow \square \xleftarrow{\text{ALI}} y$, we say that y depends (is dependent) on x. Special attention should be paid to the SKO-loop that is considered, by van den Berg and Willems, to represent the words "all",sou 3 you 3, "each",mei 3 ge, "every", mei 3 ge, and "any", ren 4 yi 4. The authors consider the four words to be slightly different so that four different graphs should be presented, although the SKO-loop, indicating something that is informationally dependent only on itself and hence can be anything, clearly stands central. In Chinese three different words are used.
Also note that the CAU-relationship, with chosen word "by" is somewhat out of line with the other relationships that seem more basic as structuring relationships. You 2 ... ying 3 qi 3, literally means "by .... cause".
Of Kant's categories we can, with some difficulty, recognize the following six: Inherence, Negation, Possibility, Necessity, Limitation and Causality. "Existence" we consider as "being

in the world", so of less basic nature than "being", "Reality" in our subjectivistic theory is an assumption about correspondence between our image in the mind and a presupposed outer world. We do not discuss the quantity categories of Kant: unity, plurality and totality. Rather, we want to focus on commonness, that we take to be synonym with alikeness, and that puts the ALI-link central.

The ALI-link might be called a primus inter pares as the process of concept formation is seen to result from discovering similarity between a set of perceived objects. A word evokes different subgraphs in different mind graphs, although probably with great similarity. Using the word for a part of its complete meaning, for the speaker, allows to use the word metaphorically. The listener may yet understand the used word in the proper way by searching for other concepts that contain as part of their word graph the same part as envisaged by the speaker. "The big bird landed on Schiphol airfield" uses "bird" for "something that flies". The listener may see this part of speaker's word graph for "bird" as the intended focus of his word choice and may see it as part of listener's word graph for "plane". Speaker is considered by listener to have used "bird" for "plane". Related to the metaphor is the pars pro toto construction. Speaker uses the word for a part of a concept to indicate the whole concept.

Like for the logic words of the first kind, there is the problem to determine the border line of the logic words of the second kind. "Alike" is the word with word graph $\square \underset{\text{ALI}}{\overset{\text{ALI}}{\rightleftarrows}} \square$ and hence a logic word of the second kind by Definition 4, but what about "like"? "A and B are alike" is a rather clean statement, but in "he works like a horse" the ALI-link stands central but in the sentence graph the words "works" and/or "horse" have to be expanded in order to localize the parts that are similar. The simple expansion of "horse" to "working horse", one of the things a horse can do, already enables the localization, although the specific feature, on which the alikeness meant by the speaker is based, namely working hard, is not yet part of the expansions. The point is that next to the ALI-link more structure is needed in order to give a proper word graph for the word "like". This is similar to the situation for the word "in", discussed in [2]. The single SUB-arc gives the word graph for "in" in English, but for the fifteen different Chinese words for "in", more complex graphs had to be given, depending of the specific context. In both examples we would like to call "like" and the Chinese words for "in" logic words of the second kind, but that implies that more complex word graphs than those existing merely of one of the basic types of arcs or parts of them are to be called word graphs of logic words. That again brings forward the problem to determine a borderline. In tense logic the word "past" clearly involves an ORD-arc to the time of the speech act from the time of the act described by the verb. In "I say (at time t_0) John worked (at time t_1)" we have $t_1 < t_0$ on the time scale, or $t_1 \xrightarrow{\text{ORD}} t_0$, in short notation. We say that the verb "work" has the past tense. The essence in "past" is as much the ordering as the fact that we are talking about time. In "the lower floors of the building" the word "lower" involves an ordering too, but now concerning spatial coordinates. Both "past" and "lower" may be considered to be logic words of the second kind as the ORD-arc stands central. However, it is increasingly becoming more difficult to classify the word as a logic word when the graph becomes more complex. The linking of concepts by one of the eight basic types of arcs is predominant in prepositions, see [2], that form the glue for the many nouns, verbs and adwords, see [3]. So prepositions are logic words of the second kind according to Definition 4. It will also be clear that the classification formed by the authors by subjective coding and leading to the classes C_{11} to C_{55} mixes up logic words of the first kind and logic words of the second kind

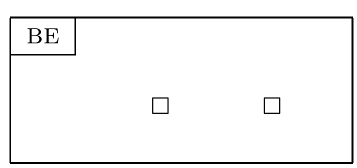
as distinguished in this section. Apparently in our minds no sharp borderline is present to distinguish logic words from other words. One might even take the stand that all words that are not noun, verb or adword are forming the material used to link these basic words. Here one should remark that with the noun its, named, instantiations are included. The number "2" is an instantiation of the noun "number". Likewise all plant names are instantiations of the noun "plant name". All plants in the extension of "plant", however indicated, would also belong to the set of basic words. An objection to this stand is that there are verbs like "be", "can" and "must" that are clearly pure logic words. But then, these verbs are, not without reason, called *auxiliary* verbs. In fact in Chinese "can" and "must" are not considered to be verbs. The verb "be", shi 4, is often missing in a sentence, no other verbs being present. The nouns indicating the categories in Kant's ontology, as far as we already interpreted them in the tables, would have to be excluded as well and this poses a more serious problem, as for example "possibility" was considered a typical logic word of the first kind. But then, it might be considered to be a concept that expresses something attributed by the mind, like "beauty".
Due to our view on word formation a classification should be based on the concept. Although representing a concept is affected by sociology, philosophy, psychology and so on, it is independent of the difference in languages such as Chinese and English. Whether we choose Chinese or English to express a concept, that is the same in the minds of a Chinese or an Englishman, the structure of the concept is the same. There may of course be language specific concepts. Before taking a more firm stand on the choice of the classification principle, we will try to give knowledge graphs for some potential logic words, as knowledge graphs are specifically designed for representing the structure of concepts.

5 Word graphs for logic words

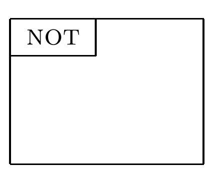
For our discussion about the classification problem for logic words it is useful to give a survey of the actual word graphs. For the process of structural parsing we need to have word graphs for all words anyhow.
We already considered as first set the prepositions, [2], then Chinese classifiers and adwords [3] next to the many nouns and verbs that, in first instance, have very simple word graphs, just describing the genus of the word. In the following listing we distinguish types of structures of logic words. Types of frames are indicated in the left upper corner.

5.1 Proposition operators

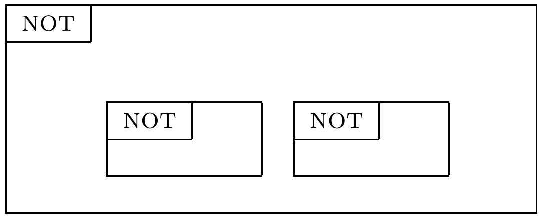
AND:



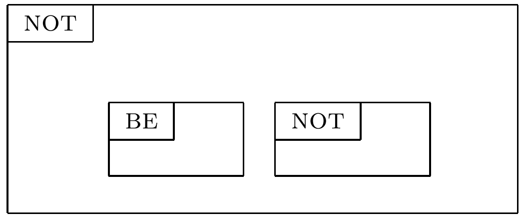
NOT:



OR:



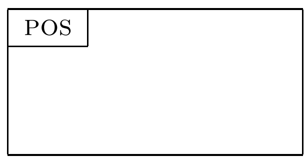
IF .... THEN ....:



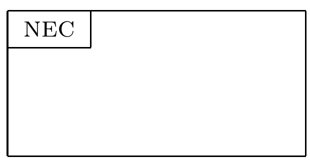
According to Definition 3 these are word graphs for logic words of the first kind.

5.2 Modal logic operators

POSSIBLE:

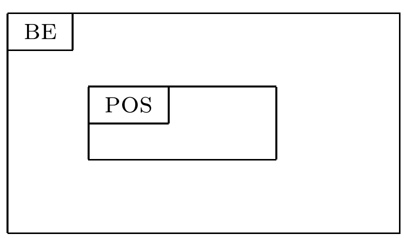


NECESSARY:

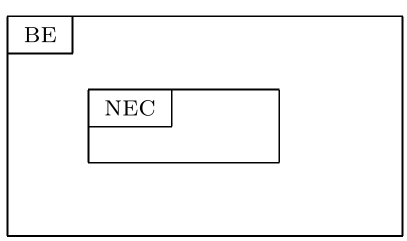


The nouns “possibility” and “necessity” have word graphs, corresponding to “being possible” and “being necessary”.

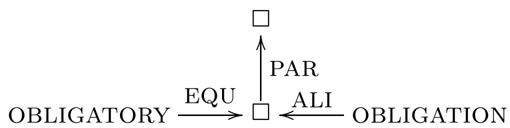
POSSIBILITY:



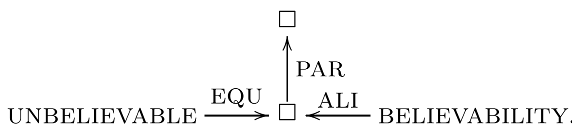
NECESSITY:



Within the frames, graphs, that describe propositions, can be present and we have a formalism, strictly parallel to "normal" notation for logic, see van den Berg [1]. If we would allow two other types of frames, according to "obligatory" and "believable", analogous graphs could be given for "obligation" and "believability". However, whether something is obligatory or believable is a subjective matter. Hence a PAR-arc, for attribution, seems more appropriate and we would have word graphs like



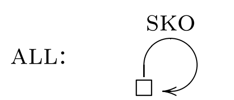
and



The adjectives are seen as instantiations of the nouns. If "possibility" and "necessity" are seen as judgments, analogous word graphs could be given, but we would clearly get away from the formalism of logic.

5.3 Quantification

Existential quantification is expressed by an explicit knowledge graph in which a variable is instantiated, i.e. a token has been valuated. Unvaluated tokens correspond to free variables. Note that there may be many graphs of the same structure only differing in the instantiation. All these graphs correspond to "there is an x such that........" by explicitly mentioning the "x". We follow van den Berg and Willems and represent universal qualification by a SKO-loop:

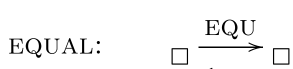


5.4 Logic words based on set comparison

As we distinguish eight types of binary relationships, we would have eight classes of logic words of the second kind to consider. However, as was discussed in [2], the types are to be divided into three groups, four based on set comparison, two based on the structure of space-time and two based on mind processes.

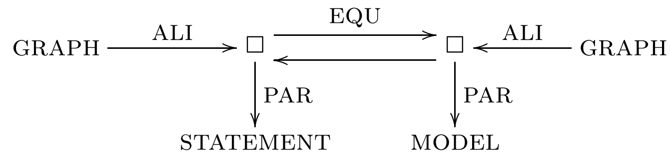
5.4.1 The equ-link

The basic word here is of course



With a BE-frame we obtain the word graph for "equality". There are quite a few words in which the word equal occurs, like e.g. equivalent. The only word for which we would like to give a word graph here is "true". The graph of the statement compared with the graph of the model in which the statement is interpreted should be *equal* to make the statement true. Hence

TRUE:



could be given as the word graph for “true”. Note the central position of the EQU-link. “Truth” is then again obtained by putting a BE-frame around the word graph for “true”.

5.4.2 The sub-link

$$\text{IN:} \qquad \square \xrightarrow{\text{SUB}} \square\,.$$

This preposition was extensively discussed in [2]. The SUB-arc is typically structuring entities. There are many words of which “sub” is a part. Should all these words be called logic words? Is “subset” mainly describing a set or is the word mainly expressing a “part of” relationship? In a general classification, according to Definition 2, the specific purpose of the classifier is decisive. When we want to gather all words showing structuring aspects we should include all “sub”- words in the set of logic words of the second kind and give predominance to the “part of” aspect.

5.4.3 The ali-link

The basic word is of course “alike”

$$\text{ALIKE:} \qquad \square \underset{\text{ALI}}{\overset{\text{ALI}}{\rightleftarrows}} \square.$$

We recall that this type of link may be considered the primus inter pares as it is considered to be at the basis of the framing and naming process. Having discovered the alikeness in the examples of a species the mind may form a prototype structure and frame and name this.
We mention the ALI-relationship as playing an important role in metaphors. It occurs in the word graphs of word like “similar”, which is almost a synonym of “alike”, or a word like “seem”.

5.4.4 The dis-link

The example of two sets with empty intersection, *dis*joint sets, is appropriate here.

$$\text{DISPARATE:} \qquad \square \underset{\text{DIS}}{\overset{\text{DIS}}{\rightleftarrows}} \square.$$

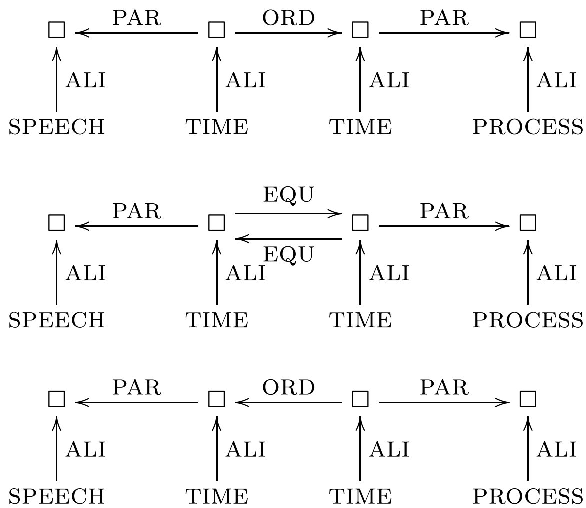
Like for “sub”, there are many words including “dis”. For the same reason, if we want to list all structuring words as logic words of the second kind, all these words are considered to be so, even if this means including verbs like “disrupt” or “discover”.

5.5 Logic words referring to space and time

For the reflection upon space and time we assume that two basic types of relationships suffice, the ORD-link and the CAU-link, although the CAU-link may be a composite according to the philosopher Hume, whom we are inclined to follow.

5.5.1 The ord-link

Here again we refer to the first paper on prepositions, in which the ORD-link is the main link. Words like "from", "to", "before", "after", "under", "behind", "above" etc. all have word graphs in which the ORD-link stands central. The words used in temporal logic mainly refer to some kind of ordering. Interesting examples are the tenses of a verb; present, past and future. Like we said before, two values of time are involved, the time of speaking and the time of the process described by the verb.



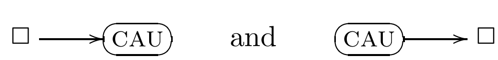
are describing that the process occurs after, during, respectively before the speech act. In a rather complicated way these graphs will be present when "future tense", "present tense", respectively "past tense" would have to be described, as "tense" refers to the form of the verb that describes the process. We do not give such graphs but rather mention that these three graphs without the token for process, so in smaller form, would be word graphs for "at a time after speech", "at the time of speech", respectively "at a time before speech", which is usually described by "later", "now" respectively "earlier". These words, and analogous triples like "tomorrow", "today" and "yesterday" are considered to be logic words of the second kind.
The ORD-link typically occurs in word graphs for words that are used in comparisons.

5.5.2 The cau-link

We do not have a good word for the graph

$$\square \xrightarrow{\text{CAU}} \square.$$

The word "causing" seems to come closest. With a BE-frame around it we might have the word graph for "causation". This again sheds some doubt on the CAU-link being basic, now in a different way. Subgraphs of the total graph like



however, may be seen as word graph, for “cause” and “effect”, “something that is causing” and “something that is caused”. Causal relationships are very important in building expert systems or decision support systems. First expert systems in medicine were rule-based systems, where the rules were formulated in the “if A then B” form instead of “A causes B”. For this reason and because we chose the CAU-link in our ontology, the words with word graphs in which the CAU-link stands central are considered to be logic words of the second kind as well.

5.6 Logic words due to mental processes

The last two types are the PAR-link for attribution and the SKO-link for informational dependency.

5.6.1 The par-link

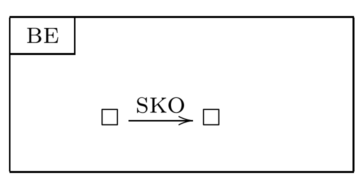
Next to the FPAR-link and the SUB-link, this is the third mereological relationship. All three occur in the word graphs of prepositions like “of” and “with”. The PAR-link, with BE-frame describing “attribution”, is typically used for adjectives and adverbs, see [3]. It is structuring thought like the other types of link do, but also has syntactical aspects, in that it links certain types of words, adwords, to other types of words like nouns and verbs. The CAU-link, that we use for describing the functioning of verbs, has such a syntactical aspect as well, in that it determines subject and object (if present). Although the ending “–ly” in adverbs refers to the attribution to verbs, and hence to the PAR-link, we are not inclined to say that the PAR-link stands central in adverbs and do not call these words logic words.

5.6.2 The sko-link

The SKO-loop was used for universal quantification. The typical example for the SKO-arc is mapping or function, as known from mathematics. If $y = f(x)$ indicates that y is a function of x, we say that y is informationally dependent on x, which is essentially described by

DEPENDENT ON: $\square \xrightarrow{\text{SKO}} \square$,

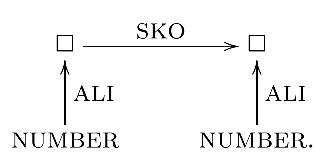
DEPENDENCY being described by:



.

In natural language words for which the SKO-link stands central in the word graph are rather rare. The concept “function”, as mapping from numbers to numbers, is used in mathematical language and has word graph:

FUNCTION:



Like in the case of "functional" and "mapping", for mapping of arbitrary objects to numbers respectively mapping of arbitrary objects to objects, the SKO-link stands central in the word graph for "function". Words like these are called logic words of the second kind as well.

5.7 Words used in other logics

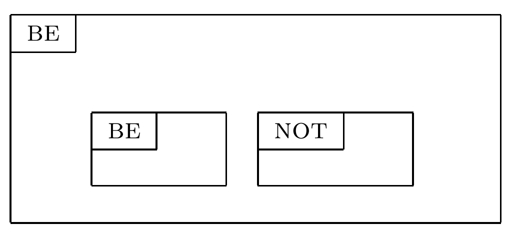
We have seen in Section 5.5 that words used in temporal logic would not be called logic words in our classification according to Definitions 3 and 4. However, the important role played by the ORD-link makes us say that many of these words fall in the category of logic words of the second kind. There are other logics like deontic logic or fuzzy logic, where according to our classification we would not speak of logic words as none of the basic types of n-ary or binary relationships plays an important role, although we have mentioned that for deontic logic an OBL-frame might be introduced. In that case words like obligatory would be logic words of the first kind.

We prefer not to do this, as ethical valuations are essentially subjective. Likewise, for words with a fuzziness aspect like "youth", "old" or "somewhat" we do not have the structuring aspect that other logic words have. A word that seems to stand central here is "extent", which is basically a measure for inclusion. The value of this measure may be called the fuzziness of the inclusion. With "inclusion" we may think of sets, but the word is used in a broader sense here. The quantitative aspect stands central. For this reason we would not speak of logic words in case of fuzziness.

5.8 Words linking sentences

The last group of logic words is special in that they link sentences. Examples are "however", "nevertheless" or "but". These words are frequently used in a reasoning process. Within the same sentence the word "although" links subsentences in the same way. Let p and q represent two sentences connected by the word "but". Then the meaning is considered to be "it is not so that p implies q", or $\neg(p \rightarrow q)$ in logical notation. As this is equivalent to $\neg(\neg(p \wedge \neg q))$, which is equivalent to $(p \wedge \neg q)$, the word graph would be

BUT:



and therefore we should classify such words as logic words of the first kind, although two types of frames are occurring.

6 Conclusion

It will be clear for which type of classification we have chosen; type c), classification by using knowledge graph ontology. In Section 5 we have tried to apply our Definitions 3 and 4 for classifying pure logic words and other logic words. The main problem that we encounter is to make a decision about the demand "stands central" with respect to a specific type of link. The problem is similar to the problem of delineating the meaning of a word. In knowledge graph theory the structure, of the graph, is the meaning, but what are the boundaries of the specific subgraph of the mind graph? The extreme stand was taken that the whole graph focussed upon, that contains the word graph, is the meaning. This means the word and all its associations, i.e. background knowledge. If the same stand would be taken with respect to logic words, their word graphs would not have natural boundaries as well.
The taken stand is rather off line with the mathematical logical tradition in which it is the goal to give a meaning that is as concise as possible. In knowledge graph theory interpretation, this means that the word graph given, should be the smallest graph still considered to give the meaning of the word, one would say the essential meaning.
For our classification problem we face the problem that we would like to call a word a logic word or not. Here some frame or some basic type of link may be embedded in a larger graph, that may still be the smallest graph giving the essential meaning of the word. The question is: does the word graph contain the link like any other type of link or is its structure so that the link grasps the essentials of the meaning. The smaller the graph, the easier it is to answer this question. We have made choices in Section 5, calling some words logic words and other words not. This unsatisfactory feature is accepted because for the process of structural parsing we have in mind the distinction between logic words and other words becomes irrelevant. We have indicated a third set of words, next to those considered in [2] and [3] and that is what really matters.
The words and linguistic elements remaining, the fourth set, will have to contain graphs for linguistic elements like question marks, exclamation signs, to name a few. A paper in preparation by Hoede and Liu is focussing on these elements of language, that play an important role in the description of a dialogue, for which knowledge graphs are particularly well suited. Words in the fourth set, the rest group so to say, will be discussed in an other paper in preparation by Hoede and Zhang.

References

[1] Berg, H. van den, *Knowledge Graphs and Logic: One of Two Kinds*, Dissertation, University of Twente, The Netherlands, ISBN 90–9006360–9 (1993).

[2] Hoede, C. and X. Li, Word Graphs: The First Set, in *Conceptual Structures: Knowledge Representation as Interlingua*, Auxiliary Proceedings of the Fourth International Conference on Conceptual Structures, Bondi Beach, Sydney, Australia (P.W. Eklund, G. Ellis and, G. Mann, eds.), ICCS'96 (1996) 81–93.

[3] Hoede, C. and X. Liu, Word Graphs: The Second Set, in *Conceptual Structures: Theory, Tools and Applications*, Proceedings of the 6th International Conference on Conceptual Structures, Montpellier, ICCS'98 (M,-L. Mugnier, M. Chein, eds.) Springer Lecture Notes in Artificial Intelligence 1453, (1998) 375–389.

[4] Hofland, K. and S. Johansson, *Word Frequencies in British and American English*, The Norwegian Computing Centre for the Humanities, Bergen, Norway, (1982) 43–53.

[5] Peirce, C.S., On the algebra of logic, *American Journal of Mathematics*, vol. 7, (1885) 180–202.

[6] Sowa, J.F., *Knowledge Representation: Logical, Philosophical, and Computational Foundations*, Draft of a book scheduled to the published by the PWS Publishing Company, Boston Massachusetts, (1994) 99–100.