On a number theoretic property of optimal maintenance grouping

A. van Harten, G.C. van Dijkhuizen & J.G.M. Mars

WP-9[illegible]

BETA-publicatie	:	WF [illegible]
ISSN	:	1386-9213;
NUGI	:	684
Enschede	:	May 1999
Keywords	:	Maintenance grouping, number theory
Submitted to	:	European Journal of Operations Research

ON A NUMBER THEORETIC PROPERTY OF OPTIMAL MAINTENANCE GROUPING

A. van Harten
G.C. van Dijkhuizen
University of Twente, Dept. of Technology & Management,
P.O. Box 217, 7500 AE Enschede
J.G.M. Mars
University of Utrecht, Dept. of Mathematics,
P.O. Box 80010, 3508 TA Utrecht

Abstract

In this paper we consider the problem of preventive maintenance of a failure prone system, for which a number of maintenance actions has to be executed on a regular basis. For each action i the frequency is prescribed. Between consecutive actions of type i there is an integer interspacing of T(i) time units. The set-up costs are activity dependent. The set-up structure is supposed to be tree-like and additive over the set-up nodes involved in the action or group of actions. Hence, for different activities with common setup nodes joint execution leads to set-up costs reduction. The question is how the actions should be arranged in time in order to exploit this set-up costs reduction effect maximally. It is shown that the time averaged set-up costs are **minimal** if a main peak clustering property is satisfied: **all maintenance actions are combined at one moment in time**. Intuitively, this property is appealing, but it asks for some interesting and non-trivial applications of number theory and inductive reasoning, to prove it.

Contents

1. INTRODUCTION

For a production system the objective of maintenance is to ensure that the equipment necessary for production retains in or is restored to a state in which it can perform adequately. For maintenance management it is logical to maximize the availability of the production system, while keeping overall maintenance efforts at an acceptable level. For a survey of the sort of decision problems inherent to maintenance management we refer to Gits, 1987; Nakajima, 1988; Pintelon, Gelders and van Puyvelde, 1997; van Dijkhuizen, 1998. From a managerial point of view preventive maintenance actions contribute to reduction of possible breakdowns or other system failures such as degraded output quality. A wealth of maintenance optimisation models exists to support the choice of a preventive strategy with age replacement and block replacement models as some of the bestknown examples, cf. Gertsbakh, 1977. In such models frequencies for preventive maintenance actions are determined considering costs of prevention against failure costs.

Here we assume that in some way a choice for the frequency of each preventive maintenance action has been made. As a generalisation of block replacement this is implemented by performing each maintenance action i with a constant interspacing of T(i) time units, i.e. at t= a(i)+n.T(i) with n integer. As a consequence we assume that the time averaged costs due to direct (i.e operational, non set-up related) costs of the preventive maintenance actions and due to corrective maintenance in case of failures, are known and independent of the choice of the a(i)'s. Note however, that even in this situation where T(i) is prescribed, the a(i)'s still remain as control variables. Their choice provides flexibility for combining maintenance actions at certain moments in time and this leads to the question how this can be done best in view of set-up costs reduction. Here the set-up structure comes into the picture.

Maintenance activities usually require one or more preparatory set-up activities. This can be due to dismantling, delivery of maintenance equipment or -crew. Now in several cases there is a perspective of significant gains, if maintenance actions are carried out simultaneously, so that set-up costs can at least partly be shared. In this respect the following grouping possibilities can be distinguished: static grouping, dynamic grouping and opportunistic grouping, cf. Gertsbakh, 1977;Wildeman, 1996; Wildeman, Dekker and Smit 1997; Dekker and Smeitink, 1994; van Dijkhuizen, van Harten, 1997a,b. In our context here of prescribed frequencies for maintenance actions the policy is static, but dynamically maintenance actions are carried out in different clusters. The question is how to choose the control variables a(i) in order to minimize the time averaged set-up costs in the long run. In this problem the time horizon is set as infinitely long. Or equivalently, as we suppose the T(i)'s are integer, we can restict ourselves to an interval in discrete time t=0, ...,$\boldsymbol{L}$-1 with $\boldsymbol{L}$=lcm{T(i)}, where lcm represents the least common multiple of the numbers within the parentheses. Of course the answer to this question depends on the set-up structure. Here we consider the situation of a multi-component system with a multiple set-up structure. We assume that the set of set-up activities can be ordered hierarchically into a tree-like structure, where each node corresponds with a different set-up activity. For node j the estimated costs of the corresponding set-up activity are denoted as C(j). A maintenance activity is associated with precisely one of the nodes and in order to carry it out, all set-up activities corresponding with that specific node or one of its predecessors have to be carried out. A predecessor can be interpreted as a step earlier in the dismantling process constituting the set-up. This set-up structure was introduced in van Dijkhuizen, van Harten, 1997a, b and it extends Gertsbakh, 1972. This framework gives rise to a rich scala of realistic modelling of set-up structures with possibilities for partly shared set-ups. For an example we refer to Sculli and Suraweera, 1979.

Now the result of this paper can be stated in a quite simple way:

Irrespective of the number of maintenance actions I, the structure of the set-up tree and the values of the T(i)'s and the C(j)'s, an optimal solution is obtained by choosing:

a(i)=0 for i=1, ... ,I

It is clear that this choice corresponds with a main peak at t=0, since all activities are carried out then. Of course, there might be alternative choices of the a(i)'s leading to the same optimal time averaged costs.. As far as we know this statement "a(i)=0 is optimal" is not mentioned explicitly in the literature. In their work van Dijkhuizen, van Harten, 1997a restrict themselves a-priori to a class of candidate optimizers for frequency-constrained clustering with such a property. The result which is proven here shows that optimality of the MILP solution in that paper holds in a wider sense. Besides that the result is interesting in its own respect and in the sequel we shall try to convince the reader that the proof has some intrinsic beauty.

The main line of the proof is that first in section 3 the general case of a set-up tree is reduced to the common set-up case and next the common set-up case is dealt with in number theoretic terms in section 4. In section 2 some notation is introduced and an example is given. In section 5 a sufficient condition for minima with some of the a(i)'s≠0, is discussed and it is shown that this generalisation can easily be related to the main peak clustering property discussed before by applying a shift in time. Moreover, it is shown that also situations with alternative optima which violate the main peak clustering property, can occur.

2. MODEL FORMULATION AND DESCRIPTION OF THE OPTIMISATION PROBLEM

To start with we give an example to illustrate the set-up structure and to show the effect of different choices of a(i)'s. Consider a case with 6 maintenance actions and a set-up structure with 5 nodes. In figure 1 maintenance actions are depicted with squares labeled by their identification number. The set-up nodes are depicted as circles labeled with their identification number. The relation of a node with a predecessor is indicated with a continuous line, the association of a maintenance action with its set-up node with a dotted line. The values of the prescribed frequencies of the maintenance actions and the estimated costs of set-up actions are given in table 1.

T(1)=2	T(2)=4	T(3)=6	T(4)=4	T(5)=6	T(6)=5
C(1)=1	C(2)=3	C(3)=1	C(4)=2	C(5)=2	

Table 1: values of T(i) for i=1, ... ,6 and C(j) for j=1, ... ,5 in the example.

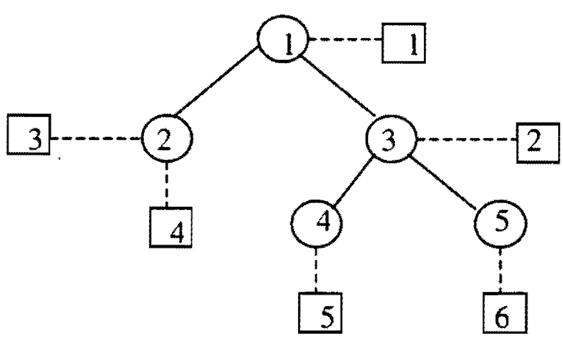

Figure 1: an example of a set-up tree with associated maintenance actions.

It is clear that in this example we obtain a cycle time L=lcm(3,4,5,6)=60. In figure 2 the set-up combinations of two different choices for the a(i)'s are compared.

```
Costs A:   741444144540441742147510461442444514441472144710444442164510
Action 6   6____6____6____6____6____6____6____6____6____6____6____6____
Action 5   _5_____5_____5_____5_____5_____5_____5_____5_____5_____5____
Action 4   4___4___4___4___4___4___4___4___4___4___4___4___4___4___4___
Action 3   ___3_____3_____3_____3_____3_____3_____3_____3_____3_____3__
Action 2   _2___2___2___2___2___2___2___2___2___2___2___2___2___2___2__
Action 1   1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_

Time →     012345678901234567890123456789012345678901234567890123456789

Action 1   1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_1_
Action 2   2___2___2___2___2___2___2___2___2___2___2___2___2___2___2___
Action 3   3_____3_____3_____3_____3_____3_____3_____3_____3_____3_____
Action 4   4___4___4___4___4___4___4___4___4___4___4___4___4___4___4___
Action 5   5_____5_____5_____5_____5_____5_____5_____5_____5_____5_____
Action 6   6____6____6____6____6____6____6____6____6____6____6____6____
Costs B:   901054705040701450707010741050905014701070705410704050745010
```

Figure 2: comparison of different choices of the a(i)'s, in case A: a(1)=0, a(2)=1, a(3)=3, a(4)=0, a(5)=1, a(6)=0; in case B: a(i)=0 for i= 1, ... ,6

The costs for carrying out the set-up for a group of maintenance actions at a certain moment in time is found in a straight forward way. It is the sum of the costs of each of the set-up activities necessary for that group of maintenance actions. For example, in case B at time 20 the group consists of {1,2,4,6} and the necessary set-up activities are (1,2,3,5} with costs 1+3+1+2=7.

Note that in case B the peak in the costs is higher than in case A, but the time averaged costs are lower, namely 168/60 versus 203/60 per unit of time. Hence a considerable reduction of the set-up costs (over 20% in this case) is possible by exploiting the freedom in the a(i)'s.

However, one can imagine that nevertheless one could in practice not immediately discard case A, since lower peak values might be an intrinsic other management criterium to strive for, say because of work balancing of the maintenance staff. This aspect will not be explored further in this paper, but it gives rise to many additional interesting research questions .

Let us now first introduce some notation useful in the treatment of the general problem.

First we introduce **a** and **T** as a notation for the transposed vectors (a(1), ... ,a(I)) and (T(1), ... ,T(I)). We denote **S** to indicate the tree structure of the set-up activities and the maintenance actions. *Note that* without loss of generality we can assume that **S** has the following property: for each set-up activity node there is at least one maintenance action directly associated with it. The reason is that if this were not the case then consider a set-up node k without direct maintenance actions. This node can either be ignored, if it is a leaf of the tree or else it can be incorporated in its children by augmenting for each child

the set-up costs with C(k). As a consequence the number S of set-up nodes of **S** is smaller than I. Therefore **S** is nothing else then a tree in the sense of graphs for which each leaf corresponds with a maintenance action and each other node corresponds with a set-up activity. **S** consists minimally of of the root node and one leaf.
Hence in this sense the example in figure 1 is representative.
Let us refer to the triple **a;S,T** as a maintenance policy.
Now we define:

F(**a;S,T**)= time averaged costs of the maintenance policy **a;S,T**

$$= \sum_{t=0,\dots,L-1} \sum_{j \in G(t)} C(j)/L$$

Here in the range of the summation L is a shorthand notation for lcm(**T**) and $G(t)$ is the set of set-up activities corresponding with the group of maintenance actions $M(t)$ to be executed at time t. Note that $M(t)= \{ i \mid t \bmod (T(i))=a(i) \}$ and $G(t)= \{ j \mid$ for some action i in $M(t)$ node j in **S** is a predecessor of action i $\}$. What we want to demonstrate is that

$$F(\mathbf{0};\mathbf{S},\mathbf{T}) \leq F(\mathbf{a};\mathbf{S},\mathbf{T})$$

where **0** denotes the null vector.

3. THE MAIN PEAK CLUSTERING PROPERTY AND ITS NUMBER THEORETIC BACKGROUND

In the special case of a so-called common set-up the structure of F can be immediately related to a counting exercise in number theory. For a common set-up there is only one set-up activity node with set-up costs C and all maintenance actions require that setup-activity. Let us denote this set-up structure as **1**. Then it can immediately be derived that

$$F(\mathbf{a};\mathbf{1},\mathbf{T})=Cf(\mathbf{a};\mathbf{T})/L$$

The interpretation of f is as follows:

$$f(\mathbf{a};\mathbf{T})=\#\{0\le t<L \mid \text{for some } i=1,\ldots,I \text{ it holds true that } t \bmod T(i)=a(i)\}$$

Here # denotes the cardinality of the set to which it is applied.
The statement

$$f(\mathbf{0};\mathbf{T})\le f(\mathbf{a};\mathbf{T})$$

expresses the fact that the density of the numbers $t=a(i)+nT(i)$ as a subset of the natural numbers is minimal for **a=0.** This number theoretic result will be proved in the next section. Let us now first show that the result for a general set-up tree follows from this number theoretic result.

Proof (reduction of the general case for S to the case of a common set-up)

This part of the proof follows by induction with respect to the number of set-up activity nodes S. The clue for the induction step where given the validity of the result for any **S*** with S*<S set-up nodes the validity for **S** with S set-up nodes is shown, is the following recursion formula:

$$F(\mathbf{a};\mathbf{S},\mathbf{T})=C(1)f(\mathbf{a};\mathbf{T})/L+\sum F(\mathbf{a}^*;\mathbf{S}^*,\mathbf{T}^*)$$

The summation runs over the sub-trees of **S** which arise by omitting root node of the set-up tree and the maintenance actions directly connected with that node.
For example, in the situation of figure 1 there are two sub-trees **S***. The first one consists of set-up activitiy 2, which is now the root of this sub-tree and connected with it the maintenance actions 3 and 4. The second one consists of the set-up node 3 as a root node and the nodes 4 and 5 as its children and, as before, the maintenance actions 5 and 6 connected with them. These sub-trees are depicted in the following figure.

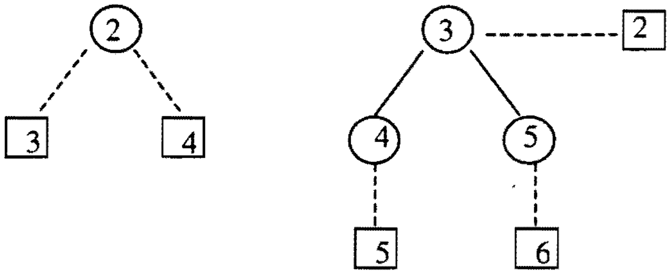

Figure 3: sub-trees in the induction step for the set-up tree in figure 1

The definition of **a***, **T*** for a sub-tree **S*** follow immediately from the reduced set of maintenance actions corresponding with it. Note that either **S** is a tree with a common set-up and S=1 and there are no sub-trees or a sub-tree has strictly less set-up nodes than the original tree. Given that the result also holds for the common set-up case as found in the first term of the recursion formula (see the next section) the induction step can now easily be completed using a sandwich procedure:

$$F(\mathbf{0};\mathbf{S},\mathbf{T}) \geq \min F(\mathbf{a};\mathbf{S},\mathbf{T}) = \min[\, C(1)f(\mathbf{a};\mathbf{T})/\boldsymbol{L} + \sum F(\mathbf{a}^*;\mathbf{S}^*,\mathbf{T}^*)\,] \geq$$

$$\min[\, C(1)f(\mathbf{a};\mathbf{T})\,] + \sum \min[\, F(\mathbf{a}^*;\mathbf{S}^*,\mathbf{T}^*)\,] = C(1)f(\mathbf{0};\mathbf{T})/\boldsymbol{L} + \sum F(\mathbf{0}^*;\mathbf{S}^*,\mathbf{T}^*) = F(\mathbf{0};\mathbf{S},\mathbf{T})$$

Due to the enclosure as found we have indeed equality of min $F(\mathbf{a};\mathbf{S},\mathbf{T})$ and $F(\mathbf{0};\mathbf{S},\mathbf{T})$.
Let us now have a closer look at the common set-up case.

4. ON THE UNDERLYING NUMBER THEORY.

Let us now have a closer look at the before mentioned property f(**0**;**T**)=min f(**a**;**T**). Somewhat surprisingly, we could not find an accessable reference for this property. In principle this property can be checked by just counting numbers, but some ingenuity is needed to give a convincing general proof. Below we give some of the details.

Proof (number theoretic details for the common set-up case)

It is somewhat more convenient to work with g(**a**;**T**) =*L*-f(**a**;**T**). Note that in contrast with the definition of f and the number density interpretation of f/*L*, the function g is related to counting gaps, since

$$g(\mathbf{a};\mathbf{T}) = \#\{0 \le t < L \mid t \bmod T(i) \ne a(i) \text{ for all } i=1, \ldots, I\}$$

As a matter of fact we shall demostrate the following more general property:

$$g(\mathbf{b};\mathbf{T}) = \max g(\mathbf{a};\mathbf{T}) \quad \text{if the following compatibility condition holds: } b(i)=b(j) \bmod \gcd(T(i),T(j)) \text{ for all } i,j$$

Here gcd refers to the greatest common divisor. Of course **b=0** is an example where the compatibility condition on the b(i)'s is satisfied. Of course the result for g does immediately imply that under the given compatibility condition on **b** we have f(**b**;**T**)=minf(**a**;**T**). In order to demonstrate this more general property of g we shall use induction with respect to *L*.

First there is a trivial situation where T(i)=*L* for all i=1, ... ,I. If *L*>1 then it is clear that in order to get a maximum b(i) mod*L* should have the same value for each i. If *L*=1 then **b=0,** g=0 is the only possibility anyway.

Hence from now on we can restrict ourselves to the situation I>1 and T(i)<*L* for some i. Then, after renumbering the elements of **T** we can suppose that there exists a splitting of the following sort: **T=(T*,T**)** where *L**=lcm(**T***)<*L* and for each element p of **T**** we have *L*=lcm(**T***, p). By **a*** we denote those elements of the transposed vector of a(i)'s corresponding with **T*.** Now define **T'** as the transposed vector of the same length as **T,** but with elements T'(i)=gcd(T(i), *L**). Note that lcm(**T'**)=*L**<*L*. As a shorthand notation we introduce ***M=L/L****. Now the following inequality is crucial:

$$g(\mathbf{a};\mathbf{T}) \le M g(\mathbf{a};\mathbf{T}') + (M-1) g(\mathbf{a}^*;\mathbf{T}^*) \qquad \text{(lemma1)}$$

In this inequality equality holds for transposed vectors **b** satisfying the compatibility condition given hereabove.

The proof of this lemma will be discussed later on. Let us first show that this inequality produces the induction step using a sandwich procedure as before in section 3.

$$g(\mathbf{b};\mathbf{T}) \le \max g(\mathbf{a};\mathbf{T}) \le \max[\, M g(\mathbf{a};\mathbf{T}') + (M-1) g(\mathbf{a}^*;\mathbf{T}^*) \,] \le$$

$$\max[\, M g(\mathbf{a};\mathbf{T}') \,] + \max[\, (M-1) g(\mathbf{a}^*;\mathbf{T}^*) \,] \le M g(\mathbf{b};\mathbf{T}') + (M-1) g(\mathbf{b}^*;\mathbf{T}^*) = g(\mathbf{b};\mathbf{T})$$

Note that in the last step of this sandwich procedure we really use the equality part of lemma 1. So, it remains to prove lemma 1.

Let t be an element of the set $\boldsymbol{G}=\{0 \leq t<\boldsymbol{L} \mid t \bmod T(i) \neq a(i) \text{ for all } i=1, \ldots, I\}$ of which g counts the cardinality. Then in a unique way we can represent $t=k+y.\boldsymbol{L}^*$ with $0 \leq y<\boldsymbol{M}$ and $0 \leq k<\boldsymbol{L}^*$. Now there are two cases of k-values that we can distinguish:

1. For all elements of **T*** we have kmod $T(i) \neq a(i)$ and for all elements of **T**** we have the stronger property k mod $\gcd(T(i),\boldsymbol{L}^*) \neq a(i)$ mod $\gcd(T(I),\boldsymbol{L}^*)$. In this case $k+y.\boldsymbol{L}^*$ is in $\boldsymbol{G}$ for all values of y. This gives rise to the first term in the inequality without over estimation.
2. For all elements of **T*** we have k mod $T(i) \neq a(i)$ but for at least one element of **T**** we have a violation of the stronger property, say for $i=i^{**}$. Then $k+y.\boldsymbol{L}^*=a(i)+n.T(i^{**})$ for some values of y and n due to some elementary number theory, cf. Anderson and Bell, 1997. Hence in $k+y.\boldsymbol{L}^*$ at most $\boldsymbol{M}$-1 of the y-values give rise to an element of $\boldsymbol{G}$. The second term in the inequality takes care of this factor, but it introduces some potential overestimation by ignoring that more than one y-value can drop out.

Herewith the inequality has been derived. However we still have to check that the stronger result with an equality holds true in case of a transposed vector **b** for which the compatibility condition is satisfied. In order to exclude overestimation for the second term of the inequality arising from the second class of k-values introduced above we only have to prevent that for two different elements T(i) and T(j) of **T**** we have two different solutions of the equations

$$k+y.\boldsymbol{L}^*=b(i)+n.T(i) \qquad \text{and} \qquad k+z.\boldsymbol{L}^*=b(j)+m.T(j)$$

This means solutions with the same k-value and $y \neq z \bmod \boldsymbol{M}$. If we assume $b(i)=b(j) \bmod \gcd(T(i),T(j))$ then $b(i)+rT(i)=b(j)+l.T(j)$ and the conclusion would be that

$$(y-z)\boldsymbol{L}^*= (n+r)T(i)-(m+l)T(j)$$

Solutions can only exist if $(y-z)\boldsymbol{L}^* \bmod \gcd(T(i),T(j)) = 0$, cf. Anderson and Bell, 1997. However by construction of **T*** and **T**** this can only be the case if $(y-z)=0 \bmod \boldsymbol{M}$.

This completes the proof of the lemma and consequently the proof of the main peak clustering property in the general setting.

5. CONCLUDING REMARKS

We conclude with the remark that also in the setting of a general set-up tree a **minimum** of the time averaged costs is found **not only for a=0, but also for any** other transposed vector **b satisfying the compatibility condition**. However, as a matter of fact cases with such a **b** are equivalent to the case **a=0** concerning the main peak clustering property. The only difference is that instead of t=0 the main peak occurs at a different moment in time. In Anderson and Bell, 1997 this result is given as an exercise to generalize the wellknown Chinese remainder theorem (p.156, exc.6). For completeness sake we include here a few lines to sketch the idea of the proof. The result can easily be shown using induction with respect to I. Suppose that the property holds for I-1 maintenance actions with

$$\tau=b(1)+n(1)T(1)= \ldots\ldots = b(I-1)+n(I-1)T(I-1)$$

let us then demonstrate that the compatibility condition implies that the property also holds for I maintenance actions. To do this we make a shift in time to $t'=t-\tau$. By doing so we can work with **b'**=(0, ... ,0,b'(I)) where b'(I)modT(i)=0 for i=1, ... ,I-1. Hence

$$b'(I)=m(1)T(1)= \ldots\ldots = m(I-1)T(I-1)$$

Note that all maintenance actions upto I-1 are carried out simultaneously at t'-values which are multiples of lcm(T(1), ... ,T(I-1)). Maintenance action I is carried out at

$$t'(n)=b'(I)+n.T(I)$$

By choosing n=lcm(T(1)/gcd(T(I),T(1)), ... ,T(I-1)/gcd(T(I),T(I-1))) it can easily be seen that t'(n) is an integer multiple of each of the T(i)'s for i=1, ... ,I-1 and consequently it is also a multiple of lcm(T(1), ... ,T(I-1)). Indeed all maintenance actions coincide at a this moment in time and this completes the induction step.

The conclusion is that **b**'s satisfying the compatibility condition describe the same main peak clustering property as **a=0**.

However, the compatibility condition is not a necessary condition for optimality. An example is given by a situation with one common set-up and three maintenance actions with T(1)=1, T(2)=2, T(3)=4 and b(1)=b(3)=0, b(2)=1.

6. REFERENCES

Anderson, J.A., Bell, M.B. (1997) *Number theory with applications.* Prentice Hall, Upper Saddle River, New Jersey, USA

Dekker, R. and E. Smeitink (1994). Preventive maintenance at opportunities of restricted duration. *Naval Research Logistics 41*, 335-353.

Gertsbakh, I. (1972). Optimum choice of preventive maintenance times for a hierarchical system. *Automated Control Computer Sciences 6*, 24-30.

Gertsbakh, I. (1977). *Models of Preventive Maintenance.* Oxford: North-Holland.

Gits, C. (1987). On the maintenance concept for a technical system III: design framework. *Maintenance Management International 6*, 223-237.

Nakajima, S. (1988). *Introduction to TPM: Total Productive Maintenance*, Portland: Productivity Press.

Pintelon, L., L. Gelders, and F. van Puyvelde (1997). *Maintenance Management*, Leuven: Acco.

Sculli, D. and A. Suraweera (1979). Tramcar maintenance. *Journal of the Operational Research Society 30*, 809-814.

Van Dijkhuizen, G.C. (1998). *Maintenance meets Production.* Thesis, University of Twente, The Netherlands.

Van Dijkhuizen, G. and van Harten, A. (1997a). Coordinated planning of preventive maintenance in multi-setup multi-component systems. Technical report, University of Twente, The Netherlands,. To be published in *Management Science.*

Van Dikhuizen, G. and van Harten, A. (1997b). Optimal clustering of frequency-constrained maintenance jobs with shared set-ups. *European Journal of Operations Research 99*, 552-564.

Wildeman, R. (1996). *The art of grouping maintenance.* Thesis, Erasmus University Rotterdam, The Netherlands.

Wildeman, R., Frenk, J. and Dekker, R. (1997) A dynamic policy for grouping maintenance activities. *European Journal of Operations Research 99*, 530-551.