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1. Introduction

The situation we consider is an airline company running a continental hinterland hub. The main function of such a hub is to facilitate connections between relatively short haul feeder flights to hinterland destinations with long haul intercontinental flights. In order to increase the connection potential of flights the schedule design consists of a number of waves at the hub. Each wave clusters incoming and outgoing flights to and from the hub in some time interval. Given a wave centre T, IC (=intercontinental)arrivals and C (=hinterland) arrivals are concentrated in ($T-T_i+\frac{1}{2}T_c$, $T-M_i+\frac{1}{2}M_c$) and ($T-\frac{1}{2}T_c$, $T-\frac{1}{2}M_c$), respectively, i.e. in intervals at the left of T. Here T_i, T_c denote the maximal connection times for intercontinental and continental flights and M_i, M_c the minimal connection times. Analogously IC – and C-departures are concentrated in ($T+M_i-\frac{1}{2}M_c$, $T+T_i-\frac{1}{2}T_c$) and ($T+\frac{1}{2}M_c$, $T+\frac{1}{2}T_c$).
In practice a wave-system structure for the hub can be determined from the desired feeder destinations, their frequencies and their round trip times. Here this structure is taken for granted and we consider the following question (IFSP):
how can the intercontinentalflights be scheduled optimally within a given (new) wave-system structurefor a given intercontinentalnetwork and flight-structure?

The wave-system structure specifies a set of arrival and departure windows, in which the intercontinental flights have to be scheduled in view of connections between the intercontinental feeder flights. In addition we introduce the concept of commercial scheduling windows or windows of opportunity for intercontinental flights. Such a scheduling window describes the time-window at the hub in which an intercontinental flight has to be scheduled to ensure the flights to arrive at and depart from the intercontinental spokes at both commercially and operationally attractive times of the day.

In character IFSP is a multi-objective problem with multiple constraints. Altogether, IFSP can be summerised as: determine the optimal timings for intercontinental departures and arrivals at the hub, such that:

- Total intercontinental seat production is as much as possible equally distributed over the various connection waves;
- Flights are scheduled as much as possible to the centre of the waves;
- Connections between intercontinental flights to and from priority regions are maximised;
- Ground-time at the intercontinental spokes is minimised,

while taking into account the constraints, that:

- There is a minimum ground-time at the turn-around station as necessary for the type of aircraft assigned to the flight;
- The total inflow of aircraft is greater than or equal to the total outflow of aircraft per wave per subtype;
- Each flight is scheduled within its predefined commercial scheduling window;
- Each flight is scheduled within the arrival and departure windows as specified in the wave system structure.

In this paper we shall discuss how IFSP can be approached applying simulated annealing techniques to a relaxed version of the problem where the hard constraints are replaced by penalisations in the objective. The multi-criteria character of the problem is reduced to a single objective by assigning weights factors to various components. By varying the weight sets the quality of the solution can be improved interactively with a decision team. This research was applied to the redesign of wave system structure of the Dutch airline company KLM with its hub at Amsterdam and Europe as hinterland. Results of this application will be shown.

2. Model formulation and relaxation

Formulation of the constrained model
Let us start by introducing I as the set of all flights departing from the hub to some destination at least one day per week and K as the set of all flights arriving at the hub. As mentioned, we like to optimize schedules for both departing and arriving flights at the hub. For flight i departing at time t from the hub to a certain destination we introduce the concept of a return interval at the hub as t + [δ(i),Δ(i)]. Here δ(i) denotes the minimal time required for the intercontinental round-trip, and
$\Delta(i)\geq\delta(i)$ denotes an upperbound. By making use of the flexibility in turn-around times at the turn-around stations, a departing flight from the hub can usually be scheduled independently from the returning flight to the hub. A *minimum* turn-around time should be guaranteed as part of

$\delta(i)$. Since we minimize ground-time at the intercontinental spokes anyway, one can put $\Delta(i)$ at infinity in such flexible cases. For a specific set of flights, so called circle-flights, we allow no flexibility to schedule the departing flight independently from the returning flight, i.e. $\delta(i)=\Delta(i)$. C is the set of circle-flights, where C is a subset of I.

One of our objectives is to maximize connections between flights to and from priority regions. Hence, we introduce:

I_1 = subset of I, departing flights to priority region 1;
I_2 = subset of I, departing flights to priority region 2;
K_1 = subset of K, arriving flights from priority region 1;
K_2 = subset of K, arriving flights from priority region 2;
W = connection quality function, assigning a quality value to a connection between two flights from the priority regions based on the connection time between the two flights;

Our overall objective is to find optimal departure and arrival times for all flights. Flights can be scheduled to depart or arrive at the hub at every quarter of an hour of the day:
J = set of all possible arrival and departure times at the hub.

Principle decision variables indicate whether a departing flight i is scheduled at time j (X_{ij}) and whether arriving flight k is scheduled to arrive at time l (Y_{kl}). Both decision variables are binary. One of the constraints of IFSP is to schedule flights such that they fit within their commercial scheduling windows SW_i and SW_k, but also the fit into the arrival and departure windows of the connection waves is an objective. Connection waves are indicated by variables a and b ($a,b=1...N$), where N indicates the total number of waves. Both scheduling windows and arrival and departure windows of connection waves are subsets of J:

A_b = feasible arrival times in wave b;
D_b = feasible departure times from wave b;
SW_i = feasible departure times from the hub for departing flight i;
SW_k = feasible arrival times at the hub for arriving flight k;
M_b = time of the day indicating the centre of wave b ($M_b \epsilon J$);
H = connection quality function, determining the quality of the timing of an intercontinental flight, in view of the resulting connection times to European feeder flights; the value of H decreases as the flights are scheduled further away from the centre of the connection waves (see 2.1);

Each flight is operated by one or more types of aircraft. The following variables are associated to the various types of aircraft and the operating frequency of flight/aircraft-type combinations:

E_i = set of types of aircraft for departing flight i;
I_e = set of departing flights operated by aircraft type e;
E_k = set of types of aircraft for arriving flight k;
K_e = set of arriving flights operated by aircraft type e;
$f(e,i)$ = weekly operating frequency of departing flight i with aircraft type e;
$f(e,k)$ = weekly operating frequency of arriving flight k with aircraft type e, note that $f(e,i)=f(e,i+|I|)$.
m_e = minimum turn-around time for aircraft type e;
s_e = number of seats of aircraft type e;

S_i = total weekly number of seats produced by departing flight i;
S_k = total weekly number of seats produced by arriving flight k.
$V_{b,e}$ = aircraft balance for aircraft type e in connection wave b;

In order to determine the turn-around time at turn-around stations, we need to calculate arrival and departure times at turn-around stations, given scheduled departure and arrival times at the hub. The out-station arrival time of flight i (oa_i) is determined as the sum of the departure time from the hub and the flying time of flight i (ft_i). Similarly, the departure time from the out-station of returning flight k (od_k) is determined as the difference of the scheduled arrival time at the hub and the flying time of the returning flight k (ft_k). Let us now introduce a model describing the situation. Our problem has four objectives and three major constraints.

Maximize:

$$c_1\cdot\{\sum_{b=1}^{N}(\sum_{j\in D_b}\sum_{l\in A_b}(\sum_{k\in K_1}\sum_{i\in I_2}Y_{kl}\cdot X_{ij}\cdot W(lY_{kl}-jX_{ij}) + \sum_{k\in K_2}\sum_{i\in I_1}Y_{kl}\cdot X_{ij}\cdot W(lY_{kl}-jX_{ij})))\} \quad (1)$$

$$-c_2\cdot G\{\sum_{a=1}^{N-1}\sum_{b=a+1}^{N}((\sum_{i\in I}(\sum_{j\in D_a}X_{ij}\cdot S_i - \sum_{j\in D_b}X_{ij}\cdot S_i))^2 + (\sum_{k\in K}(\sum_{l\in A_a}Y_{kl}\cdot S_k - \sum_{l\in A_b}Y_{kl}\cdot S_k))^2)\} \quad (2)$$

$$-c_3\cdot G_6\{\sum_{i\in I\setminus C}\mathrm{mod}((od_{k=i+|I|}-oa_i),96)\} \quad (3)$$

$$+c_4\cdot\{\sum_{b=1}^{N}((\sum_{i\in I}\sum_{j\in D_b}H(j\cdot X_{ij}-M_b)) + (\sum_{k\in K}\sum_{l\in A_b}H(M_b-l\cdot Y_{kl}))\} \quad (4)$$

while:

$$\sum_{j\in SW_i} X_{ij} = 1 \ , \ \forall \ i\in I \tag{5}$$

$$\sum_{l\in SW_k} Y_{kl} = 1 \ , \ \forall \ k\in K \tag{6}$$

$$Y_{k=i+|I|,\ l=oa_i} - X_{ij} = 0 \ , \ \forall \ i\in C \tag{7}$$

$$\begin{aligned} V_{1,e} &= V_{N,e} + \sum_{k\in K_e}\sum_{l\in A_1} Y_{kl}{\cdot}f(e,k) \\ &- \sum_{i\in I_e}\sum_{j\in D_1} X_{ij}{\cdot}f(e,i) \ , \quad \forall \ e\in E \\ V_{b,e} &= V_{b-1,e} + \sum_{k\in K_e}\sum_{l\in A_b} Y_{kl}{\cdot}g(e,k) \\ &- \sum_{i\in I_e}\sum_{j\in D_b} X_{ij}{\cdot}f(e,i) \ , \ \forall \ e\in E, \ b>1 \end{aligned} \tag{8}$$

$$V_{1,e}\geq 0 \ ,\ldots, \ V_{N,e}\geq 0 \ , \ \forall \ e\in E \tag{9}$$

$$S_i = \sum_{e\in E_i} f(e,i)\cdot s_e \ , \ S_k = \sum_{e\in E_k} f(e,k)\cdot s_e \ , \quad \forall \ i\in I \ , \ \forall \ k\in K \tag{10}$$

$$\mathrm{mod}(od_{k=i+|I|} - oa_i, \ 96) \geq \underset{e\in E_i}{MAX} \ m_e \ , \quad \forall \ i\in I\setminus C \tag{11}$$

$$\begin{aligned} oa_i &= \mathrm{mod}(\sum_j X_{ij}{\cdot}j + ft_i, \ 96), \ \forall \ i\in I \\ od_k &= \mathrm{mod}(\sum_l Y_{kl}{\cdot}l - ft_k, \ 96), \ \forall \ k\in K \end{aligned} \tag{12}$$

$$X_{ij} = \begin{cases} 1, & \textit{if flight i departs at time j} \\ 0, & \textit{otherwise} \end{cases} \ , \quad \forall \ i\in I \ , \ \forall \ j\in SW_i \tag{13}$$

$$Y_{kl} = \begin{cases} 1, & \textit{if flight k arrives at time l} \\ 0, & \textit{otherwise} \end{cases} \ , \quad \forall \ k\in K, \ \forall \ l\in SW_k \tag{14}$$

Part (1) of the objective describes the goal of maximizing connections between flights to and from the identified priority regions. For the structure of W we refer to (23) Part (2) describes the objective of equally distributed intercontinental seat production over the various connection waves. The constraints (9) describe how this weekly seat production can be calculated. The function G_2 describes a shape-function.
At this point, we mention that the formulation of objective part (2) depends on the wave-system structure chosen. Sometimes, an obvious adaption is necessary, cf. [Bootsma, 1997].
The partial objective (3) describes the goal of minimizing total ground-time at the turn-around stations. The constraints (11) describe how the out-station arrival and departure times are related to scheduled arrival and departure times at the hub and the flights' flying times. G_6 will be chosen as in (22). The *minimum* turn-around time constraints should be met (cf. 10). For each flight the minimum turn-around time is based on the *maximum* required turn-around time amongst the various types of aircraft by which the flight is operated.

Flights can be scheduled at only one time of the day and must be scheduled within the commercial windows of opportunity (constraints 5 and 6). In case multiple daily flights are scheduled to the same intercontinental destination, then *beforehand* windows of opportunity for each of these flights are determined. The partial objective (4) describes this goal of minimizing the total time between both scheduled departure times and the wave-centres (first component of (4)) and wave-centres and scheduled arrival times (second component of (4)). The constraints (7) describe that for circle-flights there is no flexibility to optimize departure and arrival times of intercontinental flights independently from each other. Once the departure time has been scheduled, the arrival time is given. The constraints (8.1) and (8.2) describe the inflow-outflow constraints for each wave-aircraft type combination. Finally, constraints (12) and (13) show both principle decision variables X_{ij} and Y_{kl} to be binary.

Characteristics of the application

Our problem instance was provided by KLM and involves the rescheduling of the intercontinental flights according to a new wave-system structure. Until recently KLM operated a 3-wave system structure at Amsterdam Airport Schiphol. The problem instance we used for testing our solution algorithm for IFSP is focused at rescheduling the intercontinental flights according to a 5-wave system

structure. We used KLM's intercontinental schedule for summer 1997. This flight-set contains 209 flights: 92 round-trips and 25 circle-flights. The problem instance contains 26 flights to priority region 1 and 31 flights to priority region 2.
Considering an average size for scheduling windows of 6 hours, the problem reduces to ±5.000 decision making variables.The total number of constraints within this instance of IFSP equals 1.014. Given the size and structure of the application, we have to pursue a heuristic approach to solving the initial formulation of this problem.

Relaxation of IFSP

In this subsection we will discuss a relaxation of IFSP and the structure of the reward- and penalty functions used to assign an overall score to a schedule. We will first give a formulation of the total relaxed problem and then consecutively discuss the elements of the new objective function.

Maximize

$$SCORE = \lambda_1 \cdot BALANCE + \lambda_2 \cdot DISTR + \lambda_3 \cdot POSITION + \lambda_4 \cdot TURN + \lambda_5 \cdot CONNECT \tag{15}$$

where:

$$BALANCE = \sum_{e \in E, 1 \leq b \leq N} G_1\{V_{b,e}\} \tag{16}$$

$$POSITION = \sum_{b=1}^{N} \Big(\big(\sum_{i \in I} G_4\{H(\sum_{j \in D_b} j \cdot X_{ij} - M_b),\ SW_i\}\big) + \big(\sum_{k \in K} G_5\{H(M_b - \sum_{l \in A_b} l \cdot Y_{kl}),\ SW_k\}\big)\Big) \tag{17}$$

Furthermore the terms come from the original multi-objective: DISTR = (2), TURN = (3), CONNECT = (1). In addition the non-relaxed constraints mentioned in (5) - (13) remain valid. Note that the relaxation enhances connectivity of the search space in our simulated annealing algorithm later on. The constants c_i mentioned previously in (i)-(iv) are in corporated in the λ_is from now on.

Let us now describe the structure of valuation functions as used in this relaxed formulation of IFSP. For valuation function $G_1(x)$ we take:

$$G_1(x) = \begin{cases} -M_1, & for\ x<0 \\ p_1, & for\ x \geq 0 \end{cases} \tag{18}$$

with $M_1 \gg 0$ and $p_1>0$. Hence, the schedule will get a large penalty for each violation of an aircraft balancing constraint, while a small reward of p_1 is given for each aircraft balancing constraint which is satisfied.

$G_2(x)$ rewards the schedule for minimization of the sum of quadratic differences in arriving and departing seat production levels for each pair of connection waves.

$$G_2(x) = M_2 - \frac{x}{M_3} \tag{19}$$

Here is a scaling factor, as the sum of quadratic differences for arriving and departing seat production levels for all pairs of connection waves may result in a very large number (see section 3.4). M_2 is used in order to get a high score for well balanced schedules, i.e. $M_2 \gg 0$.

Equation (16) is a combination of objective (4) and a relaxation of constraints (5) and (6) from the constrained formulation. For all departing flights ($i \varepsilon I$) holds:

$$G_4 = \begin{cases} H(\sum_{j \in D_b} j \cdot X_{ij} - M_b), & if \sum_{j \in D_b} j \cdot X_{ij} > 0 \wedge \sum_{j \in D_b} j \cdot X_{ij} \in SW_i \\ -p_3, & if \sum_{j \in D_b} j \cdot X_{ij} > 0 \wedge \sum_{j \in D_b} j \cdot X_{ij} \notin SW_i \\ 0, & if \sum_{j \in D_b} j \cdot X_{ij} = 0 \end{cases} \tag{20}$$

Similarly, for all arriving flights $k \varepsilon K$:

$$G_5 = \begin{cases} H(M_b - \sum_{l \in A_b} l \cdot Y_{kl}), & if \sum_{l \in A_b} l \cdot Y_{kl} > 0 \wedge \sum_{l \in A_b} l \cdot Y_{kl} \in SW_k \\ -p_3, & if \sum_{l \in A_b} l \cdot Y_{kl} > 0 \wedge \sum_{l \in A_b} l \cdot Y_{kl} \notin SW_k \\ 0, & if \sum_{l \in A_b} l \cdot Y_{kl} = 0 \end{cases} \tag{21}$$

Hence, a penalty of p_3 is used in case flights are scheduled inside the window of the connection wave considered, but outside their individual scheduling window. G_4 and G_5 equal 0, in case the flight considered is not scheduled in the window of the connection wave considered. In all other cases, $H(x)$ assigns a positive score to the schedule.

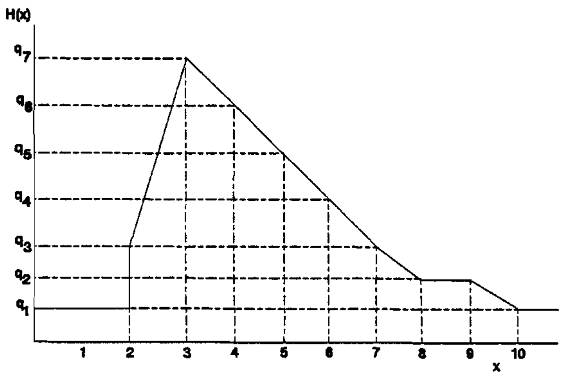

Figure 1: Structure of positioning valuation function H(x)

Next $G_6(x)$ assigns a score to each turn-around. $G_6(x)$ is structured as follows (22):

$$G_6(x) = \begin{cases} -M_4 \ , \ if\ x < -1 \\ -M_5 \ , \ if\ x < 0 \\ -x \ , \ if\ x \geq 0 \end{cases} \qquad (22)$$

Violations of the minimum turn-around constraints get high penalties (M_4 and M_5), while schedules using the flexibility in lengthening turn-around times at intercontinental spokes for optimizing arrival and departure times at the hub get a score of $-x$, where x is the difference between the actual turn-around time and the minimum turn-around time required.

Finally, $W(x)$ values the quality of a connection as a linear function of the connection time x. For $W(x)$ holds:

$$W(x) = \begin{cases} p_4(1-(x-4)/8) \ for\ 4 \leq x \leq 12 \\ 0 \ , \ otherwise \end{cases} \qquad (23)$$

Hence, connections between intercontinental flights to and from priority regions get a maximum score of p_4 in case the connection time is exactly one hour. The score gradually decreases to 0, as the connection time increases to a maximum of 3 hours. Connection times smaller than the minimum connection time of 1 hour or larger than the maximum connection time of 3 hours are not rewarded.

3. Design of an annealing algorithm

Simulated annealing

Simulated annealing [Aarts, 1988], [Van Laarhoven, 1989], [Romeijn and Smith, 1993] is a heuristic, which exploits the advantages of general local search techniques, but tries to eliminate the major disadvantage of deterministic local search.

Given a starting solution, a simulated annealing procedure starts by choosing a next solution from the so called *neighbourhood-structure.* The annealing procedure stochastically chooses a next solution from the neighbourhood-structure, by random or semi-random lottery with a certain chance (p_a). This chance depends on the difference between the score of the current solution and the next solution considered and the current value of T. Many different formulations can be chosen for this acceptance chance. Eglese and Rand [1987] and Dowsland [1990] propose for example p_a=*exp(difference/temperature)*. During the annealing procedure the temperature is gradually decreased and by consequence the acceptance chance of worse solutions decreases. The manner in which the temperature decreases is determined by the so called *cooling schedule*. Examples of a cooling schedule are: $T_{i+1}=\alpha \cdot T_i$ or $T_{i+1}=T_i/(1+b \cdot T_i)$ where the value of b may or may not depend on the current value of T_i. Another control variable within a simulated annealing algorithm is the number of iterations (K) per temperature-step. The algorithm stops at a certain value of T, or when no new solutions have been accepted during a relatively large number of iterations.

A simulated annealing algorithm for the relaxed version of IFSP

In IFSP a solution is defined by the timings of all intercontinental arriving and departing flights. Initially, the neighbourhood of any solution is defined as *all potential solutions, resulting from an adjustment of the arrival time at the hub or the departure time from the hub of one single flight by one quarter of an hour in forward or backward direction.* Now, a simulated annealing algorithm for IFSP can be formulated as indicated in the box below:
The highest score found is stored in memory, in case the algorithm does not find the highest score at the end of the simulated annealing procedure [Wright, 1989].

In a standard annealing algorithm all choices with respect to (i) which flight to reschedule (ii) rescheduling direction and (iii) rescheduling step-size, will be made randomly. In variations of the standard annealing algorithm, one or more of these choices will be made more cleverly. In [Bootsma, 1997] several options with respect to more clever choosing of the flight to be rescheduled, the rescheduling direction and the rescheduling step-size are discussed.

Pseudocode for simulated annealing algorithm for IFSP:

```
{Read existing intercontinental schedule}
{Read scheduling windows for each of the
intercontinental flights}
{Determine score of current schedule}
{Determine T0}

WHILE T>0 (or T>Ts, where Ts is stop-
temperature)
   FOR k=1 TO K
         {Choose a flight to reschedule}
         {Choose rescheduling direction}
```

In simulated annealing literature, the importance of a good starting solution for a problem is often emphasized. Besides the annealing algorithm we developed a "pre-process" program, in order to improve the score of the starting solution, before the simulated annealing procedure was started.

4. Results of annealing experiments

Let us start by giving the values we used for the parameters of functions G_1,..,G_6, H and W.

Parameter values used for G_1,..G_6, H and W	
G_1	M_1=100, p_1=10
G_2	M_2=4.000, M_3=1·10^4
G_4	p_3=0
G_5	p_3=0
G_6	M_4=100, M_5=50
H	q_1=10, q_2=11, q_3=12, q_4=13, q_5=15, q_6=17, q_7=18
W	p_4=9

Table 3: Parameter values used for G_1,..G_6, H and W

In the experiments described in this subsection, we used the following initial values for the weighting coefficients in the objective function: [λ_1,..,λ_5] = [20,5,10,15,1]. We used the KLM's current (3-wave) system schedule as a starting solution for our rescheduling algorithm.

It can be concluded this starting solution is not very good. It suffers from very negative scores for the distribution of flights over the various connection waves and violations of minimum turn-around constraints.

SCORE	Weighting Coeff.	Initial Score	Weighted score
BALANCE	20	420	8.400
DISTR	5	-3.090	-15.450
POSITION	10	2.812	28.120
TURN	15	-1.782	-26.730
CONNECT	1	1.058	1.058

Table 4: Total score for starting solution

According to our simulated annealing experience the initial temperature, T_0 should be such that the initial chance of accepting a worse solution than the starting solution equals approximately 0,95, while the end-temperature T_s should be such that the final chance of accepting a worse solution than the last solution found equals 0,05. For the acceptance chance we used p_a=exp(difference/temperature). Initial experiments indicated that the average difference between two consecutive solutions was approximately 50 points. Hence, T_0=-50/ln(0,95)≈1.000 while T_s=-50/ln(0,05)≈15. Given these indications for values of T_0 and T_s, we experimented with values for T_0 of 1.000, 800 and 600. Initially, T_s was set at 10 and K at 100, while variations of the starting temperature were combined with variations of cooling parameter α. Table 2 shows the results of these experiments.

However, given the poor initial solution we have indication that the algorithm was captured in a local minimum before reaching a much better optimum. This was mended by changing the initial solution.

T_0	T_s	K	α	BA-LANCE	DISTR	POSITION	TURN	CONNECT	SCORE
1.000	10	100	0,6	420	-665	3061	-400	1165	30855
			0,7	420	-535	3054	-398	1155	31450
			0,8	420	-402	3037	-399	1148	31923
			0,9	420	73	3020	-452	1164	33349
			0,95	420	1210	3007	-451	1237	38992
			0,98	420	2218	2893	-433	1208	43163
			0,99	**420**	**2525**	**2774**	**-323**	**1221**	**45141**
			0,999	420	-355	2994	-498	1121	30316
800	10	100	0,9	420	73	3020	452	1164	33349
			0,95	420	1791	2934	475	1132	40702
			0,98	420	1694	2899	-422	1181	40411
			0,99	**420**	**2291**	**2842**	**-382**	**1197**	**43712**
			0,999	420	1868	2611	-914	1172	31312
600	10	100	0,9	420	-542	3086	-408	1213	31638
			0,95	420	447	2951	-432	1298	34963
			0,98	420	1591	2901	-469	1276	39631
			0,99	**420**	**2124**	**2811**	**-381**	**1213**	**42583**
			0,999	420	3215	1799	533	1851	36321

Table 5: Some annealing results for various values of T_0 and α

	$\lambda_1..\lambda_5$	S_0	$S_{0,w}$	S	S_w	S_{max}	$S_{max,w}$	S_{ref}	$S_{ref,w}$
BALANCE	20	420	8400	420	8400	420	8400	420	8400
DISTR	5	-1664	-8320	3498	17490	3498	17490	3000	15000
POSITION	10	3087	30870	3095	30950	3225	32250	3510	35100
TURN	15	-349	-5235	-262	-3930	-61	-915	-276	-4140
CONNECT	1	1210	1210	1616	1616	2371	2.371	2.000	2000
SCORE			26925		**54526**		59596		56360

Table 6: Total (weighted) scores for the improved starting solution (S_0,$S_{0,w}$); total (weighted) scores for the best solution found by the simulated annealing algorithm (S,S_w); total weighted observed maximum scores (S_{max},$S_{max,w}$); total weighted reference scores (S_{ref},$Sref_{,w}$)

A further significant improvement came from an improved starting solution. A separate program was written in order to generate better initial turn-around and commercial performance.

Based on the results shown in table 6, we can conclude that the simulated annealing algorithm designed in this chapter works rather well. The score of the improved starting solution has been more than doubled (+103%).

The best solution found by the algorithm for the objective function is close to the maximum observed values for each of the individual elements separately. The best solution found by the algorithm is 92% of this assesed theoretical maximum.

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