Defect grating modes as superimposed grating states

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For a symmetric grating structure with a defect, we show that a fully transmitted defect mode in the band gap can be obtained as a superposition of two steady states: an amplified and an attenuated defect state. Without scanning the whole band gap by transmission calculations, this simplifies the direct calculation of the defect wavelength as the eigenvalue in a non-standard eigenvalue problem.

Introduction

Grating structures are often used in integrated optical devices and in sensor applications. A defect grating structure as will be considered here has defect modes with full transmission at a specific (defect) wavelength in the bandgap of the gratings. See e.g. [2] for bandgap in gratings, and [3, 4] for gratings as one-dimensional versions of photonic bandgap materials or photonic crystals; a Fabry-Perot cavity as described e.g. in Refs. [2, 5, 6, 7] can be viewed as a result of a 'defect' in a Bragg grating. We have calculated transmittance in linear and nonlinear defect grating structures by means of finite-element techniques in detail in [8, 9]. The narrow bandedness and the sensitive dependence of the defect wavelength on the material or geometric properties of the defect region is exploited in devices with functionality as optical filters or sensors.

The defect modes are found in transmission experiments at discrete defect wavelength(s) that appear as resonances inside the BG (bandgap). Instead of finding the defect wavelengths and properties of the defect mode in this way, it is desirable to have a direct characterization that makes the scanning of the full BG unnecessary. Besides these defect *modes* there will exist defect *states*: resonant, non-propagating states (which correspond to standing wave patterns in the dynamic electric field). In this paper we will consider a special grating structure: a symmetric structure placed in air consisting of two finite gratings that enclose a linear defect structure. All material properties are assumed to be linear, loss-less, and non-dispersive. We restrict to TE-polarization since TM can be treated analogously. We will start by characterizing the optical properties of the gratings inside the BG by their effect on the boundary. This makes it easy to find the boundary conditions for the pairs of defect states which can be found by looking at the defect region only. Afterward it is shown that a suitable superposition of the defect states leads to the defect modes. Amongst others, this will give a direct characterization of the defect wavelength. In another paper we present these results in a more general setting, including nonlinear defects and varying index changes [1].

In the following we use the notation $u(z)$ for the spatial dependent part of the harmonic electric field; with ω the frequency, the governing equation reads

$$\partial_z^2 u + \omega^2 n^2(z) u = 0$$

where $\omega = 2\pi c/\lambda$ with λ the vacuum wavelength and where we have introduced a normalization such that $c = 1$.

Characterization of optical grating properties

In this section we study the gratings that constitute important building blocks of the considered structures. We consider gratings with two layers, and the same optical path length in each layer

$n_1\ell_1 = n_2\ell_2$; at the 'design' frequency ω_0 such that $\omega_0 n_1 \ell_1 = \pi/2$ this is a quarter wavelength stack.

We restrict the description to frequencies inside the first BG. As a consequence of Bloch's (Floquet) theorem for periodic structures, any solution is then a linear combination of two independent real solutions $G^\pm$ that are of the form

$$G^\pm(\omega;z) = w^\pm(z)e^{\pm\rho z}, \quad w(z+p) = -w(z), \quad \rho > 0.$$

Since the quotient $\partial_z G/G$ is p-periodic, this value of this quotient is the same at each period-facet. Hence we can define real valued numbers associated with these solutions by

$$\kappa^\pm(\omega) := \frac{\partial_z G^\pm}{G^\pm} \text{ evaluated at (any) period-facet.}$$

Denoting by $g(\omega) := \exp(\rho(\omega)p)$ the 'gain-factor' per period (Floquet multiplier), we have $G^+(z+p) = -gG^+(z)$ and $G^-(z+p) = -G^-(z)/g$. The three quantities $g, \kappa^\pm$ can be used to characterize the effect of the gratings surrounding the defect region through efficient boundary conditions at the interfaces in transfer matrix techniques. Here we will only consider one special case that is particularly illuminating and can be understood without algebraic manipulations. For more general cases we refer to [1].

Using transfer matrix technique the the basic quantities can be calculated; see plots below for the case that $n_2 = 2n_1$ for which at the design frequency the maximal gain factor is $g = 2$ and $\kappa^+ = 0, \kappa^- = \infty$, corresponding to $\partial_z G^+ = 0, G^+ = 0$, and $\partial_z G^- = 1, G^- = 0$ at the period facet.
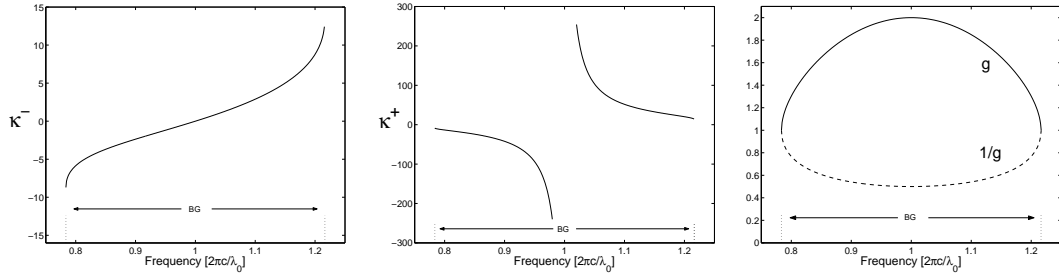
Defect states

In an infinitely long grating, both solutions $G^\pm$ considered above are unbounded. When the grating is finite, and a defect region is introduced, the defect region makes it possible to connect a solution in the left grating to one in the right grating for certain resonant frequencies that depend on properties of the gratings and of the defect region. Here we will consider two such states: an amplified and a attenuated state in a symmetric structure.

The grating structure is placed in air, index n_0 for $|z| > M$, the defect region for $|z| < L$ has index n_d and the gratings enclosing the defect region in $L < |z| < M$ consists of N layers, say. For illustration, the defect region is taken to be a high index region, but with twice the width as the high index regions in the gratings. At the design frequency this means that the defect layer supports a half-period wave. Then there are a skew-symmetric and a symmetric mode, connecting, respectively, a growing/decaying grating solution in the left grating to the decaying/growing grating solution in the right grating. These solutions, the amplified state S^+ and the attenuated defect state S^- respectively, are illustrated in the plots below. Note, in particular that both states exist at the same (design) frequency.

It should be observed that both defect states are states of zero powerflow, the dynamic behaviour is a standing wave pattern. At the exterior regions, the solution consists of a pure real harmonic,

indicating influxed waves that are reflected with the same strength, as symbolically indicated in the next plot below.

In this and in more general cases, these states can be found from a formulation as a eigenvalue problem on the defect region, say $[-L, L]$, only:

$$\partial_z^2 u + \omega^2 n_d^2 u = 0, \quad \partial_z u = \mp \kappa(\omega) u \text{ at } z = \pm L$$

Note that the EVP is nonstandard since the eigenvalue ω to be looked for, the resonant frequency, is also present in the boundary condition.

Defect modes

The states considered above are non-travelling, but 'steady, standing waves'. In that sense the defect states cannot be 'produced' by light influxed from one side only. We will now consider this so-called transmission problem which is often studied by numerical methods. Defect modes can exist for certain resonant frequencies in the BG and are solutions for which the influxed wave is completed transmitted so that in both exterior intervals there is only a right travelling wave.
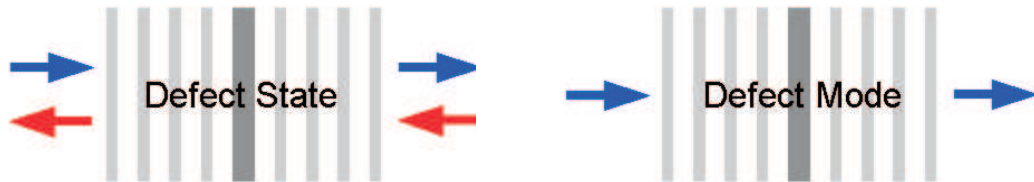
Such defect modes may look very similar to defect states, which is remarkable because the behaviour in the influx and outflux regions are completely different, and the mode has nonzero powerflow; see the plots below of the amplified defect state at the left, and the defect mode at the right.
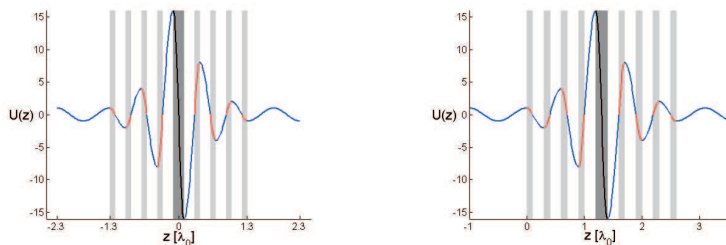

In fact, the defect mode arises as a superposition of the defect states. Since both S^+ and S^- has a standing wave at the influx region, but with different phase, a suitable complex combination will produce a pure influx wave from the left, and the defect mode M is obtained for a suitable linear superposition $\alpha, \beta \in \mathbb{C}$

$$M := \alpha S^+ + \beta S^-.$$

In the example given, $\alpha = 1$ and $\beta = i$; the small contribution from S^- in the defect region explains why the defect mode and the amplified defect state look so much alike.

The given argument can be extended in many ways; for instance in the plot below defect modes are shown to exist in the band gap (the dark region) for various values of the defect index; more generalizations are described in [1].
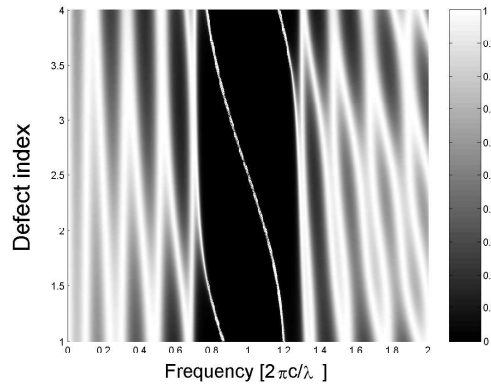
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