Dispersion of (Light) Inertial Particles in Stratified Turbulence

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Abstract We present a brief overview of a numerical study of the dispersion of particles in stably stratified turbulence. Three types of particles are examined: fluid particles, light inertial particles ($\rho_p/\rho_f = \mathcal{O}(1)$) and heavy inertial particles ($\rho_p/\rho_f \gg 1$). Stratification suppresses the vertical dispersion of all three types of particles compared to isotropic turbulence. The horizontal dispersion, on the other hand, is enhanced. The importance of the forces that act on inertial particles are examined and the results are shown for particles with $\rho_p/\rho_f = 10$. The inertia of the particles results in a nonuniform particle distribution over the domain. This preferential concentration effect is presented here for heavy particles, and it is found that the effect is stronger in isotropic turbulence than in stratified turbulence.

1 Introduction

The turbulent dispersion of inertial particles plays an important role in stratified oceanic and atmospheric flows, such as estuaries, coastal areas or the nocturnal atmospheric boundary layer. The final applications that we have in mind are oceanographic environments, where the density of the particles (plankton, algae, sand) is of the same order as that of the surrounding fluid. In stably stratified turbulent flows a negative vertical density gradient is present, which suppresses vertical fluid motions. The flow typically displays large horizontal vortical structures and thin, sheared layers in vertical direction [6]. Previous dispersion studies in these stratified flows mainly focused on fluid particles, see for example, Refs. [2, 7, 8, 10].

2 Numerical approach

We study a model system by means of direct numerical simulations (DNS): the dispersion of particles in statistically stationary homogeneous stably stratified turbulent flows. The flow field is solved using the Eulerian approach. The

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Navier–Stokes equations with Boussinesq approximation are solved on a triple-periodic domain [2, 13]. A linear stable background density stratification is imposed which is kept constant throughout a simulation. Density fluctuations are present on top of the linear profile, and the total density is given by $\rho_f = \rho_0+\overline{\rho}(z)+\rho'(x,y,z,t)$ (ρ_0 a reference value, $\overline{\rho}$ the background profile, ρ' density fluctuations). From the density gradient the buoyancy frequency $N^2 = -\frac{g}{\rho_0}\frac{\partial \overline{\rho}}{\partial z}$ can be computed, with g the gravitational acceleration. Large-scale forcing is applied in the horizontal direction to obtain a statistically stationary state. A description of the resulting flows can be found in Refs. [1, 2].

The Lagrangian approach is applied to study the particle trajectories. Three types of particles are tracked in the flows: fluid particles, light particles ($\rho_p/\rho_f = \mathcal{O}(1)$) and heavy particles ($\rho_p \gg \rho_f$), with ρ_p the particle density. Particle trajectories are obtained from $\frac{d\mathbf{x}_p}{dt} = \mathbf{u}_p$, with $\mathbf{x}_p$ the particle position and $\mathbf{u}_p$ its velocity. For fluid particles their velocities are derived from cubic spline interpolation of the velocity field at the particle position [2]. The velocities of the inertial particles (light and heavy) are obtained by solving the Maxey–Riley equation [9]:

$$\begin{aligned} m_p\frac{d\mathbf{u}_p}{dt} &= 6\pi a\mu\left(\mathbf{u}-\mathbf{u}_\mathrm{p}+\frac{1}{6}a^2\nabla^2\mathbf{u}\right)+m_f\frac{D\mathbf{u}}{Dt} \\ &+(m_p-m_f)\,\mathbf{g}+\frac{1}{2}m_f\left(\frac{D\mathbf{u}}{Dt}-\frac{d\mathbf{u}_p}{dt}+\frac{1}{10}a^2\frac{d}{dt}\nabla^2\mathbf{u}\right) \\ &+6\pi a^2\mu\int_0^t d\tau\frac{d\mathbf{u}/d\tau-d\mathbf{u}_p/d\tau+\frac{1}{6}a^2 d\nabla^2\mathbf{u}/d\tau}{[\pi\nu(t-\tau)]^{1/2}}. \end{aligned} \tag{1}$$

The particle mass is given by m_p, a is the radius of the particle and m_f is the mass of a fluid element with a volume equal to that of the particle. The fluid velocity is denoted by $\mathbf{u}$, ν is the kinematic viscosity and $\mu = \nu\rho_f$ is the dynamic viscosity. The forces on the right-hand side of this equation are viscous drag, a local pressure gradient in the undisturbed fluid, gravitational forces, added mass and the Basset history force, successively. For the added mass term the form described by Auton et al. [4] is used.

Equation (1) reduces to $\frac{d\mathbf{u}_p}{dt} = \frac{1}{\tau_p}(\mathbf{u}-\mathbf{u}_p)+\mathbf{g}$ in the limit of small heavy particles. For the results presented in this paper gravity is set to zero ($\mathbf{g}=0$). A measure of the particle inertia is the particle response time $\tau_p = \frac{d_p^2\rho_p/\rho_f}{18\nu}$, with $d_p = 2a$ the particle diameter. The particle inertia will be expressed in the following using the Stokes number $St = \tau_p/\tau_K$ (τ_K the Kolmogorov time). Fluid particles can be seen as particles in the limit of $\rho_p = \rho_f$ and $St \to 0$. The particles are released when the flow has reached a stationary state and velocity and position time series of $\mathcal{O}(10^4)-\mathcal{O}(10^6)$ particles are collected for about 40 eddy turnover times.

It has been tested how many previous data points are needed for the Basset force to reproduce this force accurately. Since the particles are small, the

smallest scales of the flow are the most important for the strength of the forces that act on a particle. It is found that the history term has to be calculated over a time interval of at least one Kolmogorov time. For the runs presented here a history of about $2\tau_K$ (about 800 time steps) is chosen; increasing this time does not significantly change any of the forces acting on a particle.
The computation of especially the Basset force is highly memory-demanding. This makes the study of light particle behavior in turbulent flows a computational challenge.

3 Results

The results presented in this paper are obtained in either a moderately (case N10, $N = 0.31\ \mathrm{s}^{-1}$) or a strongly (case N100, $N = 0.98\ \mathrm{s}^{-1}$) stably stratified flow [1,2]. For several cases also the results for isotropic turbulence are shown for comparison. Several quantities are studied; the main interest here is on single-particle dispersion, on the effect of preferential concentration and on the relative importance of the different forces that are acting on the particles.

3.1 Single-particle dispersion

The horizontal and vertical dispersion (mean-squared displacement, $\delta_i^2 = \overline{[x_{p,i} - x_{p,i}(0)]^2}$) of both fluid particles and inertial particles in isotropic and stratified turbulence (case N10) are shown in Fig. 1. For isotropic turbulence the classical initial ballistic t^2-regime and long-time linear diffusion limit are retrieved [12]. For stratified turbulence the vertical dispersion is clearly

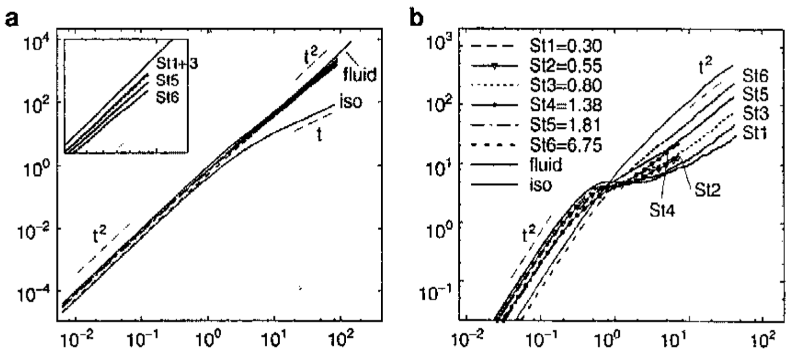

Fig. 1. Horizontal (a) and vertical (b) single-particle dispersion as a function of time for case N10. Results for fluid particles, light particles (St2 and St4) and heavy particles. The axes are scaled using the horizontal ($\overline{u_h'^2}$) and vertical ($\overline{w'^2}$) rms-velocities, the Lagrangian time scale T_L and the buoyancy frequency N. For reference, also the result for isotropic turbulence is added, in the right plot this graph is shifted and the axes are scaled using $\overline{w'^2}$ and T_L.

suppressed. Fluid particles that are displaced from their original equilibrium height tend to return to that equilibrium height. After the t^2-regime a plateau is found that scales as N^{-2} [7]. For long times again a transition towards a diffusive t-regime can be seen, which is caused by molecular diffusion of the density [2, 11]. In the horizontal direction it can be seen (Fig. 1a) that the dispersion of fluid particles is enhanced in stratified turbulence. This is a result of strong local shear between large-scale horizontal vortical structures in different vertical layers [2].

When looking at the transport of more realistic particles, inertial effects have to be considered. Inertial particles do not exactly follow the flow like fluid particles, as will be seen in the next section, and they follow biased trajectories. The effect of inertia, expressed using the Stokes number, can be seen for six different St in Fig. 1. Particles denoted with St2 and St4 have density ratios of 10 and 25, the others are heavy particles with $\rho_p/\rho_f = \mathcal{O}(10^4)$. Under the assumption that the particles are small (diameter not exceeding $\mathcal{O}(\eta)$), particles wit $\rho_p/\rho_f = \mathcal{O}(1)$ have Stokes numbers smaller than about 0.1 and it is found that their results resemble the results obtained for fluid particles. The horizontal long-time dispersion of heavy particles is slightly smaller than that of fluid particles, but the slope remains of order t^2. Results for St2 and St4 are omitted in Fig. 1a for clarity, but they show similar behavior. In the vertical direction a clear difference is found for the dispersion of fluid particles and that of inertial particles. With increasing St the long-time dispersion is enhanced. The typical plateau becomes less pronounced and the transition to a final linear diffusion limit sets in at earlier times. Here, the Stokes number is the determining factor. The density ratio only has a minor effect on the vertical dispersion in stably stratified turbulence, as becomes clear from the strong resemblance between the light (St2 and St4) and heavy particle results.

3.2 Preferential concentration

The distribution of the particles over the flow domain can give an impression of the mixing properties of the flow. Several previous studies (for example, Bec et al. [5]), looking at heavy particles in isotropic turbulence, observe the so-called effect of preferential concentration. The particle distribution is found to be highly nonuniform, and the particles collect in regions of high strain rate and low vorticity. Also for stratified turbulence we obtain this nonuniform distribution of particles [1]. Strong local heavy particle accumulation is shown in Fig. 2 for isotropic turbulence and for strongly stratified turbulence. The particle distribution in stratified turbulence differs from that obtained in isotropic turbulence, and a clear difference can be seen between the horizontal and the vertical direction. The particle distribution reflects the anisotropy of the flow. In the horizontal direction particles cluster on larger scales than in isotropic turbulence, whereas in the vertical direction thin, sheared layers are observed. The correlation dimension $\mathcal{D}_2$ as used by Bec et al. [5] is applied to

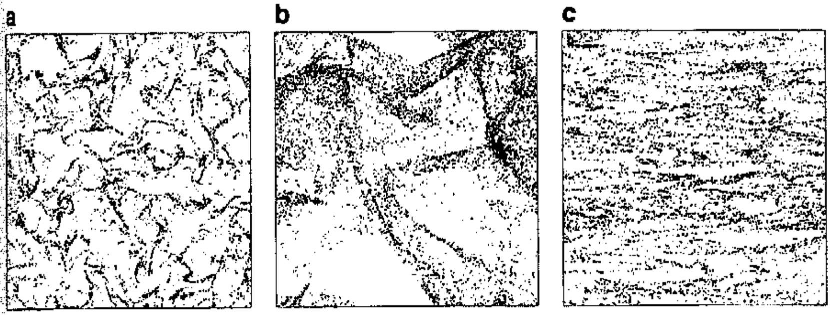

Fig. 2. Particle distributions in horizontal (b) and vertical (a,c) cross-sections of the flow domain. Results are shown for isotropic turbulence (a) and for stably stratified turbulence (b,c) (case N100), from runs with 10^6 particles and Stokes number around the optimum for clustering. No gravitational forces are acting on the particles.

quantify the preferential concentration. This leads to the conclusion that in stably stratified turbulence the effect of preferential concentration decreases with increasing stratification [1].

3.3 Forces acting on the particles

Elaborate studies on the importance of the different forces that are acting on inertial particles are scarce in the literature. The topic is studied for a turbulent channel flow by Armenio and Fiorotto [3]. It is mainly of interest for light particles. All forces in equation (1) are taken into account, except for the gravitational force (to avoid a mean drift velocity). The forces act at the smallest scales of the flow. At these scales the flow is more or less isotropic, also for the moderately stratified case N10. Therefore, a strong resemblance is obtained between the results for isotropic turbulence and for stratified turbulence. For particles with density ratios of order one, all forces are relevant except for the added mass Faxèn correction term. With increasing ρ_p/ρ_f the importance of the different forces decreases. For particles with $\rho_p/\rho_f = 10$ and $St = 0.55$ in a case N10 flow the probability density functions (PDFs) of the ratio of the different forces and the Stokes drag are shown in Fig. 3. The dispersion results for these particles are shown previously in Fig. 1. It can be seen that for this combination of ρ_p/ρ_f and St, apart from the Stokes drag, only the pressure gradient and the Basset force play a significant role. In a separate simulation with the same St but $\rho_p/\rho_f = 144$ these two forces are found to be smaller by a factor of ten and two, respectively. It can therefore be concluded that the density ratio is an important parameter in the study of the different forces.

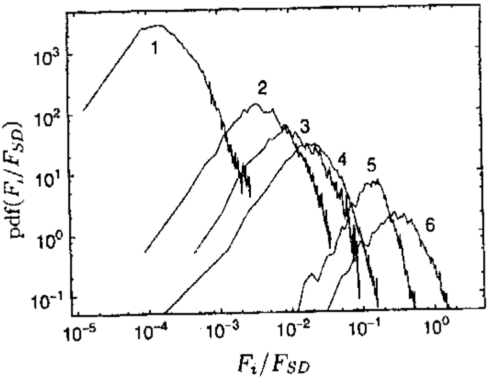

Fig. 3. PDF of the ratio of different forces F_i and the Stokes drag F_{SD}; F_1 = added mass Faxèn correction, F_2 = Basset force Faxèn correction, F_3 = Stokes drag Faxèn correction, F_4 = added mass, F_5 = pressure gradient, F_6 = Basset force. Particles with $St = 0.55$ and $\rho_p/\rho_f = 10$ are tracked in a case N10 flow.

4 Concluding remarks

In this work we showed the behavior of light and heavy particles in stably stratified turbulence. Three topics are discussed; single-particle dispersion, the effect of preferential concentration and the importance of the different forces that are acting on the particles. In future work we will study in more detail the influence of the different forces on the dispersion of (light) inertial particles in stably stratified turbulence. Furthermore, we will examine the influence of gravitational forces that are acting on the particles on the presented results.

Acknowledgments

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