SPHERICITY AND TWIST AS FUNCTIONAL PARAMETERS TO REPRESENT SURFACE GEOMETRIES

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ABSTRACT

Topographical measurements of surfaces yield a lot of data which are usually presented by tables of height coordinates or by contour maps. Here a method is described to characterize the surface geometry by just a few parameters which quantify the functional properties: **sphericity, twist** and **waviness**. Examples of applications are given from the field of surface plate measurement. Some measurement techniques based on electronic levels, autocollimators and laserinterferometers are briefly discussed. A resulting accuracy of 0.1 μm on a one square meter surface is claimed. With the characteristic parameters, even small geometrical changes due to the environmental conditions temperature and moisture could be recorded and explained.

1. INTRODUCTION

The paper deals with the measurement and representation of semi-flat surfaces which mostly are applied in the precision industry. Up to about 500 mm diameter, holistic methods can be used to measure the whole surface very accurately. For optical surfaces, interferometers are the most accurate instruments with a resolution up to 0.01 μm. For larger surfaces autocollimators, electronic levels and laserinterferometers are applied. We see in particular for surface plate calibration a growing interest in computer aided measurement techniques. In this way reproducible measurements have been obtained with 0.05 μm standard deviation on typical 1 m wide surfaces. This makes that changes in geometry due to external influences can clearly be seen. In that case, however, coordinate tables or contour maps as commonly used to represent the geometry do not conform to the requirements, because changes in geometry may be small compared to the profile heights itself.

The most important components of a surface geometry are sphericity and twist. They are very helpful to characterize a surface by just a few numbers which describe functional properties of the surface. They are also directly related to environmental conditions. Sphericity is generally related to that type of distortion caused by temperature gradients perpendicular to the surface while twist is connected to external load or changing support conditions. Naturally, both effects can also be related to the fabrication process. In the field of optics, sphericity of optical surfaces is the best known geometry. It will be pointed out that twist of surfaces can be described also in terms of curvature. Some useful relations will be given which may be applied both in the field of precision- and optical engineering.

3. QUADRIC SURFACES

After several years of practice with surface plate calibrations the need was felt for some parameters which characterize the general shape of the surface. An obvious approach is to describe the surface in terms of a quadric surface represented by the general second order equation. This equation contains 10 independent coefficients from which 4 independent quantities which are invariant with respect to the translation and rotation can be calculated. They define properties of the quadric which do not depend on the position and can be applied to specify the surface (Korn, 1968). There are, however, reasons for a different approach. It is difficult to obtain the 4 invariants from the measured data and moreover their meaning is hard to visualize.

While examining many contour maps, often more or less spherical or twisted surfaces have been encountered. Both quadric forms are easily understandable and are significant from a technical point of view.

Suppose as a result of measurement a set of height coordinates H_{xy} (or H_{ij} when measured on an equidistant grid). Because the reference plane is usually arbitrarily chosen, a first step is to transform the heights to the regression plane as a new reference. Compared to all other planes the regression plane gives the closest approximation of the measured surface. It is the mean plane through the surface and can be seen as the natural reference. The relation between the reference plane and the regression plane is given by:

$$h_{ij} = H_{ij} - a - j\alpha - i\beta \tag{1}$$

where h_{ij} is the distance from the regression plane in a given point. The coefficient a is the translation term while α and β describe the rotation with respect to both axes. The coefficients can easily be obtained by the least square method giving:

$$\begin{bmatrix} a \\ \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \Sigma 1 & \Sigma j & \Sigma i \\ \Sigma j & \Sigma j^2 & \Sigma ij \\ \Sigma i & \Sigma ij & \Sigma i^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Sigma H_{ij} \\ \Sigma j H_{ij} \\ \Sigma i H_{ij} \end{bmatrix} \tag{2}$$

Sphericity and twist

The magnitudes of sphericity and twist can be found in a similar way. With Q as the regression plane and S a spherical plane (Fig. 1) the distance s_{ij} can be expressed by:

$$s_{ij} = \rho S_{ij} \tag{3}$$

where $1/\rho = R$ the radius of curvature and where, if s_{ij} is small compared to R, the geometrical factor is found to be:

$$S_{ij} = -\frac{1}{2}(x_i^2 + y_j^2) \tag{4}$$

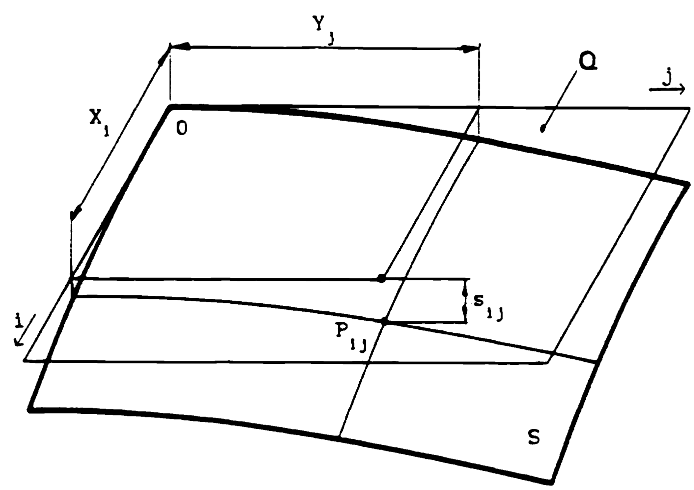

Figure 1. Height difference s_{ij} in case of sphericity.

The height difference due to twist or torsion can be expressed considering a rotation axis in a reference plane Q (Fig. 2). The rotation of that axis is assumed to be proportional to the length of that axis and to the parameter τ quantifying the amount of twist. The height difference in a given point P_{ij} is proportional to the distance AP to the twist axis and the rotation of the line AP in point A, on a distance OA from an arbitrarily chosen origin. With the twist axis at an angle φ from the x-axis, it can be derived from the figure:

$$t_{ij} = \tau T_{ij} \tag{5}$$

where τ the amount of twist [rad/m] and the geometrical factor T_{ij} = OA · AP or (Eq.6):

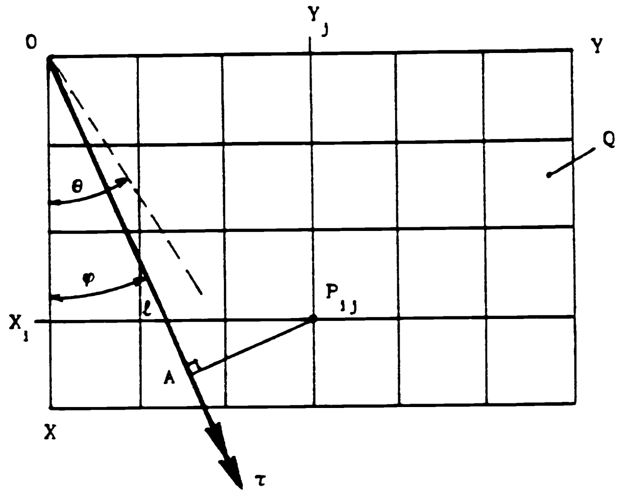

Figure 2. Twist along the axis l causes a height difference t_{ij}.

$$T_{ij} = \frac{1}{2}(y_j^2 - x_i^2)\sin 2\varphi + x_i y_j \cos 2\varphi \tag{6}$$

The parameters ρ and τ can be found, similar to the parameters of the regression plane, by the least square method minimizing the quare sum $\Sigma\ \Delta^2$, where:

$$\Delta = h_{ij} - T_{ij}\tau - S_{ij}\rho \tag{7}$$

The parameters for which the square sum is minimum (best fit) are found from Eq. 8.

$$\begin{bmatrix} a \\ \alpha \\ \beta \\ \tau \\ \rho \end{bmatrix} = \begin{bmatrix} \Sigma 1 & \Sigma j & \Sigma i & \Sigma T_{ij} & \Sigma S_{ij} \\ \Sigma j & \Sigma j^2 & \Sigma ij & \Sigma j T_{ij} & \Sigma j S_{ij} \\ \Sigma i & \Sigma ij & \Sigma i^2 & \Sigma i T_{ij} & \Sigma i S_{ij} \\ \Sigma T_{ij} & \Sigma j T_{ij} & \Sigma i T_{ij} & \Sigma T_{ij}^2 & \Sigma T_{ij} S_{ij} \\ \Sigma S_{ij} & \Sigma j S_{ij} & \Sigma i S_{ij} & \Sigma T_{ij} S_{ij} & \Sigma S_{ij}^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Sigma H_{ij} \\ \Sigma j H_{ij} \\ \Sigma i H_{ij} \\ \Sigma T_{ij} H_{ij} \\ \Sigma S_{ij} H_{ij} \end{bmatrix} \tag{8}$$

in which the summation is taken over all points (i,j) where a value H_{ij} has been measured (in case of a non-equidistant grid the numbers i and j should be replaced by the actual distances x and y). The direction φ of the twist axis is not directly found from eq. 8. This is done iteratively. A minimum for $\Sigma\ \Delta^2$ can be found in each quadrant. From the Eqs. 5 and 6 follows:

$$\begin{aligned} \tau(\varphi \pm \pi) &= \tau(\varphi) \\ \tau(\varphi \pm \tfrac{\pi}{2}) &= -\tau(\varphi) \end{aligned} \tag{9}$$

The minimum is found by the technique of interval intersection at an 0 - 90° interval. For $|\theta-\varphi| < 0.1°$ (Fig. 2) no significant change in characteristic parameters has been observed. This requires about 10 steps.

4. NORMALIZED BOWRISES

Both parameters ρ and τ represent an angle per unit of length and both can be related to a bowrise of the surface (Fig. 3). When the bowrises are considered to be relatively small linear equations will be obtained.
In case of sphericity it is easily found:

$$b_{ls} = \frac{\rho l^2}{8} \tag{10}$$

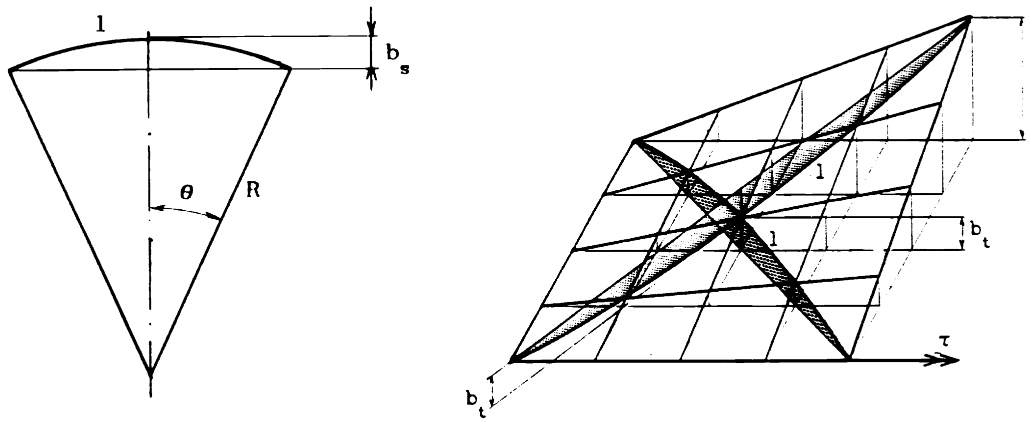

Figure 3. Bowrise b_s due to pure sphericity and b_t due to pure twist of the surface.

In case of twist also bowrises are found. Extreme values (concave and convex) are found in directions $\varphi \pm 45°$ both with $b_t = z/4$ where $z = \tau\, l^2/2$ and l the length of the bow. Then it follows similar to Eq. 10:

$$b_{lt} = \frac{\tau l^2}{8} \tag{11}$$

The length l in Eqs. 10 and 11 may be the length or diagonal of the measured surface. In general we take for l the unit of length (1 m) and define the normalized bowrises as:

$$b_s = \frac{\rho}{8} \quad \wedge \quad b_t = \frac{\tau}{8} \quad [\mu m/m^2] \tag{12}$$

These quantities describe the shape of the surface itself and are not related to any dimension of that surface. A measurement on a part of a surface will now deliver the same normalized bowrises as a measurement over the entire surface. Such a surface can be described by a function $z = z\,(b_s, b_t, \varphi)$. The quantities b_s, b_t and φ are in that view the parameters of a second degree surface containing a spherical as well as a twist component and which is the best approximation of the measured surface. The difference between that approximation and the measured surface is given by the higher order waviness i.e. the Root Mean Square (RMS) of the differences between both surfaces. This RMS is not a flatness parameter in the sense that it can be used to reconstruct an approximating surface, but is gives an indication of the quality of the surface and the closeness of the approximation.

Superposition rules

Suppose a surface with sphericity ρ_1 causing a height difference s_1 in an arbitrary point of the surface (Fig. 1) and another surface with sphericity ρ_2 causing a height s_2. Then it follows from Eq. 3 and 4 that a surface which is a superposition of two surfaces with sphericities ρ_1 and $\lambda\rho_2$, respectively, has in the considered point a height $s = s_1 + \lambda s_2$ with λ a freely chosen multiplier. Sphericities can thus linearly be combined.

A superposition of two twist components of differently orientated twist axes will also cause true twist. In that case, however, no linear relationship exists. Assume there is a twist (τ_1, φ_1) for surface t_1 and another component (τ_2, φ_2) for surface t_2. The surface t obtained from t_1 and λt_2 can be described in terms of one unique torsion (τ, φ). From the Eqs. 5 and 6 it can be found that:

$$\begin{aligned} \varphi &= \frac{1}{2}\arctan\left(\frac{\tau_1 \sin 2\varphi_1 + \lambda\,\tau_2 \sin 2\varphi_2}{\tau_1 \cos 2\varphi_1 + \lambda\,\tau_2 \cos 2\varphi_2}\right) \\ \tau &= \left(\frac{\tau_1 \sin 2\varphi_1 + \lambda\,\tau_2 \sin 2\varphi_2}{\sin 2\varphi}\right) \end{aligned} \tag{13}$$

From Eq. 12 it follows easily that in Eq. 13 instead of the τ's the corresponding bowrises b_t may be substituted.

A special case arises when both twist vectors are in the same direction. Then $\varphi = \varphi_1 = \varphi_2$ and $\tau = \tau_1 + \lambda\tau_2$ which is a linear relation just as in case of sphericity. When both twist components are of equal magnitude but in perpendicular direction the combination results in a true plane.

Principal directions in the surface

With $\varphi = 45^\circ$ the Eqs. 5 and 6 are reduced to:

$$t_{ij} = \frac{\tau}{2}(y_j^2 - x_i^2) \quad (14)$$

This is equivalent to the equation of a proper hyperbolic paraboloid:

$$z = \frac{y^2}{b^2} - \frac{x^2}{a^2} \quad (15)$$

where $a^2 = b^2 = 2/\tau$ (Fig. 4). The saddle point at the origin has a double curvature with:

$$R_x = -R_y = \frac{1}{\tau} \quad (16)$$

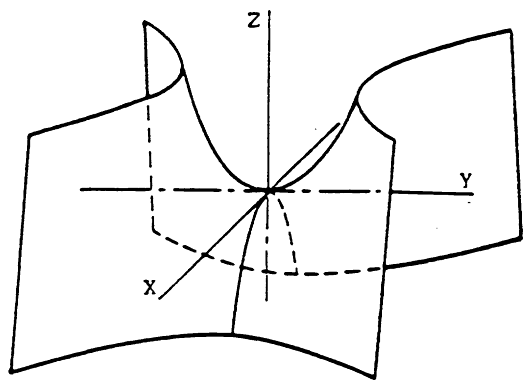

Figure 4. A hyperbolic paraboloid as a proper quadric surface.

The hyperbolic paraboloid can also be described as a double ruled surface containing two families of straight lines as shown in Fig. 3. In any arbitrary direction θ (Fig. 2) the bowrise due to twist is found to be

$$b_{t,\theta} = b_t \sin 2(\varphi - \theta) \quad (17)$$

The total normalized bowrise in a direction θ amounts to:

$$b_\theta = b_s + b_t \sin 2(\varphi - \theta) \quad (18)$$

The principal axes are the axes with extreme values of b (see x and y axis in Fig. 4). They are found to be:

$$\eta_1 = \varphi + \frac{\pi}{4} sign(b_t) \qquad (concave)$$
$$\eta_2 = \varphi - \frac{\pi}{4} sign(b_t) \qquad (convex) \quad (19)$$

If $|b_s/b_t| < 1$ there exist directions with $b_\theta = 0$, which means straight lines or directrices. The surface is then a saddle plane or hyperbolic paraboloid. Such directions are found from Eq. 17.:

$$\psi_1 = \varphi + \frac{1}{2}\arcsin\frac{b_s}{b_t}$$
$$\psi_2 = \varphi + \frac{\pi}{2} - \frac{1}{2}\arcsin\frac{b_s}{b_t} \quad (20)$$

A special case arises when $|b_s| = |b_t|$. Then the surface is a cylinder with both directrices coinciding with one principal axis. Finally, if $|b_s/b_t| > 1$ the surface is part of an ellipsoid or, if $b_t = 0$, a sphere. An example of a surface containing sphericity and twist is given in Fig. 5. The principal axes η are mutually perpendicular and are found in directions $\varphi \pm 45°$. Lines parallel to the directrices ψ are 'straight' lines in the surface.

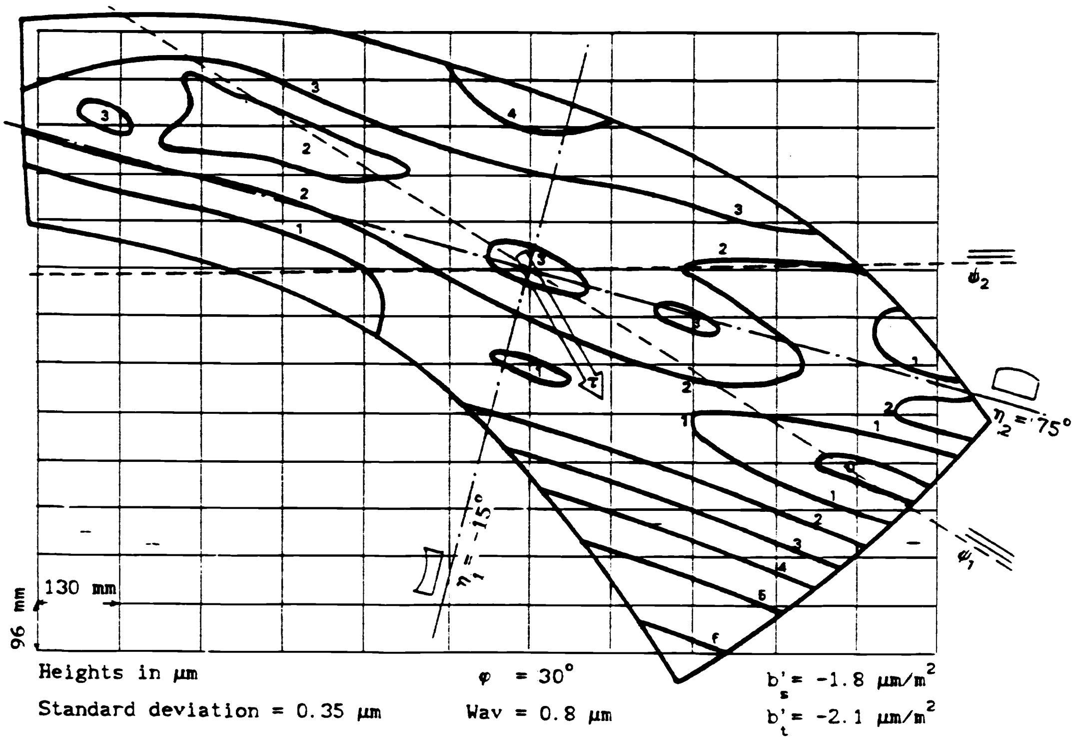

Figure 5. The twist in a 1250 x 1500 mm spectrometer plate was caused by the machine slidings.

5. RESULTS

There are different methods to measure flatness departures very accurately. This may be done directly by interferometric measurements as applied for optical flats or by using autocollimators, laserinterferometers or electronic levels on an equidistant grid, all methods deliver a set of height coordinates H_{ij} from which the flatness parameters can be obtained by Eq. 8. The measurements itself fall outside the scope of this paper.

Some results could be obtained from the BCR intercomparison flatness measurement which was organized by the PTB (1991) to evaluate the state of the art. A stationary 800 x 1300 mm section on a granite surface plate was measured by participants from European countries. The flatness parameters have been calculated from all measured results. From 7 participants was found:

Sphericity:	$b_s = -3.5\ \mu m/m^2$	Standard deviation $0.1\ \mu m/m^2$
Twist:	$b_t = -1.6\ \mu m/m^2$	Standard deviation $0.1\ \mu m/m^2$
Direction:	$\varphi = 20°$	Standard deviation 2°
Waviness (RMS)	$= 1.4\ \mu m$	Standard deviation $0.03\ \mu m$

Another intercomparison flatness measurement was organized by the Netherlands Measuring institute NMi (1992). The result is shown in Fig 6. A 600 x 800 mm wide section on a 300 mm thick granite surface plate was measured on a 100 mm pitch using Wyler electronic levels and Flattest software. The standard deviation of the measurement (which is calculated on-line) was 0.03 μm. The reproducibility was tested by a second measurement where a standard deviation of 0.04 μm was obtained. The standard deviation of the difference of both results was found to be 0.05 μm over all grid points. This is fully in accordance with the square sum of both standard deviations given before. The normalized sphericity was found to be -1.36 and -1.35 $\mu m/m^2$ respectively. Because electronic levels were used, both measurements have been automatically corrected for the earth curvature of 0.03 $\mu m/m^2$ which is in the same order as the obtained accuracy. The correlation coefficient was over 80% which means that the quadric surface fits reasonably well with the measured surface. Results of other participants are not available yet.

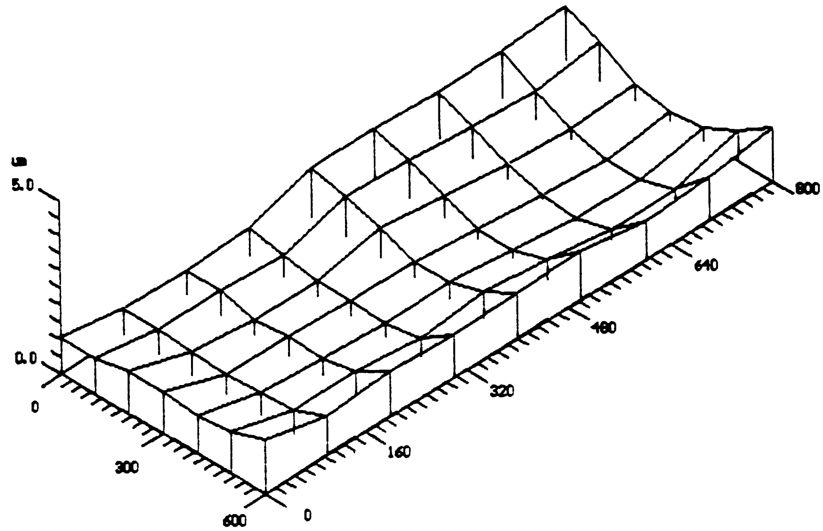

Figure 6. Test surface used for the NL intercomparison. Peak to valley 1.57 μm, sphericity -1.36 $\mu m/m^2$, twist 0.88 $\mu m/m^2$ at 39°, waviness 0.25 μm.

The example shown already in Fig. 5 was the measurement of large dipoles plates for an electron spectrometer. The measurement was done with the plates still on the grinding machine (Ettlingen, Germany). Here all (4) plates showed the same twist (b_t and φ) pointing to a systematic error in the machine slides. Although the plates were within the 10 μm tolerance nevertheless the systematic flatness deviations could be taken into account improving the performance of the spectrometer.

A similar task was the measurement of a foundation plate for a precision laser workstation. The plate was measured twice on the grinding machine, first with the plate still clamped on the machine table and second with the clamps released. The influence of clamping could be fully described by the sphericity and twist parameters (7.6 and 11.2 $\mu m/m^2$ respectively). The torsion vector was found in the direction of one of the machine slidings.

A temperature gradient over the thickness of a plate can directly be translated into a spherical bowrise by:

$$b_s = \frac{\alpha l^2 \Delta T}{8t} \tag{21}$$

where α the coefficient of thermal expansion, l the length of the bow and ΔT the temperature difference over the thickness t. Normal lighting can cause thermal errors of several micrometers. One solution is to measure the temperature gradient and correct the result using Eq. 21. A better approach is to stabilize the temperature within the required accuracy. In that case lights should not be switched off all the time otherwise a changed sphericity will be found.

An unexpected problem influencing the dimensional stability is the absorption of water by the surface of granite surface plates. This causes compressive stresses and deformation as a result. To study the geometrical stability of granite a newly fabricated plate has been measured twice a week for about one year. The sphericity was found to decrease by 5 $\mu m/m^2$ over a period of four months and stabilized then. Similar results could be obtained after wetting the top surface of a granite plate for 24 h. In that case it took 5 months to reestablish the original geometry (Fig. 7). No changes in twist have been found in that period. The process of water diffusion out of the material could be followed by the flatness parameters. Especially the sphericity proved to be very sensitive to water (and temperature gradients).

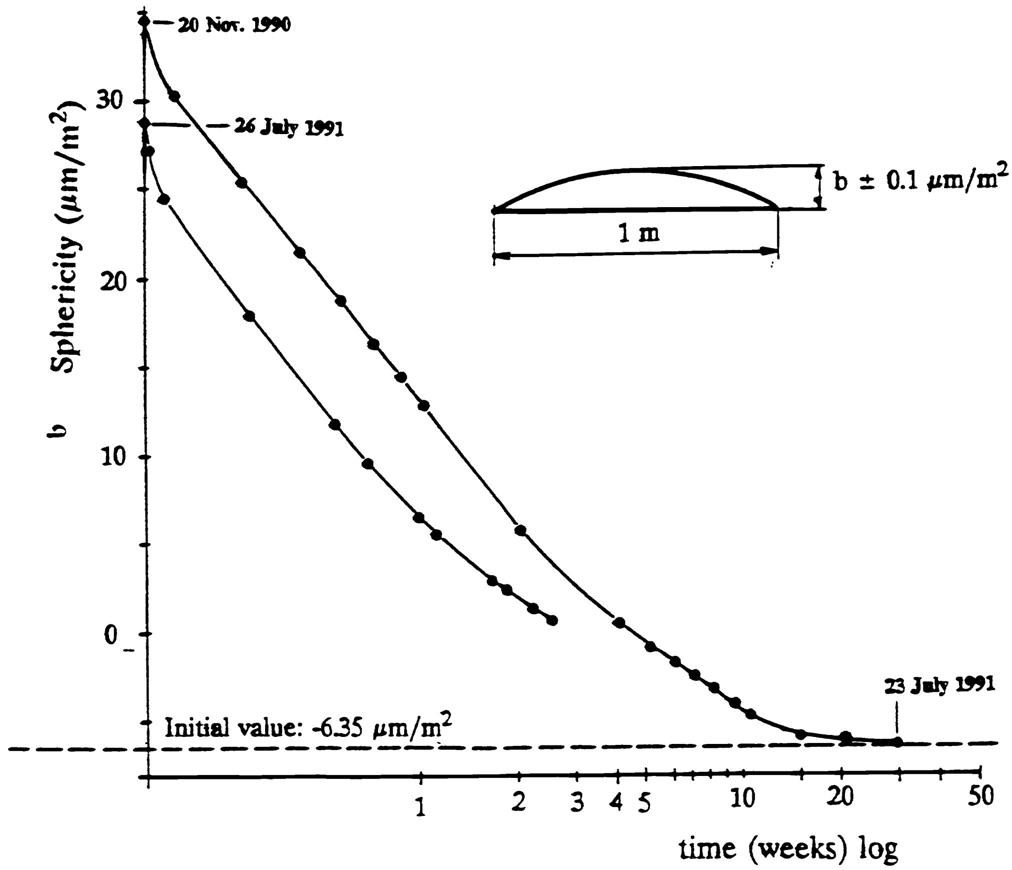

Figure 7. The sphericity parameter is very sensitive to record the drying process.

6. CONCLUSIONS

Sphericity and torsion which are often found in semi flat surfaces can easily be quantified by the (normalised) bowrises, quantifying the corresponding quadric geometry. The accuracy is very high because the result is based on all measured data which averages stochastic errors. With instruments like electronic levels standard deviations better than 0.1 $\mu m/m^2$ can easily be obtained on typical 1 m plate dimensions. This opens the way to study the influences of environmental conditions and the dimensional stability over long periods.

The flatness parameters will become of more importance with the higher accuracies as required in the precision and ultra-precision industries. Tanichuchi (1983) has shown the need of continuous improvement in accuracy by a factor two every 5 to 10 years.

7. REFERENCES

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