Guard-based Partial-Order Reduction

Alfons Laarman, Elwin Pater, Jaco van de Pol, and Michael Weber

Formal Methods and Tools, University of Twente, The Netherlands
elwin.pater@gmail.com
{laarman,vdpol,michaelw}@cs.utwente.nl

Abstract. This paper aims at making partial-order reduction independent of the modeling language. Our starting point is the stubborn set algorithm of Valmari (see also Godefroid's thesis), which relies on necessary *enabling* sets. We generalise it to a guard-based algorithm, which can be implemented on top of an abstract model checking interface.
We extend the generalised algorithm by introducing necessary *disabling* sets and adding a heuristics to improve state space reduction. The effect of the changes to the algorithm are measured using an implementation in the LTSmin model checking toolset. We experiment with partial-order reduction on a number of Promela models, some with LTL properties, and on benchmarks from the BEEM database in the DVE language.
We compare our results to the Spin model checker. While the reductions take longer, they are consistently better than Spin's ample set and even often surpass the ideal upper bound for the ample set, as established empirically by Geldenhuys, Hansen and Valmari on BEEM models.

1 Introduction

Model checking is an automated method to verify the correctness of concurrent systems by examining all possible execution paths for incorrect behaviour. The main difficulty is the *state space explosion*, which refers to the exponential growth in the number of states obtained by interleaving executions of several system components. Model checking has emerged since the 1980s [3] and several advances have pushed its boundaries. Partial-order reduction is among those.

Partial-order reduction (POR) exploits independence and commutativity between transitions in concurrent systems. Exhaustive verification needs to consider only a subset of all possible concurrent interleavings, without losing the global behaviour of interest to the verified property. In practice, the state space is pruned by considering a sufficient subset of successors in each state.

The idea to exploit commutativity between concurrent transitions has been investigated by several researchers, leading to various algorithms for computing a sufficient successor set. The challenge is to compute this subset during state space generation (on-the-fly), based on the structure of the specification.

Already in 1981, Overman [20] suggested a method to avoid exploring all interleavings, followed by Valmari's [28,31,30] *stubborn sets* in 1988, 1991 and 1992. Also from 1988 onwards, Peled [16] developed the *ample set* [23,24], later

extended by Holzmann and Peled [14,25], Godefroid and Pirottin [8,10] the *persistent set* [9], and Godefroid and Wolper [11] *sleep sets*. These foundations have been extended and applied in numerous papers over the past 15 years.

Problem and Contributions. Previous work defines partial-order reduction in terms of either petri-nets [35] or parallel components with local program counters, called processes [14,9]. While this allows the exploitation of certain formalism-specific properties, like *fairness* [24] and token conditions [33], it also complicates the application to other formalisms, for instance, rule-based systems [12]. Moreover, current implementations are tightly coupled to a particular specification language in order to compute a good syntactic approximation of a sufficient successor set. In recognition of these problems, Valmari started early to generalise the stubborn set definition for "transition/variable systems" [29,31].

To address the same problem for model checking algorithms, we earlier proposed the Pins interface [2,19], separating language front-ends from verification algorithms. Through Pins (Partitioned Interface to the Next-State function), a user can use various high-performance model checking algorithms for his favourite specification language, cf. Figure 1. Providing POR as Pins2Pins wrapper once and for all benefits every combination of language and algorithm.

An important question is whether and how an abstract interface like Pins *can support partial-order reduction.* We propose a solution that is based on stubborn sets. This theory stipulates how to choose a subset of transitions, enabled and disabled, based on a careful analysis of their independence and commutativity relations. These relations have been described on the abstract level of transition systems before [31]. Additionally, within the context of petri-nets, the relations were refined to include multiple enabling conditions, a natural distinction in this formalism [33].

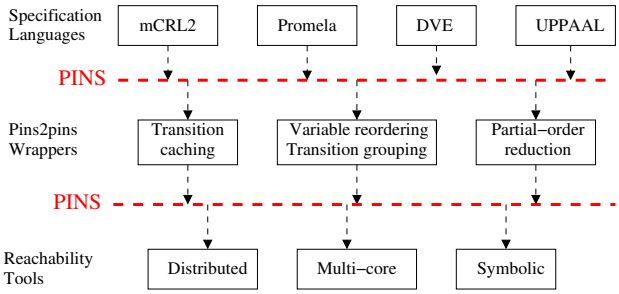


Fig. 1. Modular Pins architecture of LTSmin

We generalise Valmari's work to a complete language-agnostic setting, by assuming that transitions consist of guard conditions and state variable assignments (Section 3). In Section 4, we extend Pins with the necessary information: a do-not-accord matrix and optional; necessary enabling matrix on guards. In addition, we introduce novel *necessary disabling sets* and a new heuristic-based selection criterion. As optimal stubborn sets are expensive to compute precisely [33], our heuristic finds reasonably effective stubborn sets fast, hopefully leading to smaller state spaces. In Section 5, we show how LTL can be supported.

Our implementation resides in the LTSmin toolset [2], based on Pins. Any language module that connects to Pins now obtains POR without having to bother about its implementation details, it merely needs to export transition guards and their dependencies via Pins. We demonstrate this by extending

LTSmin's DVE and Promela [1] front-ends. This allows a direct comparison to Spin [13] (Section 6), which shows that the new algorithm generally provides more reduction using less memory, but takes more time to do so. It also yields more reduction than the theoretically best reduction using ample sets, as reported by Geldenhuys et al. [7] on the Dve BEEM benchmarks [22].

Summarising, these are the main contributions presented in this work:

1. *Guard-based partial-order reduction*, which is a language-independent generalisation of the stubborn set method based on necessary enabling sets;
2. Some improvements to efficiently compute smaller stubborn sets:
   (a) A refinement based on *necessary disabling sets*;
   (b) A *heuristic selection criterion* for necessary enabling sets;
   (c) A more *dynamic* definition of *visibility*, yielding better reduction for LTL;
3. Two language module *implementations* exporting guards with dependencies;
4. An *empirical evaluation* of guard-based partial-order reduction in LTSmin:
   (a) A comparison of resource consumption and effectiveness of POR between LTSmin [2] and Spin [13] on 18 Promela models/3 LTL formulas.
   (b) An impact analysis of necessary disabling sets and the heuristic selection.
   (c) A comparison with the ideal ample set from [7], on Dve BEEM models.

2 The Computational Model of Guarded Transitions

In the current section, we provide a model of computation comparable to [7], leaving out the notion of processes on purpose. It has three main components: states, guards and transitions. A state represents the global status of a system, guards are predicates over states, and a transition represents a guarded state change.

Definition 1 (state). *Let $S = E_1 \times \ldots \times E_n$ be a set of vectors of elements with some finite domain. A state $s = \langle e_1, \ldots, e_n \rangle \in S$ associates a value $e_i \in E_i$ to each element. We denote a projection to a single element in the state as $s[i] = e_i$.*

Definition 2 (guard). *A guard $g : S \to \mathbb{B}$ is a total function that maps each state to a boolean value, $\mathbb{B} = \{true, false\}$. We write $g(s)$ or $\neg g(s)$ to denote that guard g is true or false in state s. We also say that g is enabled/disabled.*

Definition 3 (structural transition). *A* structural transition *$t \in T$ is a tuple $(\mathcal{G}, a)$ such that a is an assignment $a : S \to S$ and $\mathcal{G}$ is a set of guards, also denoted as $\mathcal{G}_t$. We denote the set of enabled transitions by $en(s) := \{t \in T \mid \bigwedge_{g \in \mathcal{G}_t} g(s)\}$. We write $s \xrightarrow{t}$ when $t \in en(s)$, $s \xrightarrow{t} s'$ when $s \xrightarrow{t}$ and $s' = a(s)$, and we write $s \xrightarrow{t_1 t_2 \ldots t_k} s_k$, when $\exists s_1, \ldots, s_k \in S : s \xrightarrow{t_1} s_1 \xrightarrow{t_2} s_2 \ldots \xrightarrow{t_k} s_k$.*

Definition 4 (state space). *Let $s_0 \in S$ and let T be the set of transitions. The state space from s_0 induced by T is $M_T = (S_T, s_0, \Delta)$, where $s_0 \in S$ is the* initial state*, and $S_T \subseteq S$ is the set of* reachable states*, and $\Delta \subseteq S_T \times T \times S_T$ is the set of* semantic transitions*. These are defined to be the smallest sets such that $s_0 \in S_T$, and if $t \in T$, $s \in S_T$ and $s \xrightarrow{t} s'$, then $s' \in S_T$ and $(s, t, s') \in \Delta$.*

Valmari and Hansen [33, Def. 6] also define guards (conditions), which take the role of enabling conditions for disabled transitions. We later generalise this role to enabled transitions as well for our necessary disabling sets (Section 4.2).

In the rest of the paper, we fix an arbitrary set of vectors $S = E_1 \times \ldots \times E_n$, initial state $s_0 \in S$, and set of transitions T, with induced reachable state space $M_T = (S_T, s_0, \Delta)$. We often just write "transition" for elements of T.

It is easy to see that our model generalises the setting including processes (as in [7]). One can view the program counter of each process as a normal state variable, check for its current value in a separate guard, and update it in the transitions. But our definition is more general, since it can also be applied to models without a natural notion of a fixed set of processes, for instance rule-based systems, such as the linear process equations in mCRL [12].

Besides guarded transitions, structural information is required on the exact involvement of state variables in a transition.

Definition 5 (disagree sets). *Given states $s, s' \in S$, for $1 \leq i \leq n$, we define the set of indices on which s and s' disagree as $\delta(s, s') := \{i \mid s[i] \neq s'[i]\}$.*

Definition 6 (affect sets). *For $t = (\mathcal{G}, a) \in T$ and $g \in \mathcal{G}$, we define*

1. *the test set of g is $Ts(g) \supseteq \{i \mid \exists s, s' \in S : \delta(s, s') = \{i\} \wedge g(s) \neq g(s')\}$,*
2. *the test set of t is $Ts(t) := \bigcup_{g \in \mathcal{G}} Ts(g)$,*
3. *the write set of t is $Ws(t) \supseteq \bigcup_{s \in S_T} \delta(s, s')$ with $s \xrightarrow{t} s'$,*
4. *the read set of t is $Rs(t) \supseteq \{i \mid \exists s, s' \in S : \delta(s, s') = \{i\} \wedge s \xrightarrow{t} \wedge s' \xrightarrow{t} \wedge Ws(t) \cap \delta(a(s), a(s')) \neq \emptyset\}$ (notice the difference between S and S_T), and*
5. *the variable set of t is $Vs(t) := Ts(t) \cup Rs(t) \cup Ws(t)$.*

Although these sets are defined in the context of the complete state space, they may be statically over-approximated ($\supseteq$) by the language front-end.

Example 1. Suppose $s \in S = \mathbb{N}^3$, consider the transition: $t :=$ *IF* $(s[1] = 0 \wedge s[2] < 10)$ *THEN* $s[3] := s[1] + 1$. It has two guards, $g_1 = (s[1] = 0)$ and $g_2 = (s[2] < 10)$, with test sets $Ts(g_1) = \{1\}$, $Ts(g_2) = \{2\}$, hence: $Ts(t) = \{1, 2\}$. The write set $Ws(t) = \{3\}$, so $Vs(t) = \{1, 2, 3\}$. The minimal read set $Rs(t) = \emptyset$ (since $s[1] = 0$), but simple static analysis may over-approximate it as $\{1\}$.

3 Partial-Order Reduction with Stubborn Sets

We now rephrase the stubborn set POR definitions. We follow the definitions from Valmari [30] and Godefroid's thesis [9], but avoid the notion of processes.

An important property of a stubborn set $\mathcal{T}_s \subseteq T$ is that it commutes with all paths of non-stubborn transitions $t_1, \ldots, t_n \in T \setminus \mathcal{T}_s$. If there is a path $s \xrightarrow{t_1,\ldots,t_n} s_n$ and a stubborn transition $t \in \mathcal{T}_s$ such that $s \xrightarrow{t} s'$, then there exists a state s'_n such that: $s' \xrightarrow{t_1,\ldots,t_n} s'_n$ and $s_n \xrightarrow{t} s'_n$. Or illustrated graphically:

$$\begin{array}{ccccccc} s & \xrightarrow{t_1} & s_1 & \cdots & s_{n-1} & \xrightarrow{t_n} & s_n \\ \downarrow & & & & & & \\ s' & & & & & & \end{array} \quad \Rightarrow \quad \begin{array}{ccccccc} & & & & & & s_n \\ & & & & & & \downarrow \\ s' & \xrightarrow{t_1} & s'_1 & \cdots & s'_{n-1} & \xrightarrow{t_n} & s'_n \end{array}$$

Moreover a stubborn set $\mathcal{T}_s$ at s is still a stubborn set at a state s_1 reached via the non-stubborn transition t_1. Since t_1 is still enabled after taking a stubborn transition, we can delay the execution of non-stubborn transitions without losing the reachability of any deadlock states. Figure 2 illustrates this; since s is not a deadlock state, s_d is still reachable after executing a transition from $\mathcal{T}_s$. The benefit is that, for the moment, we avoid exploring (and storing) states such as $s_1, \ldots, s_n$. "For the moment", because these states may still be reachable via other stubborn paths, therefore smaller stubborn sets are only a heuristic for obtaining smaller state spaces.

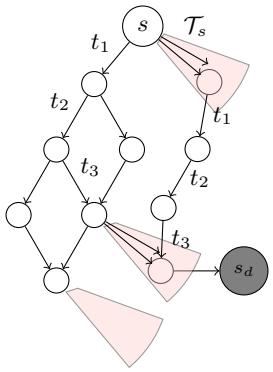


Fig. 2. Stubborn set

This theoretical notion of stubborn sets is a semantic definition. Therefore, we now present the notion of a *(static) stubborn set*, as developed by Valmari. While this definition is stronger, it efficiently (algorithmically) describes *how* to compute these sets (without referring to the entire state space). While researchers have attempted to identify even weaker notions that include more stubborn sets, increasing the chance to find one which yields a larger reduction [32, Sec. 7.4], we rely on the *strong* notion, which is still compatible for extension to LTL model checking [31] (cf. Section 5).

Definition 7 (Do not accord [30]). *First, we define* according with *as:* $A \subseteq \{(t,t') \in T \times T \mid \forall s, s', s_1 \in S\colon s \xrightarrow{t} s' \wedge s \xrightarrow{t'} s_1 \Rightarrow \exists s_1'\colon s' \xrightarrow{t'} s_1' \wedge s_1 \xrightarrow{t} s_1'\}$, *or illustrated graphically:*

$$\begin{array}{ccc} s & \xrightarrow{t'} & s_1 \\ \downarrow_{\neg} & & \\ s' & & \end{array} \quad \Rightarrow \quad \begin{array}{ccc} s & \xrightarrow{t'} & s_1 \\ \downarrow_{\neg} & & \downarrow_{\neg} \\ s' & \xrightarrow{t'} & s_1' \end{array}$$

And for do not-accord: $\mathcal{DNA} = T^2 \backslash A$. *We denote* $\mathcal{DNA}_t = \{t' \mid (t,t') \in \mathcal{DNA}\}$.

Each of the following criteria on $t, t' \in T$ is sufficient to conclude accordance:

1. shared variables $Vs(t) \cap Vs(t')$ are disjoint from the write sets $Ws(t) \cup Ws(t')$,
2. t and t' are never co-enabled, e.g. have different program counter guards, or
3. t and t' do not disable each other, and their actions commute, e.g. write and read to a FIFO buffer or performing atomic increments/decrements.

Definition 8 (necessary enabling set [9]). *Let $t \in T$ be a disabled transition in state $s \in S_T$, $t \notin en(s)$. A necessary enabling set for t in s is a set of transitions $\mathcal{N}_t$, such that for all sequences of the form $s \xrightarrow{t_1,\ldots,t_n} s' \xrightarrow{t}$, there is at least one transition $t_i \in \mathcal{N}_t$ (for some $1 \leq i \leq n$).*

Again, both relations can be safely over-approximated.

We used Valmari's definition for the do-not-accord relation instead of relying on a definition of "dependent", since it allows that transitions modify the same variable, provided they are commuting. As the definition is equivalent to Godefroid's definition of do-not-accord for enabled transitions, we can safely reuse the latter's stubborn set definition:

Definition 9 (stubborn set [9]). *A set $\mathcal{T}_s$ of transitions is* stubborn *in a state s, if $\mathcal{T}_s \cap en(s) = \emptyset \iff en(s) = \emptyset$, and for all transitions $t \in \mathcal{T}_s$:*

1. *If t is disabled in s, then $\exists \mathcal{N}_t \subseteq \mathcal{T}_s$ (multiple sets $\mathcal{N}_t$ can exist), and*
2. *If t is enabled in s, then $\mathcal{DNA}_t \subseteq \mathcal{T}_s$.*

Theorem 1. *Let $\mathcal{T}_s$ be a stubborn at a state s. Then $\mathcal{T}_s$ is dynamically stubborn at s. A search over only stubborn enabled transitions finds all deadlocks in S_T.*

Algorithm 1 from [9] implements the closure method from [32, Sec. 7.4]. It builds a stubborn set incrementally by making sure that each new transition added to the set fulfills the stubborn set conditions (Definition 9).

Example 2. Suppose Figure 2 is a partial run of Algorithm 1 on state s, and transition t_3 does not accord with some transition $t \in \mathcal{T}_s$. The algorithm will proceed with processing t and add all transitions that do-not-accord, including t_3, to the work set. Since t_3 is disabled in state s, we add the necessary enabling set for t_3 to the work set. This could for instance be $\{t_2\}$, which is then added to the work set. Again, the transition is disabled and a necessary enabling set for t_2 is added, for instance, $\{t_1\}$. Since t_1 is enabled in s, and has no other dependent transitions in this example, the algorithm finishes. Note that in this example, t_1 now should be part of the stubborn set.

To find a necessary enabling set for a disabled transition t (i.e. $\mathit{find\_nes}(t, s)$), Godefroid uses fine-grained analysis, which depends crucially on program counters. The analysis can be roughly described as follows:

1. If t is not enabled in global state s, because some local program counter has the "wrong" value, then use the set of transitions that assign the "right" value to that program counter as necessary enabling set;
2. Otherwise, if some guard g for transition t evaluates to *false* in s, take all transitions that write to the *test set* of that guard as necessary enabling set. (i.e. include those transitions that can possibly change g to *true*).

In the next section, we show how to avoid program counters with guard-based POR.

```
1  function stubborn(s)
2      T_work = {t̂} such that t̂ ∈ en(s)
3      T_s = ∅
4      while T_work ≠ ∅ do
5          T_work = T_work − t, T_s = T_s ∪ {t} for some t ∈ T_work
6          if t ∈ en(s) then
7              T_work = T_work ∪ {t' ∈ Σ | (t, t') ∈ DNA} \ T_s
8          else
9              T_work = T_work ∪ N \ T_s where N ∈ find_nes(t, s)
10     return T_s
```

Algorithm 1: The *closure* algorithm for finding stubborn sets

4 Computing Necessary Enabling Sets for Guards

The current section investigates how necessary enabling sets can be computed purely based on guards, without reference to program counters. We proceed by introducing necessary enabling and disabling sets on guards, and a heuristic selection function. Next, it is shown how the Pins interface can be extended to support guard-based partial-order reduction by exporting guards, test sets, and the do-not-accord relation. Finally, we devise an optional extension for language modules to provide fine-grained structural information. Providing this optional information further increases the reduction power.

4.1 Guard-based Necessary Enabling Sets

We refer to all guards in the state space $M_T = (S_T, s_0, \Delta)$ as: $\mathcal{G}_T := \bigcup_{t \in T} \mathcal{G}_t$.

Definition 10 (necessary enabling set for guards). *Let $g \in \mathcal{G}_T$ be a guard that is disabled in some state $s \in S_T$, i.e. $\neg g(s)$. A set of transitions $\mathcal{N}_g$ is a* necessary enabling set *for g in s, if for all states s' with some sequence $s \xrightarrow{t_1,\ldots,t_n} s'$ and $g(s')$, for at least one transition t_i $(1 \leq i \leq n)$ we have $t_i \in \mathcal{N}_g$.*

Given $\mathcal{N}_g$, a concrete necessary enabling set on transitions in the sense of Definition 8 can be retrieved as follows (notice the non-determinism):

$$\mathit{find\_nes}(t, s) \in \{\mathcal{N}_g \mid g \in \mathcal{G}_t \wedge \neg g(s)\}$$

Proof. Let t be a transition that is disabled in state $s \in S_T$, $t \notin en(s)$. Let there be a path where t becomes enabled, $s \xrightarrow{t_1,\ldots,t_n} s' \xrightarrow{t}$, On this path, all of t's disabled guards, $g \in \mathcal{G}_t \wedge \neg g(s)$, need to be enabled, for t to become enabled (recall that $\mathcal{G}_t$ is a conjunction). Therefore, any $\mathcal{N}_g$ is a $\mathcal{N}_t$. □

Example 3. Let ch be the variable for a *rendez-vous channel* in a Promela model. A channel read can be modeled as a Promela statement ch? in some process $P1$. A channel write can be modeled as a Promela statement ch! in some process $P2$. As the statements synchronise, they can be implemented as a single transition, guarded by process counters corresponding to the location of the statements in their processes, e.g.: $P1.pc = 1$ and $P2.pc = 10$. The set of all transitions that assign $P1.pc := 1$, is a valid necessary enabling set for this transition. So is the set of all transitions that assign $P2.pc := 10$.

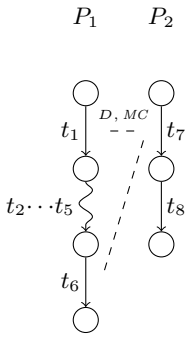
Instead of computing the necessary enabling set on-the-fly, we statically assign each guard a necessary enabling set by default. Only transitions that write to state vector variables used by this guard need to be considered (as in [21]):

$$\mathcal{N}_g^{\min} := \{t \in T \mid \mathit{Ts}(g) \cap \mathit{Ws}(t) \neq \emptyset\}$$

4.2 Necessary Disabling Sets

Consider the computation of a stubborn set $\mathcal{T}_s$ in state s along the lines of Algorithm 1. If a disabled t gets in the stubborn set, a necessary enabling set is required. This typically contains a predecessor of t in the control flow. When that one is not yet enabled in s, its predecessor is added as well, until we find a transition enabled in s. So basically a whole path of transitions between s and t ends up in the stubborn set.

Example 4. Assume two parallel processes P_1 and P_2, with $\mathcal{DNA}(t_1, t_7)$ and $\mathcal{DNA}(t_6, t_7)$. Initially $en(s_0) = \{t_1, t_7\}$; both end up in the stubborn set, since they do-not-accord and may be co-enabled. Then t_7 in turn adds t_6, which is disabled. Now working backwards, the enabling set for t_6 is t_5, for t_5 it is t_4, etc, eventually resulting in the fat stubborn set $\{t_1, \ldots, t_7\}$.



How can this large stubborn set be avoided? The crucial insight is that to enable a disabled transition t, it is necessary to disable any enabled transition t' which cannot be co-enabled with t. Quite likely, t' could be a successor of the starting point s, leading to a slim stubborn set.

Example 5. Consider again the situation after adding $\{t_1, t_7, t_6\}$ to $\mathcal{T}_s$, in the previous example. Note that t_1 and t_6 cannot be co-enabled, and t_1 is enabled in s_0. So it must be disabled in order to enable t_6. Note that t_1 is disabled by itself. Hence t_1 is a necessary enabling set of t_6, and the algorithm can directly terminate with the stubborn set $\{t_1, t_7, t_6\}$. Clearly, using disabling information saves time and can lead to smaller stubborn sets.

Definition 11 (may be co-enabled for guards). *The* may be co-enabled *relation for guards, $MC_g \subseteq \mathcal{G}_T \times \mathcal{G}_T$ is a symmetric, reflexive relation. Two guards $g, g' \in \mathcal{G}_T$ may be co-enabled if there exists a state $s \in S_T$ where they both evaluate to true: $\exists s \in S_T : g(s) \wedge g'(s) \Rightarrow (g, g') \in MC_g$.*

Example 6. Two guards that can never be co-enabled are: $g_1 := v = 0$ and $g_2 := v \geq 5$. In e.g. PROMELA, these guards could implement the channel `empty` and `full` expressions, where the variable v holds the number of buffered messages. In e.g. mCRL2, the conditions of a *summand* can be implemented as guards.

Note that it is allowed to over-approximate the maybe co-enabled relation. Typically, transitions within a sequential system component can never be enabled at the same time. They never interfere with each other, even though their test and write sets share at least the program counter.

Definition 12 (necessary disabling set for guards). *Let $g \in \mathcal{G}_T$ be a guard that is enabled in some state $s \in S_T$, i.e. $g(s)$. A set of transitions $\overline{\mathcal{N}}_g$ is a* necessary disabling set *for g in s, if for all states s' with some sequence $s \xrightarrow{t_1,\ldots,t_n} s'$ and $\neg g(s')$, for at least one transition t_i $(1 \leq i \leq n)$ we have $t_i \in \overline{\mathcal{N}}_g$.*

The following disabling set can be assigned to each guard. Similar to enabling sets, only transitions that change the state indices used by g are considered.

$$\overline{\mathcal{N}}_g^{\min} := \{t \in T \mid Ts(g) \cap Ws(t) \neq \emptyset\}$$

Using disabling sets, we can find an enabling set for the current state s:

Theorem 2. *If $\overline{\mathcal{N}}_g$ is a necessary disabling set for guard g in state s with $g(s)$, and if g' is a guard that may not be co-enabled with g, i.e. $(g, g') \notin MC_g$, then $\overline{\mathcal{N}}_g$ is also a* necessary enabling set *for guard g' in state s.*

Proof. Guard g' is disabled in state s, since $g(s)$ holds and g' cannot be co-enabled with g. In any state reachable from s, g' cannot be enabled as long as g holds. Thus, to make g' true, some transition from the disabling set of g must be applied. Hence, a disabling set for g is an enabling set for g'. □

Given $\mathcal{N}_g$ and $\overline{\mathcal{N}}_g$, we can find a necessary enabling set for a particular transition $t = (\mathcal{G}, a) \in T$ in state s, by selecting one of its disabled guards. Subsequently, we can choose between its necessary enabling set, or the necessary disabling set of any guard that cannot be co-enabled with it. This spans the search space of our new *find_nes* algorithm, which is called by Algorithm 1:

$$\mathit{find\_nes}(t, s) \in \{\mathcal{N}_g \mid \neg g(s)\} \cup \bigcup_{g' \in \mathcal{G}_T} \{\overline{\mathcal{N}}_{g'} \mid g'(s) \wedge (g, g') \notin MC_g\}$$

4.3 Heuristic Selection for Stubborn Sets

Even though the static stubborn set of Definition 9 is stronger than the dynamic stubborn set, its non-determinism still allows many different sets to be computed, as both the choice of an initial transition $\hat{t}$ at Line 2 and the *find_nes* function in Algorithm 1 are non-deterministic. In fact, it is well known that the resulting reductions depend strongly on a smart choice of the necessary enabling set [33]. A known approach to resolve this problem is to run an SCC algorithm on the complete search space for each enabled transition $\hat{t}$ [32] (but even more complicated means exist, like the *deletion algorithm* in [35]). The complexity of this solution can be somewhat reduced by choosing a 'scapegoat' for $\hat{t}$ [35].

We propose here a practical solution that does neither; using a heuristic, we explore all possible scapegoats, while limiting the search by guiding it towards a local optimum. (This makes the algorithm deterministic, which has other benefits, cf. Section 7). An effective heuristics for large partial-order reductions should select small stubborn sets [9]. To this end, we define a heuristic function h that associates some cost to adding a new transition to the stubborn set. Here enabled transitions weigh more than disabled transitions. Transitions that do not lead to additional work (already selected or going to be processed) do not contribute to the cost function at all. Below, $\mathcal{T}_s$ and $\mathcal{T}_{work}$ refer to Algorithm 1.

$$h(\mathcal{N}, s) = \sum_{t \in \mathcal{N}} cost(t, s), \text{ where } cost(t, s) = \begin{cases} 1 & \text{if } t \notin en(s) \text{ and } t \notin \mathcal{T}_s \cup \mathcal{T}_{work} \\ n & \text{if } t \in en(s) \text{ and } t \notin \mathcal{T}_s \cup \mathcal{T}_{work} \\ 0 & \text{otherwise} \end{cases}$$

Here n is the maximum number of outgoing transitions (degree) in any state, $n = \max_{s \in S}(|en(s)|)$, but it can be over-approximated (for instance by $|T|$).
We restrict the search to the cheapest necessary enabling sets:

$$\mathit{find\_nes}'(t,s) \in \{\mathcal{N} \in \mathit{find\_nes}(t,s) \mid \forall \mathcal{N}' \in \mathit{find\_nes}(t,s) : h(\mathcal{N},s) \leq h(\mathcal{N}',s)\}$$

4.4 A Pins Extension to Support Guard-based POR

In model checking, the state space graph of Definition 4 is constructed only implicitly by iteratively computing successor states. A generic next-state interface hides the details of the specification language, but exposes some internal structure to enable efficient state space storage or state space reduction.

The Partitioned Interface for the Next-State function, or Pins [2], provides such a mechanism. The interface assumes that the set of states S consists of vectors of fixed length N, and transitions are partitioned disjunctively in M partition groups T. Pins also supports K state predicates L for model checking. In order to exploit locality in symbolic reachability, state space storage, and incremental algorithms, Pins exposes a dependency matrix DM, relating transition groups to indices of the state vector. This yields orders of magnitude improvement in speed and compression [2,1]. The following functions of Pins are implemented by the language front-end and used by the exploration algorithms:

- InitState: S
- NextStates: $S \to 2^{T \times S}$ and
- StateLabel: $S \times L \to \mathbb{B}$
- DM: $\mathbb{B}_{M \times N}$

Extensions to Pins. POR works as a state space transformer, and therefore can be implemented as a Pins2Pins wrapper (cf. Figure 1), both using and providing the interface. This *POR layer* provides a new NextStates(s) function, which returns a subset of enabled transitions, namely: $\mathit{stubborn}(s) \cap \mathit{en}(s)$. It forwards the other Pins functions. To support the analysis for guard-based partial-order reduction in the POR layer, we introduced four essential extensions to Pins:

- StateLabel additionally exports guards: $\mathcal{G}_T \subseteq L$,
- a $K \times N$ label dependency matrix is added for Ts,
- DM is split into a read and a write matrix representing Rs and Ws, and
- an $M \times M$ do-not-accord matrix is added.

Mainly, the language front-end must do some static analysis to estimate the do-not-accord relation on transitions based on the criteria listed below Definition 7 While Criterium 1 allows the POR layer to estimate the relation without help from the front-end (using Rs and Ws), this will probably lead to poor reductions.

Tailored Necessary Enabling/Disabling Sets. To support necessary disabling sets, we also extend the Pins interface with an optional maybe co-enabled matrix. Without this matrix, the POR layer can rely solely on necessary enabling sets. Both $\mathcal{N}^{\min}$ and $\overline{\mathcal{N}}^{\min}$ can be derived via the refined Pins interface (using Ts and

Ws). In order to obtain the maximal reduction performance, we extend the Pins interface with two more optional matrices, called $\mathcal{N}_g^{\text{PINS}}$ and $\overline{\mathcal{N}}_g^{\text{PINS}}$. The language front-end can now provide more fine-grained dependencies by inspecting the syntax as in Example 3. The POR layer actually uses the following intersections:

$$\mathcal{N}_g := \mathcal{N}_g^{\min} \cap \mathcal{N}_g^{\text{PINS}} \qquad \overline{\mathcal{N}}_g := \overline{\mathcal{N}}_g^{\min} \cap \overline{\mathcal{N}}_g^{\text{PINS}}$$

A simple insight shows that we can compute both $\mathcal{N}_g^{\text{PINS}}$ and $\overline{\mathcal{N}}_g^{\text{PINS}}$ using one algorithm. Namely, for a transition to be *necessarily disabling* for a guard g, means exactly the same as for it to be *necessarily enabling* for the inverse: $\neg g$. Or by example: to disable the guard $pc = 1$, is the same as to enable $pc \neq 1$.

5 Partial-Order Reduction for On-The-Fly LTL Checking

Liveness properties can be expressed in Linear Temporal Logic (LTL) [26]. An example LTL property is $\Box\Diamond p$, expressing that from any state in a trace ($\Box$ = generally), eventually ($\Diamond$) a state s can be reached s.t. $p(s)$ holds, where p is a predicate over a state $s \in S_T$, similar to our definition of guards in Definition 2.

In the automata-theoretic approach, an LTL property φ is transformed into a Büchi automaton $\mathbb{B}_\varphi$ whose ω-regular language $\mathcal{L}(\mathbb{B}_\varphi)$ represents the set of all infinite traces the system should adhere to. $\mathbb{B}_\varphi$ is an automaton $(M_\mathbb{B}, \Sigma, \mathcal{F})$ with additionally a set of transition labels Σ, made up of the predicates, and accepting states: $\mathcal{F} \subseteq S_\mathbb{B}$. Its language is formed by all infinite paths visiting an accepting state infinitely often. Since $\mathbb{B}_\varphi$ is finite, a lasso-formed trace exists, with an accepting state on the cycle. The system M_T is likewise interpreted as a set of infinite traces representing its possible executions: $\mathcal{L}(M_T)$. The model checking problem is now reduced to a *language inclusion* problem: $\mathcal{L}(M_T) \subseteq \mathcal{L}(\mathbb{B}_\varphi)$.

Since the number of cycles in M_T is exponential in its size, it is more efficient to invert the problem and look for error traces. The error traces are captured by the negation of the property: $\neg\varphi$. The new problem is a *language intersection and emptiness* problem: $\mathcal{L}(M_T) \cap \mathcal{L}(\mathbb{B}_{\neg\varphi}) = \emptyset$. The intersection can be solved by computing the synchronous cross product $M_T \otimes \mathbb{B}_{\neg\varphi}$ The states of $S_{M_T \otimes \mathbb{B}_{\neg\varphi}}$ are formed by tuples (s, s') with $s \in S_{M_T}$ and $s' \in S_{\neg\varphi}$, with $(s, s') \in \mathcal{F}$ iff $s' \in \mathcal{F}_{\neg\varphi}$. The transitions in $T_{M_T \otimes \mathbb{B}_{\neg\varphi}}$ are formed by synchronising the propositions Σ on the states $s \in S_{M_T}$. For an exact definition of $T_{M_T \otimes \mathbb{B}_{\neg\varphi}}$, we refer to [34]. The construction of the cross product can be done *on-the-fly*, without computing (*and storing!*) the full state space M_T. Therefore, the NDFS [4] algorithm is often used to find accepting cycles (= error traces) as it can do so on-the-fly as well. In the absence of accepting cycles, the original property holds.

Table 1. POR provisos for the LTL model checking of M_T with a property φ

C2	No $a \in \mathit{stubborn}(s)$ is *visible*, except when $\mathit{stubborn}(s) = \mathit{en}(s)$.
C3	$\nexists a \in \mathit{stubborn}(s)$: $a(s)$ is on the DFS stack, except when $\mathit{stubborn}(s) = \mathit{en}(s)$.

To combine partial-order reduction with LTL model checking, the reduced state space M_T^R is constructed on-the-fly, while the LTL cross product and emptiness check algorithm run on top of the reduced state space [25]. Figure 3 shows the PINS stack with POR and LTL as PINS2PINS wrappers.

To preserve all traces that are captured by the LTL formula, POR needs to fulfill two additional constraints: the *visibility proviso* ensures that traces included in $\mathbb{B}_{\neg\varphi}$ are not pruned from M_T, the *cycle proviso* ensures the necessary fairness. The visible transitions T_{vis} are those that can enable or disable a proposition of φ ($p \in \Sigma$). Table 1 shows sufficient conditions to ensure both provisos (stubborn sets allow the use of the weaker conditions **V** and **L1/L2** [32]). These can easily be integrated in Algorithm 1, which now also requires T_{vis} and access to the DFS stack.

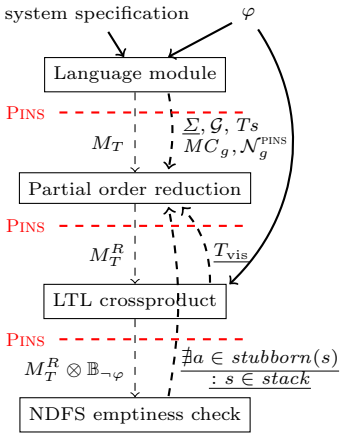


Fig. 3. PINS w. LTL POR

We extend the NEXTSTATES function of PINS with a boolean, that can be set by the caller to pass the information needed for **C3**. For **C2**, we extend PINS with T_{vis}, to be set by the LTL wrapper based on the predicates Σ in φ:

$$T_{\text{vis}}^{\min} := \{t \in T \mid Ws(t) \cap \bigcup_{p \in \Sigma} Ts(p) \neq \emptyset\}$$

Peled [23, Sec. 4.1] shows how to prove correctness. However, this is a coarse overapproximation, which we can improve by inputting φ to the language module, so it can export Σ as state labels, i.e. $\Sigma \subseteq \mathcal{G}$, and thereby obtain $\mathcal{N}/\overline{\mathcal{N}}$ for it:

$$T_{\text{vis}}^{\text{nes}} := \bigcup_{p \in \Sigma} \mathcal{N}(p) \cup \overline{\mathcal{N}}(p)$$

A novel idea is to make this definition dynamic:

$$T_{\text{vis}}^{\text{dyn}}(s) := \bigcup_{p \in \Sigma} \begin{cases} \overline{\mathcal{N}}(p) & \text{if } p(s) \\ \mathcal{N}(p) & \text{if } \neg p(s) \end{cases}$$

Finally, we can improve the heuristic (Section 4.3) to avoid visible transitions:

$$cost'(t,s) = \begin{cases} n^2 & \text{if } t \in en(s) \cap T_{\text{vis}} \text{ and } t \notin \mathcal{T}_s \cup \mathcal{T}_{work} \\ cost(t,s) & \text{otherwise} \end{cases}$$

To summarise, we can combine guard-based partial-order reduction with on-the-fly LTL model checking with limited extensions to PINS: a modified NEXTSTATES function and a visibility matrix $T_{\text{vis}} : T \to \mathbb{B}$. For better reduction, the language module needs only to extend the exported state labels from $\mathcal{G}$ to $\mathcal{G} \cup \Sigma$ and calculate the MC (and $\mathcal{N}^{\text{PINS}}/\overline{\mathcal{N}}^{\text{PINS}}$) for these labels as well.

6 Experimental Evaluation

Experimental Setup The LTSmin toolset implements Algorithm 1 as a language-independent Pins layer since version 1.6. We experimented with BEEM and Promela models. To this end, first the DiVinE front-end of LTSmin was extended with the new Pins features in order to export the necessary static information. In particular, it supports guards, R/W-dependency matrices, the do-not-accord matrix, the co-enabled matrix, and disabling- and enabling sets. Later the Promela front-end SpinS [1] was extended, with relatively little effort.

We performed experiments and indicate performance measurements with LTSmin 2.0[1] and Spin version 6.2.1[2]. All experiments ran on a dual Intel E5335 CPU with 24GB RAM memory, restricted to use only one processor, 8GB of memory and 3 hours of runtime. None of the models exceeded these bounds.

We compared our guard-based stubborn method with the ample set method, both theoretically and experimentally. For the theoretical comparison the same BEEM models were used as in [7] to establish the best possible reduction with ample sets. For the experimental comparison, we used a rich set of Promela models[3], which were also run in Spin with partial-order reduction.

BEEM Models Table 2 shows the results obtained on those models from the BEEM database [22] that were selected by Geldenhuys, Hansen and Valmari [7]. The table is sorted by the best theoretical ample set reduction (best first). These numbers (column ample) are taken from [7, column ample2 Df/Rf]. They indicate the experimentally established best possible reduction that can be achieved with the deadlock-preserving ample set method (without **C2/C3**), while considering conditional dependencies based on full information on the state space.

The amount of reduction is expressed as the percentage of the reduced state space compared to the original state space (100% means no reduction). The next three columns show the reduction achieved by the guard-based stubborn approach, based on necessary enabling sets only (nes), the heuristic selection function (nes+h), and the result of including the necessary disabling sets (nes+h+d).

The results vary a lot. For instance, the best possible ample set reduction in `cyclic_scheduler.1` is far better than the actual reduction achieved with stubborn sets (nes). However, for `cyclic_scheduler.2` the situation is reversed. Other striking differences are `mcs.1` versus `leader_election`. Since we compare *best case* ample sets (using global information) with *actual* stubborn sets (using only static information), it is quite interesting to see that guard-based stubborn sets can provide more reduction than ample sets. One explanation is that the ample set algorithm with a dependency relation based on the full state space (Df/Rf, [7]) is still coarse. However, further comparison reveals that many models yield also better reductions than those using dynamic relations (Dd/Rd, [7]), e.g. `protocols.3` with 7% vs 70%. This prompted us to verify our generated

[1] http://fmt.cs.utwente.nl/tools/ltsmin/
[2] http://spinroot.com
[3] http://www.albertolluch.com/research/promelamodels

Table 2. Comparison of guard-based POR results with [7] (split in two columns)

Model	AMPLE	nes	nes +h	nes h+d
cyclic_scheduler.1	1%	58%	1%	1%
mcs.4	4%	16%	16%	16%
firewire_tree.1	6%	8%	8%	8%
phils.3	11%	14%	16%	16%
mcs.1	18%	87%	85%	85%
anderson.4	23%	58%	46%	46%
iprotocol.2	26%	19%	17%	16%
mcs.2	34%	64%	64%	64%
phils.1	48%	60%	48%	48%
firewire_link.2	51%	24%	21%	19%
krebs.1	51%	94%	93%	93%
leader_election.3	54%	13%	12%	6%
telephony.2	60%	95%	95%	95%
leader_election.1	61%	23%	22%	11%
szymanski.1	63%	68%	65%	65%
production_cell.2	63%	26%	24%	24%
at.1	65%	96%	95%	95%
szymanski.2	66%	66%	64%	64%
leader_filters.2	66%	57%	53%	53%
lamport.1	66%	95%	95%	95%
protocols.2	68%	18%	13%	13%
collision.1	68%	88%	59%	56%

Model	AMPLE	nes	nes +h	nes h+d
driving_phils.1	69%	99%	68%	78%
protocols.3	71%	13%	7%	7%
peterson.2	72%	82%	82%	82%
driving_phils.2	72%	99%	45%	45%
collision.2	74%	75%	40%	39%
production_cell.1	74%	23%	19%	19%
telephony.1	75%	95%	95%	95%
lamport.3	75%	96%	95%	96%
firewire_link.1	79%	42%	37%	33%
pgm_protocol.4	81%	93%	56%	55%
bopdp.2	85%	90%	73%	73%
fischer.1	87%	87%	87%	87%
bakery.3	88%	99%	96%	96%
exit.2	88%	94%	94%	94%
brp2.1	88%	95%	80%	79%
public_subscribe.1	89%	81%	79%	76%
firewire_tree.2	89%	84%	63%	47%
pgm_protocol.2	89%	96%	72%	72%
brp.2	96%	76%	42%	42%
extinction.2	96%	25%	24%	21%
cyclic_scheduler.2	99%	46%	28%	27%
synapse.2	100%	93%	93%	93%

stubborn sets, but we found no violations of the stubborn set definition. So we suspect that either the relations deduced in [7] are not entirely optimal or the POR heuristic of selecting the smallest ample set fails in these cases.

We also investigated the effects of the necessary disabling sets (Sec. 4.2) and heuristic selection (Sec. 4.3). Heuristic selection improves reductions (column nes+h). For instance, for `cyclic_scheduler.1` it achieves a similar reduction as the optimal ample set method. The reduction improves in nearly all cases, and it improves considerably in several cases. Using Necessary Disabling Sets (nes+nds) in itself did not yield an improvement compared to plain nes, hence we didn't include the results in the table. Combined with the heuristic selection, necessary disabling sets provide an improvement of the reduction in some cases (column nes+h+d). In particular, for `leader_election` the reduction doubles again. Also some other examples show a small improvement.

We can explain this as follows: Although nds allows smaller stubborn sets (cf. Example 5), there is no reason why the eager algorithm would find one. Only with the heuristic selection, the stubborn set algorithm tends to favour small stubborn sets, harvesting the potential gain of nds.

We conclude that, the heuristic selection is more important to improve reductions, than the necessary disabling sets. In terms of computation time the situation is reversed: the selection heuristics is costly, but the disabling sets lower the computation time. In the next section, we investigate computation times.

Table 3. Guard-based POR in LTSmin vs ample set POR in Spin (seconds and MB)

	No Partial-Order Reduction						Guard-based POR				Ample-set POR			
					LTSmin	Spin	LTSmin				Spin			
Model	States	$\|S_T\|$	Trans	$\|\Delta\|$	time	time	$\|S_T\|$	$\|\Delta\|$	mem	time	$\|S_T\|$	$\|\Delta\|$	mem	time
garp	48,363,145		247,135,869		166	267	4%	1%	21	68	18%	9%	932	25.2
i-protocol2	14,309,427		48,024,048		28	30	16%	10%	29	31	24%	16%	240	6.0
peterson4	12,645,068		47,576,805		23	17	3%	1%	6	3	5%	2%	37	0.5
i-protocol0	9,798,465		45,932,747		29	38	6%	2%	7	21	44%	29%	362	12.3
brp.prm	3,280,269		7,058,556		6.0	5.6	29%	15%	15	14	58%	39%	161	2.4
philo.pml	1,640,881		16,091,905		9.8	10	5%	2%	1.2	4.8	100%	100%	125	10.7
sort	659,683		3,454,988		2.8	3.8	182	181	0.0	0.3	182	182	0.3	0.0
i-protocol3	388,929		1,161,274		1.0	0.7	14%	7%	0.9	0.9	26%	16%	6.6	0.1
i-protocol4	95,756		204,405		0.5	0.1	28%	18%	0.5	0.6	38%	28%	2.5	0.0
snoopy	81,013		273,781		0.6	0.2	12%	4%	0.2	0.7	17%	7%	1.2	0.0
peterson3	45,915		128,653		0.4	0.0	8%	3%	0.1	0.4	10%	4%	0.5	0.0
SMALL1	36,970		163,058		0.5	0.0	18%	9%	0.1	0.4	48%	45%	0.9	0.0
SMALL2	7,496		32,276		0.4	0.0	19%	10%	0.0	0.4	48%	44%	0.4	0.0
X.509.prm	9,028		35,999		0.4	0.0	10%	4%	0.0	0.4	68%	34%	1.1	0.0
dbm.prm	5,112		20,476		0.4	0.0	100%	100%	0.1	0.5	100%	100%	0.7	0.0
smcs	5,066		19,470		0.4	0.1	17%	7%	0.0	0.4	25%	11%	0.7	0.0

Promela *Models* Additionally, we compared our partial-order reduction results to the ample set algorithm as implemented in Spin. Here we can also compare time resource usage. We ran LTSmin with arguments `--strategy=dfs -s26 --por`, and we compiled Spin with `-O2 -DNOFAIR -DNOBOUNDCHECK -DSAFETY`, which enables POR by default. We ran the `pan`-verifier with `-m10000000 -c0 -n -w26`. To obtain the same state counts in Spin, we had to turn off control flow optimisations (`-o1`/`-o2`/`-o3`) for some models (see `ltsmin/spins/test/`).

Table 3 shows the results. Overall, we witness consistently better reductions by the guard-based algorithm (using nes+h+d). The reductions are significantly larger than the ample set approach in the cases of `garp`, dining philosophers (`philo.pml`) and `iprotocol`. As a consequence, guard-based POR in LTSmin reduces memory usage considerably more than ample-based POR in Spin (Though we included memory use for completeness, it only provides an indirect comparison, due to a different state representation and compression in LTSmin [18]).

On the other hand, the additional computational overhead of our algorithm is clear from the runtimes. This was expected, as the stubborn-set algorithm considers all transitions whereas the ample-set algorithm only chooses amongst the less numerous process components of the system. Moreover, the heuristic search still considers all enabled transitions — we do not select a scapegoat — increasing the search space. Finally, the choice to store information on a guard basis requires our implementation to iterate over all guards of a transition at times. Unfortunately, this cannot be mitigated by combining this information on a transition basis, since enabled guards are treated differently from disabled guards. However, the runtimes never exceed the runtimes of benchmarks without partial-order reduction by a great margin.

Table 4. Reductions ($\%|S_T|$) and runtimes (sec) obtained for LTL model checking

Model	States $\|S_T\|$	LTSMIN ($\%|S_T|$) $T^{\min}_{\text{vis}}$	$T^{\text{nes}}_{\text{vis}}$	$T^{\text{dyn}}_{\text{vis}}$	color	SPIN $\%|S_T|$	LTSMIN (sec) Full	$T^{\min}_{\text{vis}}$	$T^{\text{nes}}_{\text{vis}}$	$T^{\text{dyn}}_{\text{vis}}$	color	SPIN (sec) Full	POR
garp	72,318,749	35%	32.6%	25.4%	3.6%	18.3%	1,162	1,156	1,069	843	135	2,040	127
i-prot.	20,052,267	100%	32.0%	29.3%	28.1%	41.4%	193	598	152	137	132	103	37
leader	89,771,572	94%	0.1%	0.1%	0.1%	1.2%	3,558	9,493	4	4	4	1,390	5

LTL Model Checking To compare the reductions under LTL model checking with SPIN, we used 3 models that were verified for absence of livelocks, using an LTL property $\Box\Diamond$*progress*. Table 4 shows the results of POR with **C2/C3**.

In LTSMIN, we used three implementations of the visibility matrix (see Section 5) and the color proviso [6] (`--proviso=color`). To obtain $T^{\min}_{\text{vis}}$, we defined *progress* with a predicate on the program counter ($Proc.\_pc = 1$). For $T^{\text{nes}}_{\text{vis}}$, we exported an `np_` label through pins and defined $\varphi := \Box\Diamond\neg$`np_`. SPIN also predefines this label, hence we used the same property (though negated [13]).

The results in Table 4 show that approximation $T^{\min}_{\text{vis}}$ is indeed too coarse. Reductions with $T^{\text{nes}}_{\text{vis}}$ improve considerably; the novel dynamic visibility $T^{\text{dyn}}_{\text{vis}}$ and the color proviso provide the best results, also reducing more than SPIN.

7 Conclusions

We proposed guard-based partial-order reduction, as a language-agnostic stubborn set method. It extends Valmari's stubborn sets for transition systems [31] with an abstract interface (PINS) to language modules. It also generalises previous notions of guards [33], by considering them as disabling conditions as well. The main advantage is that a single implementation of POR can serve multiple specification languages front-ends and multiple high-performance model checking back-ends. This requires only that the front-end exports guards, guarded transitions, affect sets, and the do-not-accord matrix ($\mathcal{DNA}$). Optional extensions are matrices MC_g, $\mathcal{N}^{\text{PINS}}$ and $\overline{\mathcal{N}}^{\text{PINS}}$ (computing the latter merely requires negating the guards), which expose more static information to yield better reduction.

We implemented these functions for the DVE and PROMELA front-ends in LTSMIN. It should now be a trivial exercise to add partial-order reduction to the mCRL2 and UPPAAL language front-ends. Since the linear process of mCRL2 is rule-based and has no natural notion of processes, our generalisation is crucial.

We introduced two improvements to the basic stubborn set method. The first uses necessary disabling sets to identify necessary enabling sets of guards that cannot be co-enabled. This allows for the existence of smaller stubborn sets. Most of the reduction power of the algorithm is harvested by the heuristic selection function, which actively favours small stubborn sets.

Compared to the best possible ample set with conditional dependencies, the stubborn set can reduce the state space more effectively in a number of cases.

Compared to Spin's ample set, LTSmin generally provides more reduction, but takes more time to do so, probably because of the additional complexity of the stubborn set method, but also due to overhead in the guard-based abstraction.

Comparing our stubborn set computation against earlier proposals, we see the following. While other stubborn set computation methods require $\mathcal{O}(c|T|)$ [32, Sec. 7.4] using scapegoat selection and resolving the dependencies of *find_nes* arbitrarily (where c depends on the modeling formalism used), our algorithm resolves non-deterministic choices heuristically potentially reducing the search space. It would therefore be interesting to compare our heuristic algorithm to other approaches like the deletion algorithm [35], selecting a scapegoat [35] and the strongly connected components method [32], or one of these combined with the heuristics. This would provide more insight in the trade-off between time spent on finding stubborn sets and state space reductions.

Challenges remain, as not all of LTSmin's algorithmic backends can fulfill the POR layer's requirements. For example, the **C3** proviso relies on a DFS stack, and because DFS can probably not be parallelised efficiently, other methods have to be found. We partly solved this problem for a subset of LTL with the parallel DFS_{FIFO} algorithm [17, end of Sec. 5], but for other parallel algorithms, like CNDFS [5], this is still future work. One benefit for the parallel algorithms is that the heuristic selection algorithm can find small stubborn sets deterministically, which avoids well-known problems with possible re-explorations [15,27].

Acknowledgments. We are grateful to Antti Valmari, Patrice Godefroid and Dragan Bošnački for their useful feedback on this paper.

References

1. F.I. van der Berg and A.W. Laarman. SpinS: Extending LTSmin with Promela through SpinJa. In *PDMC 2012, London, UK*, ENTCS. Spinger, September 2012.
2. S.C.C. Blom, J.C. van de Pol, and M. Weber. LTSmin: Distributed and symbolic reachability. In *CAV*, volume 6174 of *LNCS*, pages 354–359, Berlin, 2010. Springer.
3. Edmund M. Clarke. The birth of model checking. In *25 Years of Model Checking*, pages 1–26. Springer, Berlin, Heidelberg, 2008.
4. C. Courcoubetis, M.Y. Vardi, P. Wolper, and M. Yannakakis. Memory Efficient Algorithms for the Verification of Temporal Properties. In *CAV*, volume 531 of *LNCS*, pages 233–242. Springer, 1990.
5. S. Evangelista, A. Laarman, L. Petrucci, and J. van de Pol. Improved Multi-core Nested Depth-First Search. In *ATVA*, LNCS 7561, pages 269–283. Springer, 2012.
6. S. Evangelista and C. Pajault. Solving the Ignoring Problem for Partial Order Reduction. *STTF*, 12:155–170, 2010.
7. J. Geldenhuys, H. Hansen, and A. Valmari. Exploring the scope for partial order reduction. In *ATVA'09*, LNCS, pages 39–53, Heidelberg, 2009. Springer.
8. P. Godefroid. Using Partial Orders to Improve Automatic Verification Methods. In *CAV*, volume 531 of *LNCS*, pages 176–185. Springer, 1990.
9. P. Godefroid. *Partial-Order Methods for the Verification of Concurrent Systems: An Approach to the State-Explosion Problem*. Springer, Secaucus, NJ, USA, 1996.

10. P. Godefroid and D. Pirottin. Refining dependencies improves partial-order verification methods. In *CAV*, volume 697 of *LNCS*, pages 438–449. Springer, 1993.
11. P. Godefroid and P. Wolper. Using partial orders for the efficient verification of deadlock freedom and safety properties. *FMSD*, 2:149–164, 1993.
12. J.F. Groote, J. Keiren, A. Mathijssen, B. Ploeger, F. Stappers, C. Tankink, Y. Usenko, M. van Weerdenburg, W. Wesselink, T. Willemse, and J. van der Wulp. The mCRL2 toolset. WASDeTT, 2008.
13. G.J. Holzmann. The model checker SPIN. *IEEE TSE*, 23:279–295, 1997.
14. G.J. Holzmann and D. Peled. An Improvement in Formal Verification. In *IFIP WG6.1 ICFDT VII*, pages 197–211. Chapman & Hall, Ltd., 1995.
15. G.J. Holzmann, D Peled, and M. Yannakakis. On Nested Depth First Search. In *SPIN*, pages 23–32. American Mathematical Society, 1996.
16. S. Katz and D. Peled. An efficient verification method for parallel and distributed programs. In *REX Workshop*, volume 354 of *LNCS*, pages 489–507. Springer, 1988.
17. A.W. Laarman and Fárago D. Improved On-The-Fly Livelock Detection. In *NFM*, accepted for publication in LNCS. Springer, 2013.
18. A.W. Laarman, J.C. van de Pol, and M. Weber. Parallel Recursive State Compression for Free. In *SPIN*, LNCS, pages 38–56. Springer, 2011.
19. A.W. Laarman, J.C. van de Pol, and M. Weber. Multi-Core LTSmin: Marrying Modularity and Scalability. In *NFM*, LNCS 6617, pages 506–511. Springer, 2011.
20. W.T. Overman. *Verification of concurrent systems: function and timing*. PhD thesis, University of California, Los Angeles, 1981. AAI8121023.
21. E. Pater. Partial Order Reduction for PINS, Master's thesis, March 2011.
22. R. Pelánek. BEEM: Benchmarks for explicit model checkers. In *Proc. of SPIN Workshop*, volume 4595 of *LNCS*, pages 263–267. Springer, 2007.
23. D. Peled. All from One, One for All: on Model Checking Using Representatives. In *CAV*, pages 409–423. Springer, 1993.
24. D. Peled. Combining partial order reductions with on-the-fly model-checking. In *CAV*, LNCS, pages 377–390, London, UK, 1994. Springer.
25. D. Peled. Combining Partial Order Reductions with On-the-Fly Model-Checking. In *CAV*, volume 818 of *LNCS*, pages 377–390. Springer, 1994.
26. Amir Pnueli. The temporal logic of programs. In *FOCS*, pages 46–57. IEEE Computer Society, 1977.
27. S. Schwoon and J. Esparza. A Note on On-the-Fly Verification Algorithms. In *TACAS*, volume 3440 of *LNCS*, pages 174–190. Springer, 2005.
28. A. Valmari. Error Detection by Reduced Reachability Graph Generation. In *APN*, pages 95–112, 1988.
29. A. Valmari. Eliminating Redundant Interleavings During Concurrent Program Verification. In *PARLE*, volume 366 of *LNCS*, pages 89–103. Springer, 1989.
30. A. Valmari. A Stubborn Attack On State Explosion. In *CAV*, LNCS, pages 156–165. Springer, 1991.
31. A. Valmari. Stubborn Sets for Reduced State Space Generation. In *ICATPN/APN'90*, pages 491–515, London, UK, 1991. Springer.
32. A. Valmari. The State Explosion Problem. In *LPN*, pages 429–528. Springer, 1998.
33. A. Valmari and H. Hansen. Can Stubborn Sets Be Optimal? In J. Lilius and W. Penczek, editors, *ATPN*, volume 6128 of *LNCS*, pages 43–62. Springer, 2010.
34. M.Y. Vardi and P. Wolper. An Automata-Theoretic Approach to Automatic Program Verification. In *LICS*, pages 332–344. IEEE, 1986.
35. Kimmo Varpaaniemi. *On the Stubborn Set Method in Reduced State Space Generation*. PhD thesis, Helsinki University of Technology, 1998.