DIRECTIONAL SENSITIVITY OF A THREE DIMENSIONAL PARTICLE VELOCITY SENSOR

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Abstract — *The omni-directional sensitivity pattern of an integrated three dimensional acoustic sensor, composed of four hot-wire particle velocity sensors is analyzed theoretically and experimentally. We investigate the influence of the presence of the probe surface on the direction of the measured particle velocity measured by the sensors, and a description of the gas flow profile around the integrated sensor is presented.*

Key Words: *particle velocity, acoustics, MEMS*

I INTRODUCTION

For the full establishment of sound fields, both pressure and particle velocity have to be determined, since an acoustic wave is characterized by both pressure, which is a scalar quantity, and particle velocity, being a vector. For the instantaneous and directional acoustic measurement, an integrated three-dimensional particle velocity sensor was realized [1,2]. By integration of four Microflowns [3,4,5] on a single silicon die it has been possible to realize a 3-D sensitive acoustic sensor with a very good sensor reproducibility in terms of directional sensitivity, and, by virtue of the small sensor to sensor distance, a high accuracy for single point measurements. The design of this monolithically integrated 3D sensor permits a single wafer fabrication process. The integrated three dimensional particle velocity sensor has opened the way both for directional measurement of acoustic fields and the localization of noise sources. Using the power spectral density of each particle velocity sensor and calculating the cross correlation signals of different pairs of the sensors, thereby eliminating the uncorrelated noise level [6], an accurate noise source localization is achievable.

Because of the importance of a very accurate determination of the direction of the involved particle velocities, a good understanding of the directional sensitivity of the probe is requisite. An important issue is the influence of the printed circuit board acts on the flow profile around the sensor wires. The particle velocity as measured by the sensor deviates from the velocity of the acoustic wave at infinity due to this obstacle. In this paper we analyze the directional sensitivity pattern of the sensor wires and obtain a model for the flow profile around the 3-D probe.

II SENSOR DESIGN

The device as shown in Fig. 1 consists of four sensors (each composed of two thin wires), mutually perpendicular and at angles of 45 degrees with the chip surface. This configuration provides at least two signals in any direction [2] and permits the calculation of cross spectral densities of two of these signals, thus reducing the uncorrelated noise sources [6]. The fabrication process out of one single wafer includes the subsequent deposition of silicon nitride, patterning of photo-resist, platinum deposition and etching [1], and, making use of both the upper and lower side of the wafer, it results in a very thin device of 250 μm thickness. The length and width of the sensor (*x*- and *z*-dimensions in Fig.1) are 5 mm.

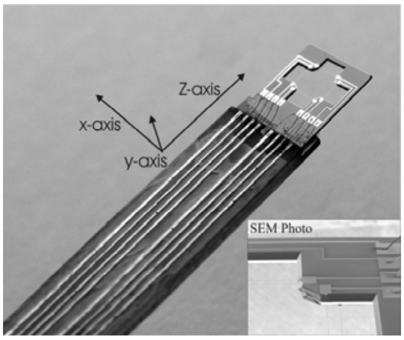

Figure 1. The integrated three-dimensional particle velocity sensor, composed of four pairs of sensor wires.

III THEORY

Let us approximate the geometry of interest by a long sheet of finite width $2c$, close to which the sensor is located. See figure 1. Since $w << 2c << l$, we assume the sheet to be infinitely thin and infinitely long. The velocity distribution in the moving liquid then depends on only two coordinates (x and y) and the flow becomes two-dimensional. For a two-dimensional, incompressible fluid, it is convenient to express the velocity in terms of the stream function ψ, defined by

$$v_x = \frac{\partial \psi}{\partial y}, \qquad v_y = -\frac{\partial \psi}{\partial x} \qquad (1.)$$

The streamlines of the flow are formed by lines of constant ψ. The flow of interest is potential (irrotational) so that we can define the velocity potential ϕ, related to ψ as follows:

$$v_x = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}, \qquad v_y = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \qquad (2.)$$

which is equivalent [7,8] to defining the complex potential w

$$w = \phi + i\psi \qquad (3.)$$

to be an analytic expression of the complex argument $z = x + i\,y$. Solving the Navier-Stokes equations now reduces to [7] solving

$$\Delta\phi = 0 \qquad (4.)$$

for the current geometry with the appropriate boundary conditions.

To calculate the stream function for a flow impinging at a certain angle on a plane lamina of length $2c$, we introduce elliptic coordinates ξ, η, connected with the normal Cartesian coordinates x,y , and consider the plane as the limiting case of an elliptic cylinder of semi-axes a and b. Define

$$x + iy = c\cosh(\xi + i\eta) \qquad (5.a)$$

or

$$x = c\cosh\xi\cos\eta, \quad y = c\sinh\xi\sin\eta \qquad (5.b)$$

where ξ may be supposed to range from 0 to ∞, and η from 0 to 2π.

The curves $\xi = \text{constant}$ are the ellipses $x^2/(c^2\cosh^2\xi) + y^2/(c^2\sinh^2\xi) = 1$, while the curves $\eta = \text{constant}$ are the hyperbolas $x^2/(c^2\cos^2\eta) - y^2/(c^2\sin^2\eta) = 1$, with common foci ($\pm\,c$, 0). See figure 3.

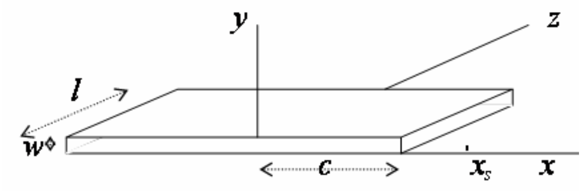

Figure 2. The model geometry

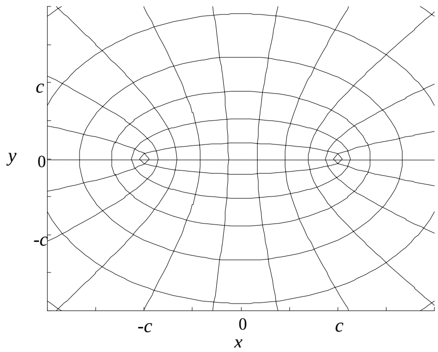

Figure 3. The elliptic coordinate system (ξ, η) according to Eq.5. The curves of constant ξ are ellipses while curves of constant η are hyperbolas with common foci (± c, 0).

In this coordinate system is included the ellipse whose parameter is ξ_0 and the semi-axes a and b of the ellipse are then given by

$$a = c\cosh\xi_0, b = c\sinh\xi_0 \qquad (6.)$$

The approach now is to calculate the fluid motion produced in an infinite liquid by an elliptic cylinder of semi-axes a and b, moving parallel to the greater axis with velocity U. We then do the same for a motion parallel to the minor axis with velocity V, sum both solutions, and then let the minor axis $b \rightarrow 0$, so that the ellipse reduces to a straight line joining the foci.

If the elliptic cylinder moves parallel to its greater axis with velocity U, the boundary condition on its surface is

$$\psi = -Uy + \text{const} \qquad (7.)$$

A solution of Eq. 4. for $w = w\,(z)$ can be found to be, for this geometry [8]

$$\phi + i\psi = Ce^{-(\xi + i\eta)} \qquad (8.)$$

where C is some real constant. This means that

$$\psi = -Ce^{-\xi}\sin\eta \qquad (9.)$$

On the boundary, the elliptic surface given by $\xi = \xi_0$, we thus have

$$Ce^{-\xi_0} = Uc\sinh\xi_0 \qquad (10.)$$

Solving for C gives $C = Ub\left(\frac{a+b}{a-b}\right)^{1/2}$, so that the solution for ψ becomes

$$\psi = -Ub\left(\frac{a+b}{a-b}\right)^{1/2} e^{-\xi} \sin\eta \qquad (11.)$$

If the motion of the cylinder were parallel to its minor axis, this expression for ψ would be

$$\psi = Va\left(\frac{a+b}{a-b}\right)^{1/2} e^{-\xi} \cos\eta \qquad (12.)$$

To find the motion of the elliptic cylinder with velocity components both U and V, one must add both Eqs. (11.) and (12.), and to describe the motion relative to the cylinder, we add to this expression for ψ:

$$Uy - Vx = c(U \sinh\xi \sin\eta - V \cosh\xi \cos\eta) \qquad (13.)$$

Letting $b \rightarrow 0$, the final expression becomes, for a (infinitely thin) plate of length $2c$,

$$\psi = c \sinh\xi (U \sin\eta - V \cos\eta) \qquad (14.)$$

For an acoustic wave with a particle velocity u_0 impinging on the plate at an angle α with the x-axis, we can write

$$U = u_o \cos\alpha, \qquad V = u_0 \sin\alpha \qquad (15.)$$

so that we obtain

$$\psi = cu_0 \sinh\xi (\cos\alpha \sin\eta - \sin\alpha \cos\eta) \qquad (16.)$$

Transforming back to Cartesian coordinates, and calculating v_x and v_y in the point where the sensor is located, $(x_s, 0)$, one finds

$$v_x(x_s, 0) = \frac{\partial\psi}{\partial y} = u_0 \cos\alpha$$
$$v_y(x_s, 0) = -\frac{\partial\psi}{\partial x} = u_0 \sin\alpha \frac{x_s}{\sqrt{x_s^2 - c^2}} \qquad (17.)$$

The angle γ of the velocity at the sensor location $(x_s, 0)$, defined as $\tan\gamma = v_y / v_x$, becomes therefore simply

$$\gamma = \arctan\left(\frac{x_s \tan\alpha}{\sqrt{x_s^2 - c^2}}\right) \qquad (18.)$$

In fig. 5, the angle γ is plotted as a function of the angle α of the incoming wave. It is seen that especially at points close to the edge of the plate, the measured direction γ deviates significantly from the angle of the incoming wave.

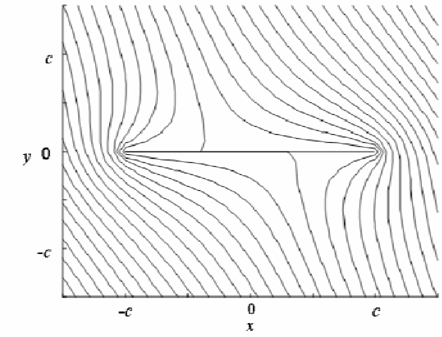

Figure 4. Streamlines of the flow around a plate of width $2c$, where the velocity at infinity makes an angle $\alpha = \pi/3$ with the x-axis. The streamlines are lines of constant ψ with ψ from Eq.(16.).

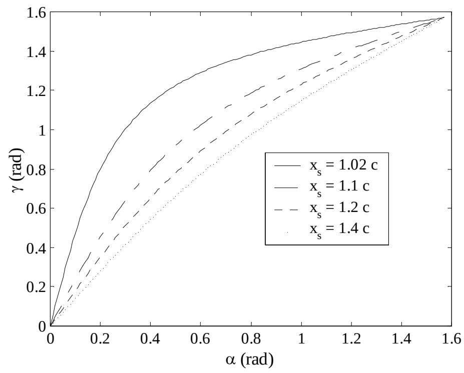

Figure 5. The angle γ of the velocity at the sensor location $(x_s, 0)$, as a function of the angle α of the incoming wave, with x_s as a parameter.

IV EXPERIMENTS

The device was placed in a measurement set up allowing rotation around the x- and z-axis (Fig.1) and subject to a 1.7 kHz sound source. As Fig. 6 shows, the measured sensitivity pattern of sensor 3 deviates from the theoretically ‘figure-of-eight’ response for a sensor in free space. Channel 3 corresponds to the sensor nearest to the printed circuit board; the deviation is almost 10 degrees. A similar behavior is observed in computational fluid dynamical simulations: for oblique sound waves with a small angle with the x-z-plane, the flow is considerably bent around the probe. See fig. 8.

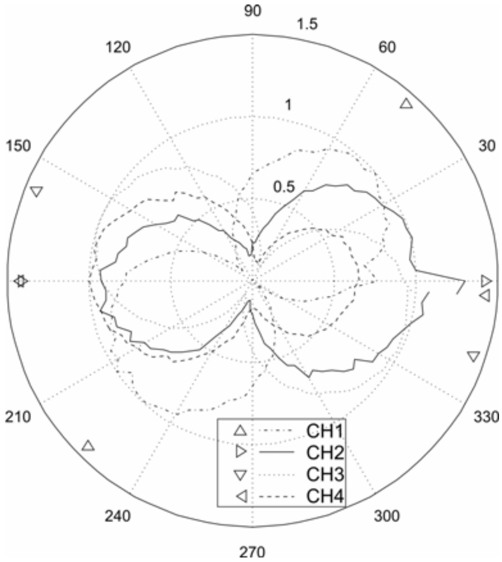

Figure 6. Sensitivity pattern of the sensors at 1.7kHz, rotated around the *x*-axis. Triangles show the measured sensitivity directions. Ideally, signals of channel 2 and 4 have directions of 0 and 180 degrees, channel 1 at 45 and 225, channel 3 at 135 and 315 degrees.

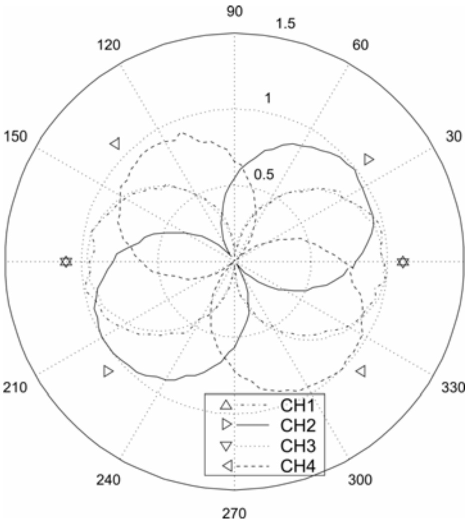

Figure 7. Sensitivity pattern of the sensors at 1.7kHz, rotated around the *z*-axis. Triangles show the measured sensitivity directions. Theoretically for sensors in free space, all channels should yield a 45 degrees angle with the *x*- or *y*-axis.

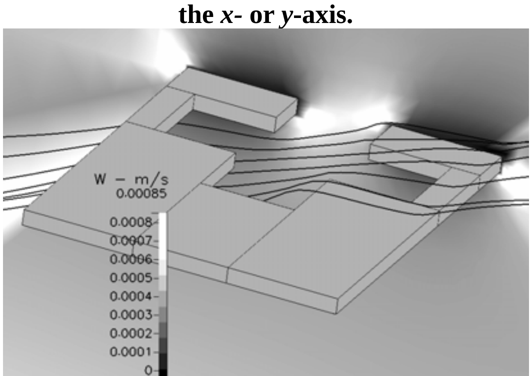

Figure 8. Visualization of results from computational fluid dynamic calculations. Streamlines are 'bent' around the edges of the probe surface.

V CONCLUSIONS

The influence of the printed circuit board that forms the plane in which the four pairs of sensor wires lay, on the omni-directional sensitivity of the integrated three-dimensional particle velocity sensor has been investigated. We developed a model describing the two-dimensional fluid flow around a plate of finite width and found that at points close to the edges, the direction of the fluid flow is considerably influenced. A similar streamline pattern was found in computational fluid dynamical simulations on the probe. The theoretic model is found to be appropriate to describe this streamline pattern and the angle-dependence of the sensitivity. Therefore, the obtained results allow processing of the different sensor signals in which this angle dependence is corrected for.

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