Perturbational evaluation of bend mode phase shifts for the tuning of cylindrical microresonators

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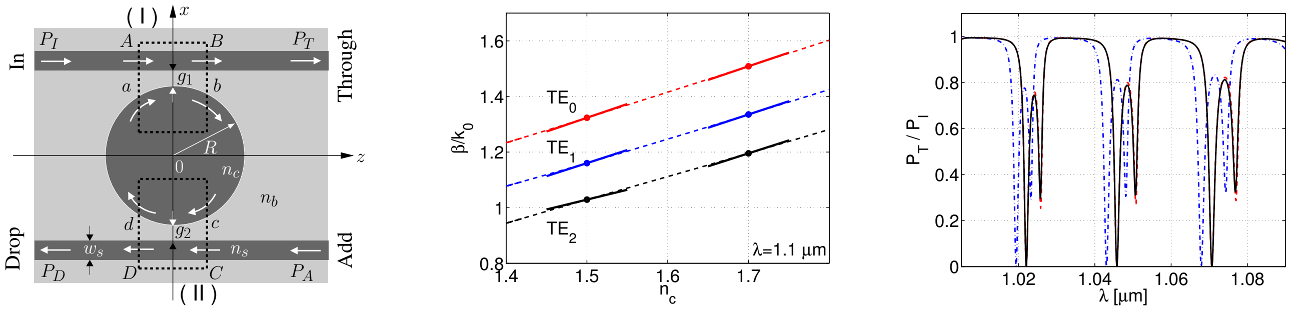
Compact cylindrical microresonators are at present discussed as building blocks for large-scale integrated optical components. The most common ringresonator model [1, 2] represents the optical fields in the cavity in terms of frequency-domain modes of bent waveguides or curved dielectric interfaces; resonances occur if specific phase relations for a complete cavity-round-trip are satisfied.

For the application of microresonator elements as tunable wavelength filters, suitable materials are introduced that permit to change the refractive index of the cavity core slightly by external mechanisms like electro- or thermo-optic effects. While expressions for the induced changes of propagation constants of modes supported by straight waveguides are quite well known (cf. e.g. [3]), in this contribution we propose and evaluate quite similar perturbational expressions for phase shifts of 2-D bend modes or whispering gallery modes. The derivation can be based on reciprocity techniques or a variational principle, respectively. The formulas are applicable to localized perturbations of the radial permittivity profile; complications due to nonconvergent integrands do not arise [4]. Extension to 3-D configurations should be straightforward.

When applied to given cavity modes of a resonator configuration, the phase shift expressions allow to evaluate analytically the wavelength tuning range for the respective resonances. Further we consider the use of the perturbational expressions in combination with a semi-analytical 2-D model for circular microresonators, based on a spatial frequency-domain coupled mode theory description for the interaction between the cavity and the bus waveguides [5, 6]. Within certain limits, the phase-shift formulas permit to predict directly how the tuning affects the entire wavelength spectrum.



Left: Schematic microresonator representation. A symmetrical multimodal 2-D setting with parameters $R = 5\,\mu$*m,* $n_s = 1.5$, $w_s = 0.4\,\mu$*m,* $n_b = 1.0$, $g_1 = g_2 = 0.2\,\mu$*m. Center: Effective mode indices* β/k_0 *of the three lowest order whispering gallery modes of the cavity disk, versus the cavity refractive index* n_c*. The tangential line segments indicate the slope of the curves according to the perturbational phase shift expressions, evaluated at* $n_c = 1.5$ *and* $n_c = 1.7$*. Right: Microresonator spectra, the relative throughput power* P_T/P_I *versus the vacuum wavelength* λ*. Dash-dotted line: Spectrum for the unperturbed cavity with* $n_c = 1.5$*. Continuous curve: Results for a perturbed resonator with* $n_c = 1.504$*, as predicted by the perturbational procedure outlined above. Dashed line (mostly shadowed): The spectral response according to a direct computation for the perturbed configuration.*

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