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ONE-DIMENSIONAL ACOUSTIC MODELING OF THERMOACOUSTIC INSTABILITIES

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Abstract

In this paper the acoustic stability of a premixed turbulent natural gas flame confined in a combustor is investigated. Specifically when the flame is operated in a lean premixed mode, the thermoacoustic system is known to exhibit instabilities. These arise from a feedback mechanism between the oscillatory flow and heat release rate perturbations in the flame and often lead to large amplitude pressure and velocity perturbations in the combustor. The acoustics of the combustor are described with a one-dimensional transfer matrix method. The feedback mechanisms that can cause instabilities are included in this method. The (complex) frequency for which the determinant of the transfer matrix goes to zero indicates an instability. An important factor in the one-dimensional acoustic model is the transfer function between the oscillatory flow and heat release rate perturbations. This transfer function is obtained from a well-stirred reactor dynamic combustion model. Results show that the one-dimensional acoustic model in combination with a well-stirred reactor model is able to describe realistic gas turbine stability behaviour.

INTRODUCTION

In modern gas turbine designs, low NO_x emissions are achieved by introducing lean premixed techniques. The downside is a higher risk of thermoacoustic instabilities, which arise from a feedback mechanism between oscillatory flow and heat release rate perturbations of the flame. These heat release rate perturbations are directly related to an acoustic mass flow source and will therefore excite an acoustic field. When

a thermoacoustic instability occurs, the amplitude of the acoustic oscillations in the combustion chamber grows until it is limited by non-linear effects. These large amplitude acoustic oscillations significantly reduce lifetime and regions of operability of the combustion system. Gas turbine acoustics are usually modeled with one-dimensional acoustic approaches.⁴ In this paper a convenient one-dimensional acoustic model in combination with a combustion model will be used in order to describe the acoustic stability of a combustion system.

The one-dimensional acoustic transfer matrix model will be presented first. This is followed by a description of a well-stirred reactor combustion model from which a flame transfer function is obtained. This transfer function is needed in the acoustic model to account for flame dynamics. Finally both models will be coupled and results will be shown.

ACOUSTIC MODEL

Non-viscid harmonic acoustic wave propagation in gases is described by the Helmholtz equation. For a long prismatic tube with rigid walls the Helmholtz equation reduces to its one-dimensional form which has the solution:

$$p(x) = \hat{p}_A e^{ikx} + \hat{p}_B e^{-ikx} \quad (1)$$

where \hat{p}_A and \hat{p}_B are the (complex) amplitudes of the acoustic pressure waves traveling with the mean speed of sound c_0 in the negative and positive x -direction, respectively. The wave number k is defined as the ratio between the angular frequency and the speed of sound, i.e. $k = \omega/c_0$. Subsequently, the linearised momentum equation can be used to obtain the velocity perturbation from the solution for the pressure perturbation $p(x)$ as:

$$u(x) = -(\rho_0 c_0)^{-1} (\hat{p}_A e^{ikx} - \hat{p}_B e^{-ikx}) \quad (2)$$

in which ρ_0 is the mean density of the gas.

Now consider a prismatic tube J , shown in figure 1. At both ends of this tube ($i = 1, 2$) the acoustic pressure p_i^J , acoustic velocity u_i^J and acoustic mass flow $Q_i^J = \pm A^J \rho_0 u_i^J$ are indicated. Note the convention for the direction of the mass flow Q_i^J . This convention is used to obtain a symmetric element matrix.

With equation 1 the (complex) pressure amplitudes \hat{p}_A^J and \hat{p}_B^J can now be written as a function of p_1^J and p_2^J and inserted into equation 2. From the resulting velocity

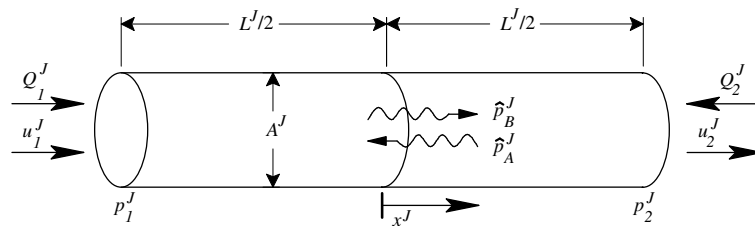


Figure 1: Symbolic conventions for the prismatic tube J .

perturbations, the mass flows Q_i^J can be calculated in terms of p_i^J . Subsequently the element matrix for a prismatic tube can be written as:⁸

$$\frac{A^J}{c_0 i \sin(kL)^J} \begin{bmatrix} \cos(kL)^J & -1 \\ -1 & \cos(kL)^J \end{bmatrix} \begin{Bmatrix} p_1^J \\ p_2^J \end{Bmatrix} = \begin{Bmatrix} Q_1^J \\ Q_2^J \end{Bmatrix} \quad (3)$$

When more of these tubes are coupled to each other, a system matrix $[M]$ can be filled with the element matrices of these tubes. This system matrix is a symmetric $n \times n$ matrix, where n is the number of coupling points (nodes). Acoustic boundary conditions can easily be included in the system matrix. When sufficient boundary conditions are imposed, the unknown pressure perturbation p at each node can be solved by inverting the system matrix, i.e. $\{p\} = [M]^{-1}\{Q\}$.

A thermoacoustic source

In a gas turbine combustion chamber, the thermoacoustic source is the flame. The main sound output from a flame results from the fluctuating heat release rate q .¹ A heat release rate can be related to an acoustic mass flow according to:

$$Q = \frac{\gamma - 1}{c_0^2} q \quad (4)$$

where γ is the ratio of specific heats. It is assumed that this acoustic source is concentrated in one point, i.e. in one node of the acoustic model.

A heat release rate perturbation can be caused by different processes. A well-known cause are fluctuations in the equivalence ratio $\bar{\phi}^\dagger$, which originate where fuel is mixed with air.^{2,3} Acoustic fluctuations at this point modulate the amount of fuel and/or air that is supplied for mixing. As a result, the equivalence ratio will fluctuate. Subsequently, this fluctuation convects to the flame front where it causes a heat release rate perturbation.

An effect of the *mean* equivalence ratio is that it can amplify the acoustic effect of a heat release rate perturbation. In the factor $\frac{\gamma-1}{c_0^2}$ in equation 4 both γ and c_0 are functions of the flame temperature, which is directly related to the mean equivalence ratio of the flame. When the factor is evaluated at the adiabatic flame temperature belonging to a certain equivalence ratio, it can be shown that for natural gas the factor between the acoustic mass flow source and the heat release rate perturbation increases with almost 70% when the mean equivalence ratio is lowered from 1 to 0.5. This implies that for an equal heat release rate perturbation input, a lean flame (low $\bar{\phi}$) causes a relatively large acoustic perturbation compared to a rich flame.

Assuming that the heat release rate perturbation is caused by an equivalence ratio fluctuation, the acoustic mass flow source can be obtained from:

$$\frac{Q}{\bar{Q}} = \frac{\gamma - 1}{c_0^2} h_f \bar{\phi} e^{-i\omega\tau_{conv}} \quad (5)$$

[†]The mean equivalence ratio $\bar{\phi} = \frac{\bar{Q}_f/\bar{Q}_a}{[Q_f/Q_a]_{st}}$, where \bar{Q} is the mean mass flow of fuel (f) or air (a) and the subscript *st* indicates the stoichiometric situation (i.e. exactly enough air for complete combustion).

in which h_f [J/kg] is the flame transfer function, i.e. the ratio between a specific heat release rate perturbation and an equivalence ratio perturbation. Furthermore, the convective time delay τ_{conv} is accounted for by the exponential factor. Assuming small perturbations around a mean value and linearising the result gives the equivalence ratio fluctuation as a function of the acoustic mass flow perturbations of the fuel and air supply:

$$\phi \approx \bar{\phi} \left(\frac{Q_f}{Q_f} - \frac{Q_a}{Q_a} \right) \quad (6)$$

The acoustic mass flow perturbations can subsequently be written as a function of the pressures at the nodes 1 (beginning of the air supply tube, see figure 2), 2 (beginning of the fuel supply tube) and 3 (mixing point), which results in the following expression for the thermoacoustic mass flow source:

$$\frac{Q}{\bar{Q}} = \frac{\gamma - 1}{c_0^2} h_f \bar{\phi} e^{-i\omega\tau_{conv}} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}^T \begin{bmatrix} -\frac{A^I}{Q_a c_0^I i \sin(kL)^I} \\ \frac{A^{II}}{Q_f c_0^{II} i \sin(kL)^{II}} \\ \frac{A^I \cos(kL)^I}{Q_a c_0^I i \sin(kL)^I} - \frac{A^{II} \cos(kL)^{II}}{Q_f c_0^{II} i \sin(kL)^{II}} \end{bmatrix} \quad (7)$$

This expression can be inserted in the system matrix [M] by subtracting it from the coefficients of the involved pressures p_1 , p_2 and p_3 . Note that all the mean values in the expression have to be known, as well as the flame transfer function h_f . The latter will be obtained from a well-stirred reactor model in the next section.

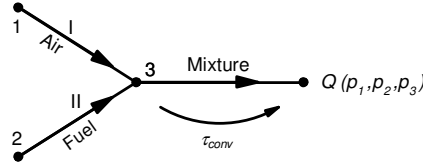


Figure 2: One-dimensional acoustic model of the thermoacoustic source.

When a thermoacoustic source is present in the system, the system might be unstable. The instabilities of the system can be found by determining the complex frequency $\omega_r + i\omega_i$ for which $|\mathbf{M}| = 0$. The real part ω_r indicates the frequency of the thermoacoustic oscillation and the imaginary part ω_i indicates its growth rate. A time dependence of $e^{i\omega t}$ was assumed, so when $\omega_i < 0$ the oscillation grows with time.

CHEMICAL MODEL

A well-stirred reactor (WSR) model will be used to obtain the flame transfer function $h_f = q/\phi$. The WSR is a simplified combustion model and its main assumption is that the properties throughout the reactor are uniform. This includes temperature, pressure and species mass fractions. The WSR takes into account the chemical reactions that can affect the species concentration and temperature. In addition to the chemical mechanism, the reactor is characterised by the reactor volume and the mass flow rate.

The conservation equations for mass and energy are derived assuming a one-step reaction mechanism.⁷ The resulting non-linear equations are solved using an explicit forward Euler time-stepping procedure.

To obtain the flame transfer function from the WSR model, a transient calculation can be performed in which an inlet quantity is fluctuated in a harmonic way.⁶ However, a calculation has to be performed for every single frequency, which is very CPU intensive. A much more attractive way is linearising the non-linear ODE's and using a state-space formulation to obtain the flame transfer function for every desired frequency. This latter procedure is used in the remainder of this paper.

In figure 3(a) the absolute value of the flame transfer function is plotted against a Strouhal number $St = f\tau$, in which f is the frequency and τ the reactor residence time. Moreover, the phase of the flame transfer function is depicted in figure 3(b). By plotting the flame transfer function as a function of St , the cross-over frequencies are independent of the reactor volume and inlet mass flow. It can be seen that the reactor acts as a low-pass filter.

Because the steady-state heat release rate is a linear function of the mean equivalence ratio, the low-frequency response of h_f should be equal for all equivalence ratio's. The small difference noticed in figure 3(a) is due to the one-step reaction mechanism, which is less accurate for lower $\bar{\phi}$.

Around the frequency that corresponds to the residence time τ a larger deviation between both curves is seen. Around this frequency the perturbations in the heat release rate at the reactor exit (which determines the reactor temperature) and equivalence ratio fluctuations at the entrance of the reactor are in phase, causing the flame transfer function to increase. Although not much, the effect is stronger for lower $\bar{\phi}$, resulting in higher values for $|h_f|$. The effect can be amplified by the presence of a re-

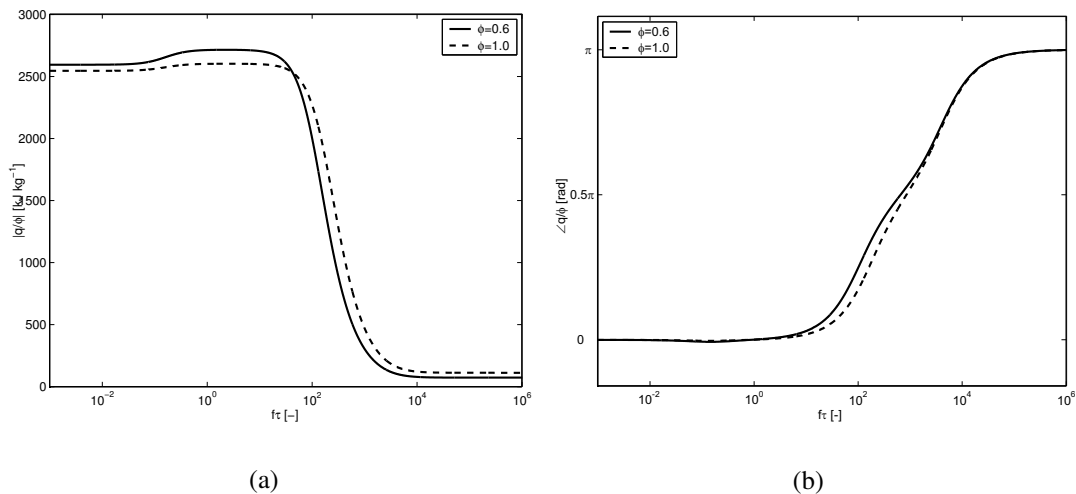


Figure 3: Absolute value (a) and phase (b) of the flame transfer function obtained from the state-space formulation of the WSR model.

circulation zone in which perturbations in the heat release rate return to the combustor entrance. When these perturbations are in phase with equivalence ratio fluctuations an increased flame transfer function can be expected. Similar effects were measured by other authors.⁵

The absolute value of the flame transfer function decays rapidly when the frequency is higher than the inverse combustion time scale, i.e. $|\omega_F/\rho|^{-1}$. In this higher frequency range, the transfer function between ϕ and q is almost zero, so the combustion system will less suffer from thermoacoustic instabilities in this range.

COUPLED MODEL

The acoustic model has been coupled to the WSR model in order to include flame dynamics. As an example, a very basic representation of a combustion system is considered (figure 4). In this model an air supply tube (element I) and a fuel supply tube (element II) come together at node 3, where both streams are mixed. Subsequently, the mixture is transported to the flame front (node 4), where the thermoacoustic source is located. A duct downstream of the source represents the remainder of the combustion chamber. The properties of the elements are listed next to the model. The convective time delay τ_{conv} was determined from the flow speed in element III and its length.

The temperature in the tubes influences the speed of sound. It is assumed that two zones exist in the model, i.e. a hot zone downstream of the thermoacoustic source, and a cold zone upstream of the source. The cold zone has a constant temperature of 300 K. The temperature in the hot zone is determined by the flame temperature, which is obtained from the WSR model.

When there is a system surrounding this basic combustion system, its acoustic influence can be represented by imposing frequency-dependent impedances on the boundaries. Two extreme cases are considered here, namely anechoic boundaries and echoic boundaries. In real gas turbine applications, the acoustic boundaries are usually close to echoic. The two cases are evaluated at different mean equivalence ratio's (carried out by changing the mean fuel inlet flow). For each mean equivalence ratio, the steady-state WSR model is solved. Subsequently, the flame transfer function is obtained from a state-space formulation. This flame transfer function is used in the acoustic model to determine the complex frequencies for which the determinant of the system matrix approaches zero. Results of these calculations are shown in figures 5(a) and (b) for anechoic and echoic boundaries, respectively.

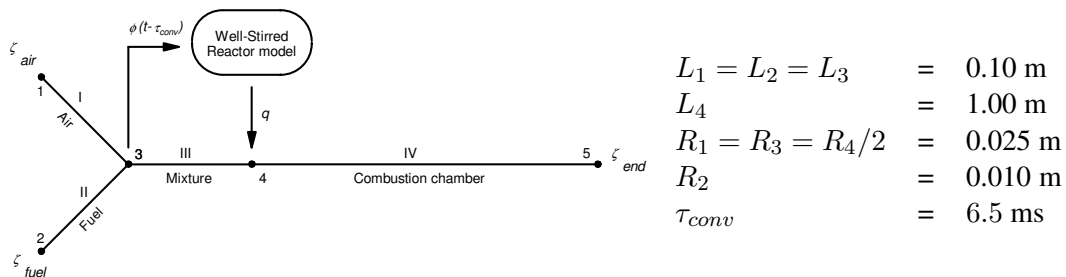


Figure 4: Acoustic model coupled with the flame dynamics model.

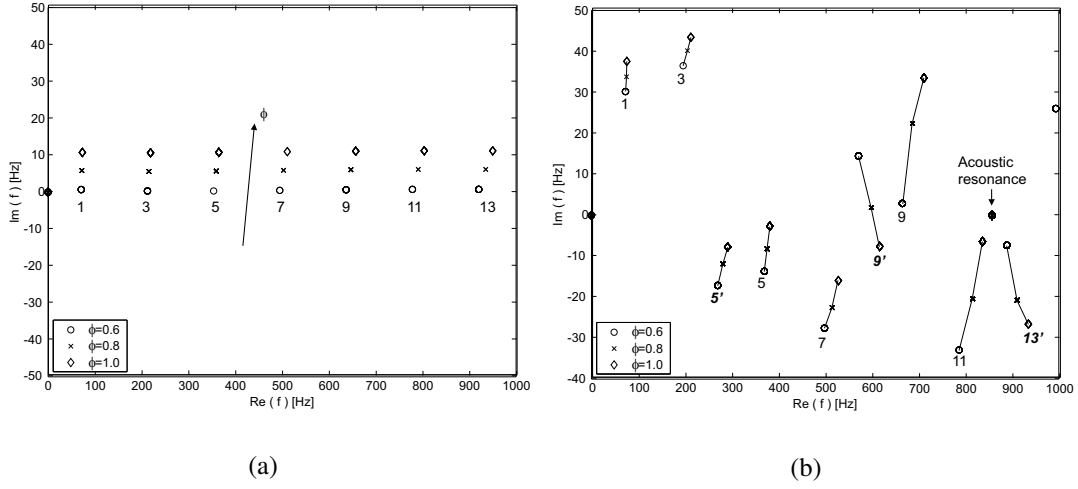


Figure 5: Complex frequencies for which $|\mathbf{M}| = 0$ for different $\bar{\phi}$. The acoustic model depicted in figure 4 is used with anechoic (a) and echoic (b) boundaries.

The real part of the frequency f in figures 5(a) and (b) can also be determined from the time delays and phase shifts involved in the thermoacoustic feedback loop:

$$\tau_{conv} + \frac{\angle h_f}{2\pi f} + \frac{\angle \frac{\phi}{Q}}{2\pi f} = \frac{n}{2f} \quad n = 1, 3, 5... \quad (8)$$

in which $\angle h_f$ and $\angle \frac{\phi}{Q}$ are the frequency-dependent phase of the flame transfer function and the phase of the transfer function between an equivalence ratio fluctuation at node 3 and the thermoacoustic mass flow source Q at node 4, respectively. For the anechoic system, $\angle \frac{\phi}{Q}$ can be determined from the acoustic time delay, i.e. $\angle \frac{\phi}{Q} = 2\pi f \frac{L_3}{c_{cold}}$. For the echoic case this is no longer true though. Standing waves will now occur in the combustion system. As a result, the factor $\angle \frac{\phi}{Q}$ in equation 8 only takes the values 0 or π , i.e. the fluctuation ϕ caused by the acoustic mass flow source Q is either in or out of phase with it.

In figures 5(a) and (b) the mode number n of equation 8 is denoted with every group of similar modes. When $\angle \frac{\phi}{Q} = \pi$, the mode number has an apostrophe. In figure 5(b) three of these modes occur. It is also observed here that depending on the mode, the stability either decreases (decreasing $\text{Im}(f)$) or increases when $\bar{\phi}$ is lowered. For the anechoic case the stability always decreases when $\bar{\phi}$ decreases. The overall stability is seen to decrease for both cases, which is also observed in real combustion systems.

The relative contributions of the three terms in equation 8 are nearly equal for all modes of the anechoic system. A very small contribution of the flame transfer function phase shift $\angle h_f$ is noted. The contribution of the acoustic phase shift $\angle \frac{\phi}{Q}$ is somewhat larger. However, the convection time delay τ_{conv} is mostly determining the frequency at which the instability occurs. This is also true for the echoic case.

CONCLUSIONS

A convenient one-dimensional acoustic model including a thermoacoustic source was used to model the acoustics of a combustion system. Flame dynamics were included using a well-stirred reactor combustion model. Both models were coupled and the instabilities of this coupled model were calculated. The model predicts a decreasing overall system stability when the mean equivalence ratio is decreased, which is also seen in real combustion systems. This effect is caused by the flame dynamics and by the factor between the heat release rate perturbation and the acoustic source.

REFERENCES

- [1] D. G. Crighton, A. P. Dowling, J. E. Ffowcs Williams, M. Heckl, and F. G. Leppington. *Modern methods in analytical acoustics*. Springer-Verlag, London, 1992.
- [2] D. A. Hobson, J. E. Fackrell, and G. Hewitt. Combustion instabilities in industrial gas turbines – measurements on operating plant and thermoacoustic modeling. *ASME*, 122, 2000.
- [3] S. Hubbard and A.P. Dowling. Acoustic resonances of an industrial gas turbine combustion system. *Journal of engineering for gas turbines and power*, 123(4):766–773, October 2001.
- [4] U. Kruger, J. Huren, S. Hoffmann, W. Krebs, P. Flohr, and D. Bohn. Prediction and measurement of thermoacoustic improvements in gas turbines with annular combustion systems. *Journal of engineering for gas turbines and power*, 123(3):557–566, July 2001.
- [5] C. Klsheimer and H. Bchner. Combustion dynamics of turbulent swirling flames. *Combustion and flame*, 131:70–84, 2002.
- [6] Timothy C. Lieuwen. *Investigation of Combustion Instability Mechanisms in Premixed Gas Turbines*. PhD thesis, Georgia Institute of Technology, August 1999.
- [7] S. R. Turns. *An introduction to combustion, concepts and applications*. McGraw-Hill, Inc., 1996.
- [8] F. J. M. van der Eerden. *Noise reduction with coupled prismatic tubes*. PhD thesis, University of Twente, 2000.