

# Simple High-order Galerkin Finite Element Scheme for the Investigation of Both Guided and Leaky Modes in Axially Anisotropic Planar Waveguides

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A simple high-order Galerkin finite element scheme is formulated to compute both the guided and leaky modes of anisotropic planar waveguides with diagonal permittivity tensor. Schemes up to 8<sup>th</sup>-order of accuracy in the effective index are demonstrated.

**Keywords:** finite element method, high-order scheme, transparent boundary conditions, leaky modes, guided modes, anisotropic waveguides, arbitrary index profile.

## Introduction

Optical waveguides that are made on a high index substrate are particularly interesting. This class of waveguides includes structures made on a semiconductor wafer. Structures that are composed of silicon compounds (silicon oxynitride, silicon nitride, silica, etc.) grown on the top of silicon substrate do not only take benefits from the low cost of silicon wafer, but also share the well developed technologies used by the microelectronics industries, and enable better integration between the optical and electronic circuits. It has been shown that structures made by these materials have a wide range of available refractive indices. Besides, arbitrary refractive index profiles can be made by precise computer control of the fabrication parameters as has been demonstrated by the realization of a rugate filter[1]. This feature gives more degree of freedom in refractive index profile engineering, i.e. tailoring of the refractive index profiles to meet certain desired properties of the waveguide, like bandwidth, mode profiles, phase matching of modes of different wavelengths, etc. Hence, the numerical investigations of this class of structure become important.

## The Scheme

In this work, we proposed a simple high-order 1-D Galerkin FEM scheme. By using transparent boundary conditions derived from the Sommerfeld radiation condition and allowing the transverse wave vector to have a complex value, it can allow light to leak into high index substrate/cladding, but decay into low index substrate/cladding, hence able to compute both the guided and leaky modes. The inclusion of Richardson extrapolation and simple mesh-adjustment scheme enable a high order of accuracy to be achieved by using only first-order-polynomial basis functions. The sparse non-linear matrix eigenvalue equation produced by the scheme can be solved using a simple iteration scheme. Hence, it turns out to be very simple, easy to implement, but highly accurate scheme. The method is suitable for leaky planar waveguides of arbitrary index profile with a diagonal permittivity tensor. Schemes up to 8<sup>th</sup>-order of accuracy in the effective index are demonstrated. Some applications examples including investigations of ARROW structures and buffer-layer optimization of silicon-compounds-based leaky waveguides are demonstrated.

## References

[1] S. Lim, S. Shih, J.F. Wager, *Thin Solid Films*, **Vol. 277**, 144-146, 1996.