

# Water circulation in non-isothermal droplet-laden turbulent channel flow

E. Russo<sup>\*</sup>, J.G.M. Kuerten<sup>\*,†</sup>, C.W.M. van der Geld<sup>\*</sup> and B.J. Geurts<sup>†,\*\*</sup>

<sup>\*</sup>*Department of Mechanical Engineering, Eindhoven University of Technology*

<sup>†</sup>*Faculty EEMCS, University of Twente*

<sup>\*\*</sup>*Department of Technical Physics, Eindhoven University of Technology*

**Abstract.** We propose a point-particle model for two-way coupling of water droplets dispersed in turbulent flow of a carrier gas consisting of air and water vapor. An incompressible flow formulation is applied for direct numerical simulation (DNS) of turbulent channel flow with a warm and a cold wall. Compared to simulations without droplets or with solid particles a significant increase in Nusselt number is found, arising from the additional phase changes. In the statistically steady state, which is reached after a long time, the flow exhibits a mean motion of water vapor from the warm wall to the cold wall. Vapor condenses on average on drops near the cold wall and evaporates from drops near the warm wall. This transport is balanced by a net mean mass transfer of droplets from the cold wall to the warm wall.

**Keywords:** droplets, diffusion, dispersion, turbulence

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## INTRODUCTION

Particle-laden flows are formed by a fluid “carrier phase” and a solid or liquid “dispersed phase”. In case of turbulent flow with large numbers of particles with sizes smaller than the Kolmogorov length, it is common practice to adopt a point-particle approach [1] using the Maxey-Riley equation [2]. In the simulations presented here we include exchange of momentum, heat and mass between the gas phase, consisting of air and water vapor, and water droplets.

Only a few studies have been reported on the modeling of heat and mass transfer in a numerical model. In particular, [3] was the first to study evaporating droplets in homogeneous turbulence with two-way coupling. In the present study, we perform DNS of inhomogeneous turbulent channel flow in which one of the walls is heated while the other is cooled. This leads to non-uniform temperature and vapor mass fraction profiles. We adopt an incompressible formulation for the carrier phase. Droplet collisions are ignored because of the small volume fraction [4].

The aim of this study is to analyze the effects of phase change on the global heat transfer properties and on droplet and water vapor motion. In an earlier study [5] it was found that the presence of a large number of inertial particles gives a significant increase in the effective heat transfer between the channel walls. In the present study it will be investigated to what extent phase changes affect heat transfer. Evaporation and condensation lead to growth of droplets near the cold wall and shrinkage near the warm wall. This results in a gradient in water vapor concentration. In the resulting statistically steady state diffusion and turbulent transport of water vapor lead to a mean flux of water vapor from the warm to the cold wall. Conservation of water mass shows that this must be compensated by an opposite mass flux of water droplets. We will quantify the magnitude of these mass fluxes.

## GOVERNING EQUATIONS AND NUMERICAL METHOD

The gas is considered in an Eulerian way and is assumed to behave as an incompressible fluid. Therefore, the gas mass density, which is the sum of the mass densities of air ( $\rho_a$ ) and vapor ( $\rho_v$ ) in the system considered, is assumed to be constant in time and space. The model for the motion of the incompressible gas consists of the continuity equation:  $\nabla \cdot \mathbf{u} = 0$  and the Navier-Stokes equations for momentum conservation:

$$\frac{\partial \mathbf{u}}{\partial t} + \boldsymbol{\omega} \times \mathbf{u} + \nabla P = \nu \Delta \mathbf{u} + \frac{\mathbf{F}}{\rho_g} + \frac{\mathcal{L} \mathbf{u}}{\rho_g}, \quad (1)$$

where  $\boldsymbol{\omega} = \nabla \times \mathbf{u}$  is the vorticity,  $P = p/\rho_g + \frac{1}{2}\mathbf{u}^2$ ,  $\nu$  is the kinematic viscosity of the gas,  $p$  the static pressure,  $\rho_g$  is the gas mass density, and  $\mathbf{F}$  is the driving force necessary to maintain a constant total gas flow rate. Finally,  $\mathcal{L}_{\mathbf{u}}$  is the term describing the momentum exchange between the two phases.

The vapor mass density changes because of convection, diffusion and two-way coupling:

$$\frac{\partial \rho_v}{\partial t} + \nabla \cdot (\rho_v \mathbf{u}) - \nabla \cdot (\mathcal{D} \nabla \rho_v) = \mathcal{L}_v. \quad (2)$$

The term  $\mathcal{L}_v$  represents the mass transfer between the droplets and the water vapor and  $\mathcal{D}$  is the diffusion coefficient.

The temperature equation is obtained from the equation expressing conservation of internal energy of the two phases and reads:

$$(\rho_a c_{v,a} + \rho_v c_{v,v}) \left( \frac{\partial T_g}{\partial t} + \nabla \cdot (\mathbf{u} T_g) \right) = k_g \nabla^2 T_g + \mathcal{L}_{\text{diff}} + \mathcal{L}_{2\text{way}}, \quad (3)$$

where  $T_g$  is the gas temperature,  $c_{v,a}$  and  $c_{v,v}$  represent the specific heat capacities of air and vapor at constant volume and  $k_g$  is the thermal conductivity of the gas. The term  $\mathcal{L}_{\text{diff}}$  in (3) represents the transport of energy due to diffusion of water vapor and  $\mathcal{L}_{2\text{way}}$  is the contribution from the coupling between the two phases.

The droplets are considered as point-particles with a uniform temperature. We do not take gravity into account. Therefore, Newton's law applied to a droplet can be written as:

$$\frac{d(m_i \mathbf{v}_i)}{dt} = m_i (\mathbf{u}(\mathbf{x}_i, t) - \mathbf{v}_i) \frac{(1 + 0.15 \text{Re}_p^{0.687})}{\tau_d} + \mathbf{u}(\mathbf{x}_i, t) \frac{dm_i}{dt}. \quad (4)$$

Here,  $m_i$  is the mass of particle  $i$ ,  $\mathbf{v}_i$  its velocity and  $\tau_d = \rho_l d_i^2 / (18 \mu_g)$  the droplet relaxation time. Moreover,  $\text{Re}_p$  is the Reynolds number based on the droplet diameter  $d_i$  and the relative velocity between the droplet and the carrier gas and  $\rho_l$  is the mass density of liquid water. The two terms on the right-hand side are the drag force and the change in momentum due to phase change.

The equation for the droplet temperature can be written as:

$$\rho_l c_l V_i \frac{dT_i}{dt} = (h_v - h_l) \frac{dm_i}{dt} + h_m A_i (T_g(\mathbf{x}_i, t) - T_i), \quad (5)$$

where  $V_i$ ,  $A_i$  and  $T_i$  denote volume, surface area and temperature of a droplet and  $h_m$  is the convective heat transfer coefficient between droplet and gas. The first term on the right-hand side is due to phase transition and contains the specific enthalpy of vapor and liquid phases. For forced convection around a sphere, the heat-transfer correlation is chosen as [6]:  $h_m d_i / k_g = 2 + 0.6 \text{Re}_p^{1/2} \text{Pr}^{1/3}$ , where  $\text{Pr}$  is the Prandtl number of the carrier gas. For the mass transfer we also follow [6]:

$$\frac{dm_i}{dt} = - \frac{m_i \text{Sh}}{3 \tau_d \text{Sc}} \ln \left( \frac{1 - x_{v,\delta}}{1 - x_{v,0}} \right), \quad (6)$$

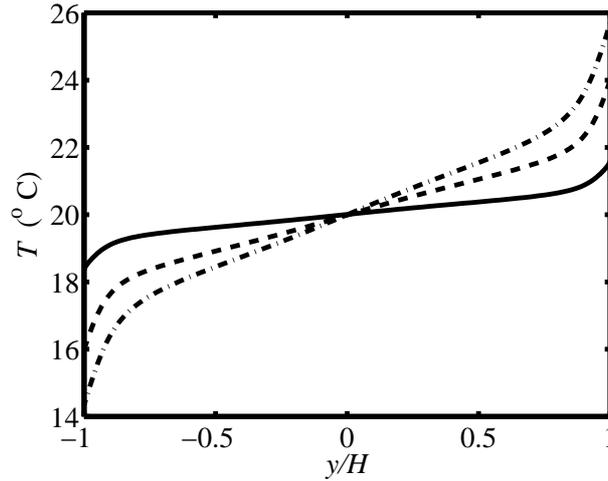
where the Schmidt number  $\text{Sc} = \mu_g / (\rho_g \mathcal{D})$  and  $x_{v,\delta}$  and  $x_{v,0}$  are the vapor mass fractions in the surroundings of the droplet and at the surface of the droplet, respectively. The Sherwood number is given by  $\text{Sh} = 2 + 0.6 \text{Re}_d^{1/2} \text{Sc}^{1/3}$  [6]. For the calculation of  $x_{v,0}$  we assume that the air at the droplet surface is saturated and apply Antoine's relation for the saturation pressure.

The two-way coupling terms in the governing equations for the gas phase satisfy the requirement that these terms do not change the total water mass, total momentum and total internal energy in the system. Moreover, we assume that the two-way coupling terms act as point forces in the governing equations for the gas. As an example, if  $N$  is the total number of droplets, the two-way coupling term in the Navier-Stokes equation for the gas phase (1) can be written as:

$$\mathcal{L}_{\mathbf{u}} = - \sum_{i=1}^N \frac{d(m_i \mathbf{v}_i)}{dt} \delta(\mathbf{x} - \mathbf{x}_i) = - \sum_{i=1}^N m_i \frac{d\mathbf{v}_i}{dt} \delta(\mathbf{x} - \mathbf{x}_i) - \sum_{i=1}^N \mathbf{v}_i \frac{dm_i}{dt} \delta(\mathbf{x} - \mathbf{x}_i). \quad (7)$$

The coupling term is split into two contributions representing the momentum transfer from the droplet to the gas due to acceleration and due to mass transfer arising from evaporation or condensation respectively.

DNS of turbulent channel flow is performed using a pseudo-spectral code as described in [7]. The application of periodic boundary conditions in the streamwise and spanwise directions allows the use of a Fourier-Galerkin approach, whereas in the wall-normal direction a Chebyshev-collocation method is used. The time integration method for the



**FIGURE 1.** Mean gas temperature in the statistically steady state (averaged over  $5,000 < t^+ < 16,000$ ); solid: droplets, dashed: particles, dash-dotted: no particles.

gas consists of a combination of a third-order accurate compact-storage explicit Runge-Kutta method for the nonlinear terms and the implicit Crank-Nicolson method for the viscous and pressure terms. The same Runge-Kutta method is applied to the governing equations of the droplets. The gas properties at the droplet location are calculated by tri-linear interpolation.

The domain has a size of  $4\pi H$  in streamwise direction and  $2\pi H$  in spanwise direction, where  $H$  is half the channel height. For the streamwise, wall-normal, and spanwise directions the notation  $x$ ,  $y$ , and  $z$  is used, respectively. The no-slip condition is adopted at the walls. On one wall a constant heat flux is supplied to the channel, which is equal to the heat flux extracted from the other wall. Simulations are performed at frictional Reynolds number approximately  $Re_\tau = 150$ . There are two million droplets or particles with diameter initially equal to  $60\mu\text{ m}$  and initial temperature  $20^\circ\text{C}$ , which are initially randomly and homogeneously distributed over the channel domain. The simulations are performed with an initial gas temperature of  $20^\circ\text{C}$  and a heat flux at the walls of  $32\text{ W/m}^2$ .

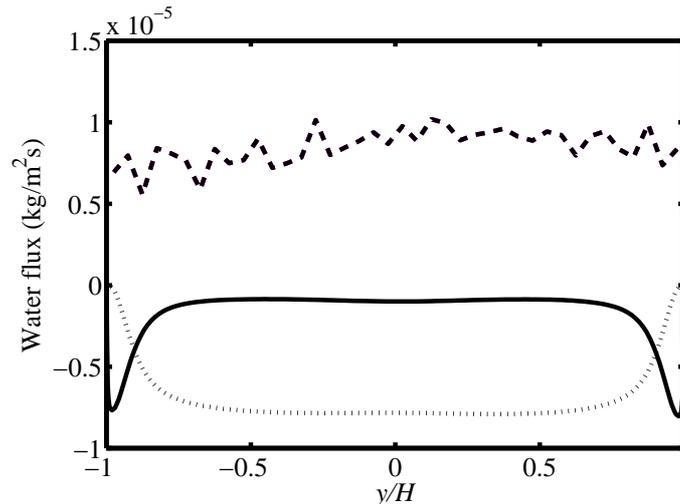
## RESULTS

The heat flux imposed at the walls creates a temperature gradient in the wall-normal direction, which leads to evaporation (condensation) in warmer (colder) regions, since initially the air is saturated. A steady temperature profile is reached around  $t^+ = 5,000$ . As a consequence of phase change the mean water vapor mass fraction also changes in time. An inhomogeneous water vapor concentration profile in the wall-normal direction develops and a steady profile is also reached around  $t^+ = 5,000$ .

In the statistically steady state the difference in temperature between the walls depends on whether there are particles present in the flow and whether the particles can evaporate. Figure 1 shows the mean temperature profiles in the statistically steady state for the case with water droplets, a case with solid particles and a case without particles. It can be seen that the temperature difference between the walls reduces in case solid particles are added to a single-phase flow and even more with droplets instead of solid particles. In order to quantify this we introduce the Nusselt number:

$$\text{Nu} = \left( \frac{d\langle T_g \rangle}{dy} \Big|_{\text{wall}} \right) / \frac{\langle \Delta T_g \rangle}{2H},$$

where  $\Delta T_g$  is the mean temperature difference between the walls. For the case of a pure fluid  $\text{Nu} = 4.4$ , for the case with solid particles  $\text{Nu} = 5.8$  and for our test case with droplets the value increases to 15.5. The increase in case solid particles are present can be attributed to the convective heat transfer between particles and gas, which decreases the temperature of the gas. The phase transition in case droplets are present further attenuates the temperature increase of the gas near the heated wall, since the latent heat required for droplet evaporation leads to a lower droplet temperature and hence a larger heat transfer between gas and droplets.



**FIGURE 2.** Water vapor and droplet mass fluxes as functions of the wall-normal coordinate in the statistically steady state (averaged over  $20,000 < t^+ < 51,000$ ); solid: water vapor mass flux due to molecular diffusion, dotted: water vapor mass flux due to turbulent transport, dashed: droplet mass flux.

There are three mechanisms by which water can have a mean motion in the wall-normal direction of the channel: molecular diffusion of water vapor by a mean concentration gradient, turbulent transport of water vapor and mean motion of droplets. In the statistically steady state, which is reached at  $t^+ \approx 2.0 \times 10^4$ , the mean droplet size and concentration do not change anymore. Therefore, the three mechanisms should cancel each other. Since the water vapor concentration is higher at the warm wall than at the cold wall, there is a net molecular diffusion flux of water vapor from the warm to the cold wall. The direction of the turbulent transport flux of water vapor is the same since the water vapor concentration is correlated to the gas temperature. This implies that, for the three mechanisms to cancel, there must be a net mass flux of water droplets from the cold to the warm wall, which balances the net mass flux of vapor from the warm to the cold wall. Figure 2 shows the mass fluxes for the three mechanisms, averaged over the two homogeneous directions and time in the statistically steady state. Although the mass flux of the droplets is still quite irregular, the results indeed underpin the picture sketched above. Both vapor mass fluxes are directed from the warm to the cold wall and are almost constant when added. The net droplet mass flux is directed from the cold to the warm wall. This shows that there is a circulation of water. Water vapor moves from the warm to the cold wall, where it condenses onto droplets. The droplets move from the cold to the warm wall, where they evaporate again.

## ACKNOWLEDGMENTS

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