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Managing Trust in Business Webs using Game Theory

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***Abstract*—Business webs are collections of enterprises designed to jointly satisfy a consumer need. We design business webs using value models that show the participating stakeholders and the value objects which they are going to exchange during a specific period. Each partner agrees to act according to the value model, however during the business, stakeholders need to be sure if their partners are acting according to the value model or not. In some cases, the best way to find out the answer of that question is to run an inspection because the value exchanges are not observable without cost. Now, the crucial question here is "how often a stakeholder needs to run an inspection and how these inspections affect the expected payoffs of the stakeholders?". Here in this paper, using game theory concepts and techniques, we aim at finding out answers for these questions.**

***Keywords*-business web; value model; game theory; trust; enterprise interoperability**

I. Introduction

A business web is a collection of enterprises designed to jointly satisfy a consumer need [1]. In a business web each enterprise contributes with its own specific products or services to satisfy a consumer need. Before going into the collaboration, each partner wants to be sure that participation in such a collaboration network is economically rational and, if so, specify the details of the coordination process.

The main goal of business value modeling is to reach agreement amongst profit-and-loss responsible stakeholders regarding the question "Who is offering what of value to whom and expects what of value in return?" Business value modeling helps the stakeholders to share their understanding regarding the collaboration and enables them to analyze economic sustainability of the business web. For modeling business webs, we use value models according to e^3value methodology [2].

A value model gives the stakeholders an indication of how profitable and sustainable a business web would be assuming that all stakeholders stick to their promises and act according to the agreements made in the value model. In fact a value model shows an ideal situation in which all partners act as promised, and it abstracts from misuses and frauds. Hence, we should provide the participating stakeholders with some tools or methods that enables them to know how trustworthy their partners are and if their value model is realistic, i.e. not too risky to implement. Parties in networks are vulnerable to the opportunistic behaviour of others. To be sustainable, a network organization needs mechanisms to govern and control the interaction among network participants and to mitigate opportunistic behaviours. Vera Kartseva [3], in her thesis, introduced an initial work on how to redesign a value model so that untrustworthy behaviors are kept under control. In our previous work [4], we proposed a model to measure the trustworthiness of the stakeholders during the collaboration in the business web. The assumption, there, was that once a stakeholder is done with a value exchange, he knows if the other party has acted according to the agreements or not and he can assign a new value to the trustworthiness of his partner by comparing the result of the collaboration with what he had been promised before. These values, then, are used in the trust calculations. This method is suitable for those situations in which value exchanges are straightforward and observable to both actors participating in an exchange e.g. the collaboration between a buyer and a webshop that sells PCs. Each time a buyer buys something from the webshop and at the end of the buying process both actors of the collaboration (webshop and buyer) can tell us how satisfied are they with the collaboration and if the other party has acted according to the promisees and agreements or not.

Nevertheless, some value exchanges are not straightforward and easy to evaluate. A well known example is the collaboration between passengers and public transportation. In some countries such as The Netherlands you are not checked for ticket upon entering a train. Instead some agents run an inspection and check, every now and then, to see if every one inside the train has bought a right ticket or not and if they catch someone without ticket they will put a penalty on him (€35 plus the price of the ticket). In this case, the ideal situation designed as a value model shows only a pair of values being exchanged between two actors namely a passenger and a transportation company. The assumption here is that every passenger buys an appropriate ticket. However, during the execution of the model, without inspection it is not clear when a value exchange happens (i.e. when somebody uses the train). Hence, for these kind of situation we can not apply our method for measuring the

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trustworthiness of the actors because as mentioned before, the assumption, there, was that the value exchanges are straightforward and easy to observe.

Therefore, we classify two types of value exchanges 1) those that are observable without cost and 2) those that are observable against an unacceptable cost; a cost that is not represented in the value model. To measure the trustworthiness of the stakeholders of the first type of value exchanges, we can use our method introduced in [4] but for the second type we need to take the cost of observation into account in our analysis. An important question here is how often a stakeholder should run an inspection because if he runs an inspection and it turned out that his partner had acted according to the agreements it would be a waste of money and if he doesn't run inspections enough, his partner might cheat and therefore he again loses money. In any case, the inspection cost will affect the final expected payoffs of the stakeholders involved in the value exchange and it should be taken into account. The contribution of this work is to provide the stakeholders with a method which helps them to find out how often they should run an inspection in a business collaboration using game theory concepts.

The rest of the paper is organized as follows: In section II, we discuss business webs and value modeling in e^3value methodology and then we show a real case business web modeled in e^3value in section III. Then, in section IV, we define some concepts of game theory briefly and after that in section V we try to find a way to deal with trust relations between stakeholders using game theory concepts. Section VI is devoted to a discussion and finally we conclude in section VII.

II. Business Webs and Value Modeling

Today, enterprises operate more and more together in networked collaborations rather than just on their own. There are many reasons for this. Among others we can refer to more complicated user needs, upward tendency toward specialization, changing demands of customers, higher customer satisfaction indexes, etc. In the literature, collections of enterprises that jointly satisfy a consumer need, are called business webs [1]. In a business web each enterprise contributes with its own specific expertise, products and services to satisfy a consumer need. For example, a web shop, logistics company, payment provider and authentication provider can jointly provide the service of on-line buying to consumers. Each partner wants to be sure that participation in such a collaboration network is economically profitable and sustainable before operational details of the coordination infrastructure are being designed. This is where business modeling comes into play. For modeling business webs we use e^3value methodology.

An e^3value model consists of a graphic part and a computational part. The graphic part is a diagram and the computational part is a spreadsheet with algorithms that can perform Net Present Value (NPV) estimations for the participating stakeholders in the diagram. In e^3value we model a business web as a graph in which the nodes represent economic actors and the edges represent economic value transfers. In addition, an e^3value model shows how a consumer need is met by a set of economic exchanges between actors in this web [2], [5], [6].

Consider the simple e^3value model (Figure 1) in which *Buyer* gives *Money* to *Seller* and receives *Good* in return. The *Seller*, in turn, gives *Money* to *Transporter* and receives *Transport*. This simple model illustrates the following modeling constructs of e^3value:

- *Contract Period.* A value model describes economic exchanges during a specific period of time, which is called contract period. The contract period should be specified in supporting documentation and the model will be used to analyze economic sustainability during this period only.
- *Actor.* An actor is an independent economic (and often also legal) entity with a specific interest in the collaboration (making profit, increasing utility, earning experience, ...). Actors in Figure 1 are *Buyer*, *Seller* and *Transporter*. The actor for whom the business web is made to satisfy his needs is called the *consumer*. We represent the consumer need by a bullet placed inside this actor (*Buyer* in Figure 1).
- *Value Object.* A value object is a service, good, money, or experience, that is of economic value to at least one actor and that is exchanged between actors. In our example value objects are *Money*, *Good*, *Money* and *Transport*.
- *Value Port.* An actor uses a value port to provide or request value objects to or from other actors. A value port is a conceptual construct indicating that during the contract period, an actor is capable of giving or receiving a value object. Value ports are represented by small triangles on the edge of the shapes representing actors.
- *Value Interface.* Value interfaces group value ports and indicate atomicity: if one value port in the interface is triggered in the contract period, all others will trigger in this period (however the model makes no statement about when this will happen: this has to be specified in a corresponding coordination model). Value interfaces are depicted by oval shapes surrounding the value ports.

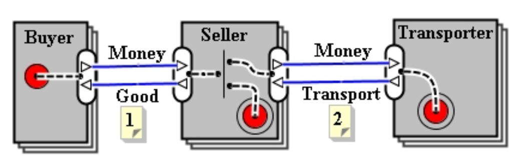

Figure 1. A simple value model

- *Value Transfer.* A value transfer connects two value ports of different actors, meaning that the actors will transfer value objects in the indicated direction.
- *Market Segment.* A market segment is a set of actors that assign economic value to objects equally. They are shown as overlapping rectangles.
- *Value Exchange.* Value transfers should come in economic reciprocal pairs, which we call value exchange.
- *Dependency Path.* In most cases an actor has multiple value interfaces and these value interfaces can be related. A dependency path connects value interfaces of the same actor together, meaning that if one of the value interfaces is triggered the connected value interfaces also must be triggered [2]. A dependency path consists of dependency nodes and connections. A dependency node is a consumer need, an AND-fork (the sign in the actor *Seller*) or AND-join, an OR-fork or OR-join, or a boundary element (Bull's eye sign). A consumer need is the trigger for the transfer of value objects. A boundary element models that no more value transfers can be triggered. A dependency is represented by a dashed line.
- *Transaction.* A transaction starts when the consumer need triggers and completes when all the value exchanges connected to that consumer need are triggered.

Given an e^3value model attributed with quantitative estimations (for example, the number of consumer needs per contract period and the valuation of objects exchanged) and a contract period, we can estimate the revenue of each actor in the specified contract period. This is a first indication whether the model at hand can be economically rational for each actor.

III. Case Study

We take an example that deals with the problem of clearing Intellectual Property Rights (IPR). It consists of two steps: collecting fees from IPR users, i.e. radio stations, bars, discotheques and so on, who play music in public spaces with the aim of getting money from it, and repartitioning the collected fees to Right Owners, i.e. artists, song writers, producers. One of the main IPR societies in the Netherlands collecting IPR fees and repartitioning it to the owners is SENA (see http://www.sena.nl/). IPR fee collection is currently done based on statistical evidence, but SENA is interested in a future business model in which fees are collected on a pay-per-play basis, in which for each music track, a track-specific business web of clearing organizations is composed. This is possible once music is broadcasted over the Internet.

Figure 2 shows one possible value model of pay-per-play fee collecting in which a Background Music Provider (BMP) delivers a stream of tracks using Internet-based technology for direct playing which is not recordable at the Receivers side. BMP and Receivers both should pay IPR societies, so both are IPR users (see Figure 2). In this value model the following actors are identified:

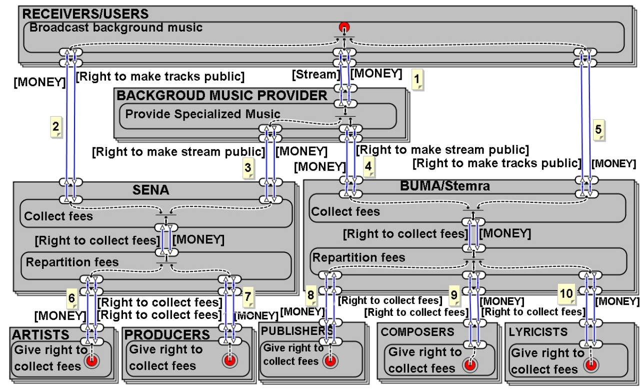

Figure 2. Value model of providing music by Streaming

Receiver A Receiver is an actor who broadcasts background music to get benefit out of it.

Background Music Provider (BMP): A BMP is an actor who provides specialized background music in exchange of a fee.

Right Owners: Right owners of a track are those who are involved in producing it, i.e., write lyrics, play a musical instrument, produce and publish tracks etc.

IPR Societies: IPR Societies collect fees for each track played in the public and repartition fees to IPR owners. SENA and BUMA/Stemra are two IPR Societies each responsible for IPR rights of some specific Right Owners. After collecting money from users it should be repartitioned between appropriate right owners. SENA repartitions fees to Artists and Producers, and BUMA/Stemra does the same for Publishers, Composers and Lyricists.

IV. Game Theory

Game theory is the formal study of conflict and cooperation. Game theoretic concepts can be applied whenever the actions of several agents are interdependent. These agents may be individuals, groups, firms, or any combination of these. The concepts of game theory provides a language to formulate, structure, analyze, and understand strategic scenarios. Game theory is the formal study of decision-making where several players must make choices that potentially affect the interests of the other players. The following concepts are the basic concepts in Game Theory:

Game: Game is a formal description of a strategic situation.

Player: A player is an agent who makes decisions in a game. A player is said to be rational if he seeks to play in a manner which maximizes his own payoff. A central assumption in many variants of game theory is that the players are rational. A rational player is one who always chooses an action which gives the outcome he most prefers, given what he expects his opponents do.

Payoff: A payoff is a number that reflects the desirability of an outcome to a player, for whatever reason. The expected payoff demonstrated the player's attitude towards risk.
Strategy: In each round of the game, each player's choice is called a strategy.
Dominance: Since all players are assumed to be rational, they make choices which result in the outcome they prefer most, given what their opponents do. If player P has two strategies A and B so that, given any combination of strategies of the other players, the outcome resulting from A is better than the outcome resulting from B for player P, then strategy A is said to dominate strategy B. A rational player will never choose to play a dominated strategy. In some games, examination of which strategies are dominated results in one single strategy.
Nash equilibrium: Sometimes, consideration of dominating strategies yields precise advice to the players on how to play the game. In many games, however, there are no dominated strategies, and so these considerations are not enough to rule out any outcomes or to provide more specific advice on how to play the game. A *Nash equilibrium* recommends a strategy to each player that the player cannot improve upon unilaterally, that is, given that the other players follow the recommendation. Since the other players are also rational, it is reasonable for each player to expect his opponents to follow the recommendation as well.

For more details about game theory interested readers are referred to [7], [8], [9].

V. Managing Trust using Game Theory: An example

Consider the value exchange labeled with 2 in the value model in section III. It shows the exchange of 'Right to make music public' for 'Money'. Receiver agrees to pay SENA for each track which he plays in his public place. This value exchange is not observable at low cost to either party i.e. SENA doesn't know if Receiver acts according the value model and pays him for all the tracks which he plays if he doesn't run an inspection. Receiver has an incentive to cheat. SENA would like to verify that Receiver is abiding by the agreement, but doing so requires inspections which are costly. If SENA does inspect and catches Receiver cheating, SENA can demand a large penalty payment for the noncompliance.

		Receiver	
		Comply	*Cheat*
SENA	*Don't Inspect*	0 0	10 −10
	Inspect	0 −20	− 90 70

Figure 3. Compliance inspection game between SENA and Receiver.

Figure 3 shows possible payoffs for such an inspection game. Payoffs are measured in an arbitrary unit and they can be multiplied by a positive number or added with any number since these two operations keep the relations between the payoffs fixed (having payoffs P_1 and P_2 if for example $P_1 \geq P_2$ then the relation holds after multiplying P_1 and P_2 by a positive number or adding them with any number).

SENA has two strategies/options; he either inspects or does not inspect. In either case Receiver has two strategies/options too; he either complies or cheats. Hence, there would be four different combinations of strategies. The case in which SENA chooses *Don't Inspect* and Receiver chooses *Comply* yields payoff zero to both SENA and Receiver. In case SENA chooses *Don't Inspect* and Receiver chooses *Cheat*, they get payoffs -10 and 10 respectively. If Receiver complies and SENA inspects, Receiver gets payoff 0, while SENA gets a negative payoff -20 because of the inspection cost. If Receiver cheats, however, inspection results in a heavy penalty (payoff -90 for Receiver) and payoff 70 for SENA.

In Figure 3, rows represent the strategies of SENA and columns those of Receiver. In general a player may have more than two strategies. Each strategy combination defines a payoff pair, like (70,-90) for (Inspect,Cheat), which is given in the respective table entry. Each cell of the table shows the payoff to SENA at the (lower) left, and the payoff to Receiver at the (right) top.

A. Solving the Game

Like any other game the question here is 'which strategy each player should play?' What is obvious here is that none of the players can play a strategy for ever because for example if SENA always chooses *Don't Inspect*, the best response for Receiver would be choosing *Cheat* strategy and this yields payoffs -10 and 10 for SENA and Receiver respectively which is to the detriment of SENA and if he always chooses *Inspect*, the best response for Receiver would be choosing *Comply* strategy with resulting payoffs -20 and 0 for SENA and Receiver respectively again to the detriment of SENA. On the other hand, if Receiver settles on *Comply*, SENA would choose *Don't Inspect* with resulting payoffs 0 for both and this is not to the best interest of Receiver since if SENA does not inspect Receiver can simply cheat and change his 0 payoff with 10. If Receiver settles on *Cheat* strategy, SENA would prefer *Inspect* with resulting payoffs -90 and 70 for Receiver and SENA respectively, which is to the detriment of Receiver. Therefor if, any of the players settles on a deterministic choice the other player can play in his own interest and to the detriment of that player.

There is a technique called max-min strategy in which each player tries to choose the strategy with the best worst payoff. The max-min strategy for SENA is to Don't Inspect (-10 is better than -20), and for Receiver it is to comply (0 is better than -90). However, as explained above this is not a

Nash equilibrium and hence not a stable recommendation to the two players, since Receiver could switch his strategy and improve his payoff. A pair of strategies, one for each player, forms a Nash equilibrium if even given knowledge of each other's strategies, neither player would have an incentive to switch to an alternate strategy [9]. Therefore, if we treat each player as simply having the two strategies, then there is no Nash equilibrium for this game.

In real life, by playing different strategies randomly, players try to make it difficult if not impossible for their opponents to predict how they play. We will refer to these as *mixed strategies* and the strategies shown in the payoff table as *pure strategies*. In a mixes strategy players rank distributions over payoffs according to their expected values to maximize their expected payoffs. There is a well-established foundation for this [10], [11], [12].

A mixed strategy for SENA is to inspect only with a certain probability. To find out which probability is the best lets examine two cases. Assume that the probability of inspection is 1%, then, irrespective of that probability, Receiver gets payoff 0 when he complies, and expects payoff

$$(0.99 * 10) + (0.01 * (-90)) = 9 \quad (1)$$

when he cheats. Since the payoff in case of cheating (9) in greater than payoff in case of complying (0), Receiver will still cheat, just as in the absence of inspection. If SENA increases the probability of inspection, for example 20%, then the expected payoff for *Cheat* is

$$(0.8 * 10) + (0.2 * (-90)) = -10 \quad (2)$$

so, Receiver prefers to comply ($0 > -10$). Therefore, if the inspection probability is either too low or too high, Receiver has a unique best response.

If Receiver was indifferent (i.e. both strategies give him the same payoff), he could choose his strategies randomly. Receiver is indifferent if and only if SENA chooses *Inspect* with probability 0.1, and *Don't Inspect* with probability 0.9 because then the expected payoffs for cheat

$$(0.9 * 10) + (0.1 * (-90)) = 0 \quad (3)$$

which is the same for *Comply*. This mixed strategy of SENA, makes Receiver indifferent between his strategies.

If SENA was indifferent (i.e. both strategies give him the same payoff), he could choose his strategies randomly. SENA is indifferent if and only if Receiver chooses *Comply* with probability 0.8 and *Cheat* with probability 0.2, because then the expected payoffs for *Dont Inspect*

$$(0.8 * 0) + (0.2 * (-10)) = -2 \quad (4)$$

and for *Inspect*

$$(0.8 * (-20)) + (0.2 * 70) = -2 \quad (5)$$

which are the same. These mix strategies make the only Nash equilibrium of the game. The resulting expected payoffs are -2 for SENA and 0 for Receiver.

Finding the mixed strategies of two players in a two strategy game using the notion of indifference, when there are no equilibria involving pure strategies, is a general and well-established principle. Each player should randomize between his strategies so that the other player becomes indifferent between his two strategies. This guarantees that neither player's behavior can be exploited by a pure strategy, and those probabilities are best responses to each other. A generalization of this principle applies to games with any finite number of players and any finite number of strategies: Nash proved that every such game has at least one mixed-strategy equilibrium. For more details interested readers are referred to [13], [14].

B. Adjusting Profitabilities in Value Models

The resulting expected payoffs of the game are in fact the best payoffs which SENA and Receiver can get out of the situation shown in the payoff table in figure 3. Note that the standard outcome, defining the reference payoff zero to both SENA and Receiver, in which SENA chooses *Don't Inspect* and Receiver chooses *Comply* is in fact the ideal case that demonstrates the value model.

Stakeholders see a profit in the value exchanges in the sense that they value what they get higher than what they give. In our case, suppose that SENA values what he gets (Money), €10 more than what he gives (Right). According to the calculations in the previous section, the resulting expected payoff is -2 for SENA. Assume also that this is equal to €-2 i.e. on average he loses €2 per exchange. Therefore the final value that SENA receives per exchange is €10 - €2 = €8.

Now, we propose the following steps to do the profitability analysis for the stakeholders involves in a business collaboration:

1) Build value model
2) Determine trust assumptions
3) Build a game for each value exchange which is not observable without cost for the trusting stakeholder. The payoff table should be build in cooperation with participating stakeholders in the value exchange. To do that we should identify all possible options/strategies for each stakeholder and then assign a payoff value to each stakeholder for the combination of all strategies.
4) Find the average payoff for each stakeholder in the game
5) Adjust the values which the stakeholders assume to get from each value exchange with the resulting average payoffs of the game

VI. Discussion

In previous section we saw a game with a mixed equilibrium in which players choose their strategies with certain probabilities. According to the payoffs of figure 3, choosing *Inspect* strategy by SENA with probability 0.1 makes Receiver indifferent between comply and cheat because the expected payoff for *Cheat*, namely $(0.9*10)+(0.1*(-90))$, is the same for Receiver as when he chooses *Comply*. Of course this calculation is valid when getting caught incurs only financial loss. In reality, getting caught in business may result in embarrassment and other undesirable consequences. Even though every thing in business boils down to money, however the exact monetary value which we assign to an event such as getting caught is not always straightforward.

Mixing seems paradoxical when the player is indifferent in equilibrium. If Receiver, for example, can equally well comply or cheat, why should he gamble? He could comply and guarantee himself payoff zero without any risk. The answer is that precisely because there is no incentive to choose one strategy over the other, a player can mix, and only in that case there can be an equilibrium [7]. If Receiver would comply for certain, then the only optimal choice of SENA is *Don't Inspect*, making the choice of complying not optimal, so this is not an equilibrium.

An interesting aspect of mixed equilibrium is that the probabilities depend on the opponent's payoffs and not on the player's own payoffs [7]. For example, one would expect that increasing the penalty for being caught (-90) lowers the probability of cheating in equilibrium, but it does not. It decreases the probability of inspection so that Receiver becomes indifferent.

This dependence of mixed equilibrium probabilities on the opponent's payoffs can be interpreted in another way. In that interpretation, Figure 3 represents an evolutionary game which demonstrate a non-symmetric interaction between SENA and a group of Receivers. In that case, the probability of a strategy implies the fraction of interactions in which that strategy is chosen. If these fractions deviate from the equilibrium probabilities, then the strategies that do better get higher probabilities. For example, if SENA chooses *Inspect* too often, the fraction of cheaters will decrease (they are more afraid of getting caught), which in turn makes *Dont Inspect* a better strategy. Again if SENA chooses *Dont Inspect* too often, Receivers are less afraid of getting caught and therefore they cheat more often (i.e. the probability of *Cheat* strategy increases) and this makes *Inspect* a better strategy for SENA. Therefore there is an obvious dynamic interplay between strategies. In this dynamic process, the long-term averages of the fractions approximate the equilibrium probabilities [7].

VII. Conclusions

In this paper we discussed the issue of trust in business webs. In some case, the value exchanges of the value model are not observable without cost. In those particular cases, a stakeholder trusts his partner and he needs to run an inspection, which costs money, to verify if his partner is abiding by the agreements made in the value model. To find out how often the trusting stakeholder should run an inspection so that the final payoff would be maximum and how this will affect the expected payoffs, we used the concepts of game theory. This is important in business because in many cases a stakeholder has no option except trusting his/her partners in the hope that they would act according to the agreements.

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