

# On the Energy Benefit of Compute-and-Forward on the Hexagonal Lattice

Zhijie Ren<sup>1</sup>, Jasper Goseling<sup>2,1</sup>, Jos Weber<sup>1</sup> and Michael Gastpar<sup>3,1</sup>,

<sup>1</sup>Department of Intelligent Systems,

Delft University of Technology, The Netherlands

<sup>2</sup>Stochastic Operations Research,

University of Twente, The Netherlands

<sup>3</sup>School of Computer and Communication Sciences,

Ecole Polytechnique Fédérale de Lausanne, Switzerland

z.ren@tudelft.nl, j.goseling@utwente.nl, j.h.weber@tudelft.nl, michael.gastpar@epfl.ch

## Abstract

We study the energy benefit of applying compute-and-forward on a wireless hexagonal lattice network with multiple unicast sessions with a specific session placement. Two compute-and-forward based transmission schemes are proposed, which allow the relays to exploit both the broadcast and superposition nature of the wireless network. The energy consumption of both transmission and reception of the nodes are taken into account. We show that with our schemes, the total energy consumption of the network is significantly reduced compared to traditional routing based transmission schemes. Moreover, our schemes also outperform the plain network coding based transmission scheme in terms of power saving as long as the receive energy of the devices is not negligible.

## 1 Introduction

Compute-and-forward [1], also known as reliable physical layer network coding, is a technique that enables to exploit the features of broadcast and superposition in wireless networks. It has been shown (e.g., [1]-[3]) that in the scenario of multiple unicast, the throughput in a network can be significantly improved by this technique, compared to traditional routing or network coding [4].

On the other hand, network coding, sometimes referred as *plain network coding* to distinguish from *physical layer network coding*, has been proven to be beneficial to a wireless network in the aspect of energy savings [5] [6]. Most of the studies consider transmit energy only, and show that energy can be saved by using plain network coding compared to traditional routing. The ratio of the energy consumption of traditional routing to advanced schemes is sometime referred as the *energy benefit*. In particular, on the hexagonal lattice with multiple unicast, the coding scheme proposed in [7] lower bounds this energy benefit by 2.4, and the lower bound is further improved to 3 by the scheme proposed in [8]. However, an interesting problem is studied in [9] that in the scenario that the receive energy (used for supporting circuit for receiving, for example) is not negligible, some plain network coding based scheme will have less energy benefit, or even no benefit at all.

Besides the above-mentioned studies, our study is also motivated by the classic example of the two-way relay channel shown in Fig. 1(a). As observed, the energy consumption is reduced by using compute-and-forward compared to traditional routing and plain network coding, since the relay node needs fewer transmissions and receptions, and thus consumes less energy.

In this paper, we study the energy benefit of applying compute-and-forward on a hexagonal lattice with specific session placement as in [8] and [9]. We consider both the

transmit and receive energy of the wireless nodes in the network, and design two energy efficient schemes based on compute-and-forward. It is shown that the energy benefit of the two schemes is between 1.5 and 3, for any value of transmit and receive energy of the nodes. Thus, by using compute-and-forward in this network, the energy consumption is at least reduced by a factor of 1.5. This result is essentially different from the energy benefit of the plain network coding based scheme in [9], which is severely degraded or completely gone when the receive energy is not negligible, since it reduces the number of transmissions of the network at the cost of increasing the number of receptions. Our schemes, on the other hand, show that compute-and-forward is able to save the energy by reducing the number of both transmissions and receptions in a wireless network with multiple unicast.

The remainder of the paper is organized as follows. In Section 2 we describe our model. In Section 3, we propose two compute-and-forward based coding schemes and prove their validity. In Section 4, we define the energy benefit, briefly introduce some previous schemes and compare the energy benefit of our schemes to previous schemes to show our contribution. At last, we conclude our work in Section 5.

## 2 Model

### 2.1 Hexagonal Lattice and Session Placement

We consider a subset of the hexagonal lattice network with nodes representing wireless terminals, defined as  $\mathcal{V} = \{v | 0 \leq v_1, v_2 \leq K, v_1 + v_2 \leq K\}$ , in which  $v$  is a node defined by a index tuple  $(v_1, v_2)$  and  $K \leq 2$  is a positive integer. The location of the node  $v \in \mathcal{V}$  in  $\mathbb{R}^2$  is given by  $vG$ , where  $G = \begin{pmatrix} 1 & 0 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$ . We then denote the interior of the network as  $\hat{\mathcal{V}} = \{v \in \mathcal{V} | 0 < v_1, v_2 < K, v_1 + v_2 < K\}$ , and define the boundary nodes as  $\bar{\mathcal{V}} = \mathcal{V} \setminus (\hat{\mathcal{V}} \cup \{(0, 0), (0, K), (K, 0)\})$  and the three borders as  $\mathcal{V}^L = \{v \in \bar{\mathcal{V}} | v_1 = 0\}$  for the left border,  $\mathcal{V}^R = \{v \in \bar{\mathcal{V}} | v_1 + v_2 = K\}$  for the right border and  $\mathcal{V}^B = \{v \in \bar{\mathcal{V}} | v_2 = 0\}$  for the bottom border. We consider local interference, i.e. for any node, the transmitted signals can only be received by its neighbors which are unit distance away. More precisely, the neighbors of node  $v$  are  $\mathcal{O}(u_1, u_2) = \{(u_1 - 1, u_2 + 1), (u_1, u_2 + 1), (u_1 - 1, u_2), (u_1 + 1, u_2), (u_1, u_2 - 1), (u_1 + 1, u_2 - 1)\} \cap \mathcal{V}$ .

Now we place  $3(K - 1)$  unicast sessions, denoted as  $\mathcal{M} = \{m^1(i), m^2(i), m^3(i) | i \in \{1, 2, \dots, K - 1\}\}$ . Sources  $s^j(i)$  and destinations  $d^j(i)$  of the sessions are positioned as follows:

$$\begin{aligned} m^1(i) : & \quad s^1(i) = (0, i), & \quad d^1(i) = (K - i, i). \\ m^2(i) : & \quad s^2(i) = (i, K - i), & \quad d^2(i) = (i, 0). \\ m^3(i) : & \quad s^3(i) = (K - i, 0), & \quad d^3(i) = (0, K - i). \end{aligned} \tag{1}$$

We assume that the source symbols for session  $m^j(i)$  are drawn from finite field  $\mathbb{F}_q$  and denote the source symbols as  $m_0^j(i), m_1^j(i), m_2^j(i), \dots$ . The hexagonal lattice with the sessions is illustrated in Fig 1(b).

### 2.2 Energy Model

In [9], an energy consumption model is used that includes both the energy for transmitting data and the energy for receiving data. The energy consumed when receiving consists of, for instance, the energy consumed by supporting circuitry. This model is, for instance, useful if the transmit energy is very small and reception energy cannot be neglected. In this paper, we study a similar energy model defined as follow: In each time slot, a symbol from  $\mathbb{F}_q$  transmitted by node  $v$  can be successfully received by node  $u$  if  $u$  is a neighbor of  $v$ ,  $v$  transmits with energy  $e_t$  and  $u$  receives with energy  $e_r$ .

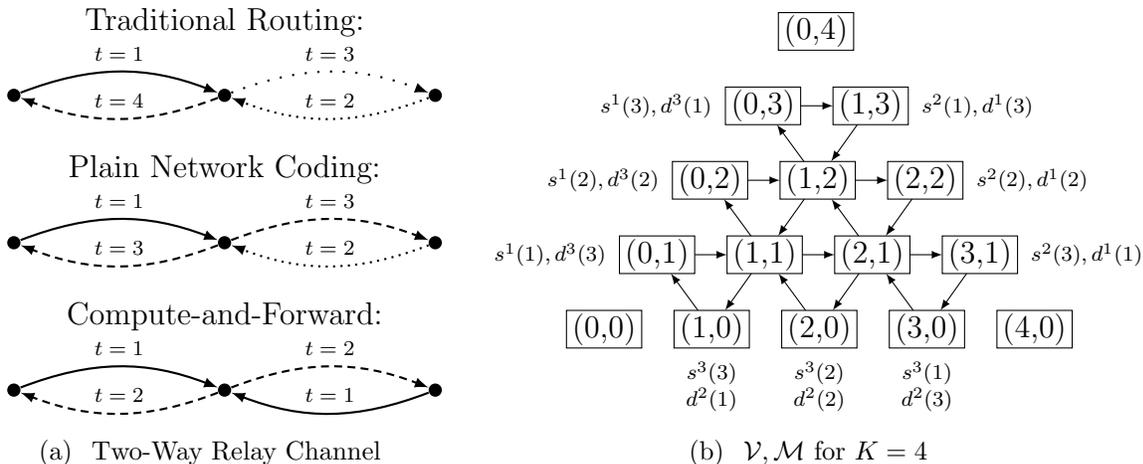


Figure 1: (a) The three transmission schemes on two-way relay channel, in which the lines of different kinds represent transmissions in different time slots and (b) the nodes and session placement of the hexagonal lattice.

### 2.3 Compute-and-forward

Compute-and-forward [1] provides a way to exploit both the broadcast and the superposition nature in wireless networks. With compute-and-forward, a node is able to retrieve a linear sum of the symbols that are transmitted by its neighbors. It has been proved that compute-and-forward achieves a rate very close to the channel capacity on the Gaussian channel, more precisely, a rate

$$R < \frac{1}{2} \log_2 \left( \frac{1}{k} + \text{SNR} \right) \quad (2)$$

is achievable [1], where  $k$  is the number of superposed symbols, which is at most 6 in our network. Comparing this rate to the capacity, only a term of  $\frac{k-1}{k}$  is missing inside the logarithm, which has only a minor effect when SNR is large. Hence, in this paper, we neglect this term and assume that in a time slot, node  $v$  can successfully retrieve the sum of the transmitted symbols by all of its non-silent neighbors if it receives with energy  $e_r$  and the non-silent neighbors transmit with energy  $e_t$ .

## 3 Compute-and-forward Based Schemes

In [8], an energy efficient network coding scheme is designed in such a way that the number of the transmissions of the interior nodes is decreased at the cost of each of the boundary nodes transmitting extra symbols for the successful decoding at the destinations. However, the extra energy consumption at the boundaries turns out to be negligible for a network with large enough  $K$ .

In this section, two compute-and-forward based coding and scheduling schemes inspired by [8] are proposed. Both of them support the multiple unicast sessions  $\mathcal{M}$  on the hexagonal network  $\mathcal{V}$ . The schemes work in rounds, in which at each destination, a new source symbol for its corresponding session is decoded after the initial startup phase. We define the notation  $x_t^i(v)$  and  $y_t^i(v)$ , respectively, as the transmission and reception of node  $v$  in time slot  $i$  of round  $t \in \mathbb{Z}^+$ .

Before starting the description of both schemes, we divide all nodes into 3 categories. We define category  $i$ ,  $i \in \mathbb{Z}_3$  as  $\mathcal{V}_i = \{(v_1, v_2) \in \mathcal{V} | v_1 \equiv v_2 + i \pmod{3}\}$ , see Fig. 2(a).

### 3.1 Scheme 1

We consider a round of 6 time slots  $i \in \{0, 1, 2, 3, 4, 5\}$ , and describe the scheme by defining the transmissions of node  $v = (v_1, v_2)$  at round  $t$ . Here, we define function  $i \oplus j$  as the summation in  $\mathbb{Z}_3$ . This notation will be used throughout this paper for simplicity.

If node  $v \in \hat{\mathcal{V}} \cap \mathcal{V}_i$ , it receives at time slot  $i \oplus 1$  and  $i \oplus 2$  and transmits

$$x_t^i(v) = y_{t-1}^{i \oplus 2}(v) - y_{t-2}^{i \oplus 1}(v) + x_{t-3}^i(v). \quad (3)$$

at time slot  $i$ .

If node  $v \in \bar{\mathcal{V}} \cap \mathcal{V}_i$ , it receives 3 times at time slot  $i \oplus 1$ ,  $i \oplus 2$  and  $(i \oplus 2) + 3$ , and transmits twice. At time slot  $i$  it transmits

$$x_t^i(v) = \begin{cases} m_t^1(v_2), & \text{if } v \in \mathcal{V}^L, \\ m_t^2(v_1), & \text{if } v \in \mathcal{V}^R, \\ m_t^3(K - v_1), & \text{if } v \in \mathcal{V}^B, \end{cases} \quad (4)$$

and at time slot  $i + 3$  it transmits

$$x_t^{i+3}(v) = \begin{cases} m_{t-v_2}^3(K - v_2) - m_t^1(v_2), & \text{if } v \in \mathcal{V}^L, \\ m_{t-v_1}^1(v_2) - m_t^2(v_1), & \text{if } v \in \mathcal{V}^R, \\ m_{t-K+v_1}^2(v_1) - m_t^3(K - v_1), & \text{if } v \in \mathcal{V}^B. \end{cases} \quad (5)$$

Scheme 1 is illustrated in Fig. 2(b).

### 3.2 Scheme 2

Scheme 2 is a dual scheme of Scheme 1, in which each interior node needs to transmit twice but only receive once in each round. Similarly, we consider the transmissions of node  $v = (v_1, v_2)$  in round  $t$ .

If node  $v \in \hat{\mathcal{V}} \cap \mathcal{V}_i$ , it receives at time slot  $i$ , transmits

$$x_t^{i \oplus 1}(v) = y_{t-1}^i(v) + x_{t-3}^{i \oplus 1}(v) \quad (6)$$

at time slot  $i \oplus 1$  and transmits

$$x_t^{i \oplus 2}(v) = -y_{t-2}^i(v) + x_{t-3}^{i \oplus 2}(v) \quad (7)$$

at time slot  $i \oplus 2$ .

If node  $v \in \bar{\mathcal{V}} \cap \mathcal{V}_i$ , it receives at time slot  $i$  and  $(i \oplus 2) + 3$ , transmits

$$x_t^{i \oplus 1}(v) = \begin{cases} m_{t-1}^1(v_2), & \text{if } v \in \mathcal{V}^L, \\ m_{t-1}^2(v_1), & \text{if } v \in \mathcal{V}^R, \\ m_{t-1}^3(K - v_1), & \text{if } v \in \mathcal{V}^B, \end{cases} \quad (8)$$

at time slot  $i \oplus 1$ , transmits

$$x_t^{i \oplus 2}(v) = \begin{cases} -m_{t-2}^1(v_2), & \text{if } v \in \mathcal{V}^L, \\ -m_{t-2}^2(v_1), & \text{if } v \in \mathcal{V}^R, \\ -m_{t-2}^3(K - v_1), & \text{if } v \in \mathcal{V}^B, \end{cases} \quad (9)$$

at time slot  $i \oplus 2$ , and transmits (5) at time slot  $i + 3$ .

Scheme 2 is illustrated in Fig. 2(c).

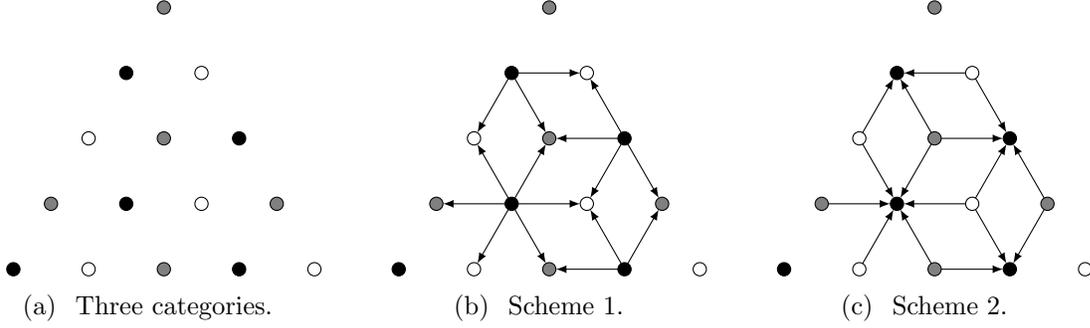


Figure 2: (a) The categories 0, 1, 2, represented by black, white, and gray circles, respectively, (b) Scheme 1 at time slot 0, and (c) Scheme 2 at time slot 0.

### 3.3 Validity of the Schemes

Firstly, we denote the extra transmission and reception in the last 3 time slots of node  $v \in \bar{\mathcal{V}}$  in round  $t$  as  $\tilde{x}_t(v)$  and  $\tilde{y}_t(v)$ . Then, we consider Scheme 1. Observe that each interior node, as well as each of the boundary nodes during the first 3 time slots, only transmit once. Hence, we use the notation  $x_t(v_1, v_2)$  for the transmitted symbol of node  $v = (v_1, v_2)$  in round  $t$  during the first 3 time slots. Then by (3) we have

$$\begin{aligned} x_t(v_1, v_2) &= x_{t-1}(v_1 - 1, v_2) + x_{t-1}(v_1, v_2 + 1) + x_{t-1}(v_1 + 1, v_2 - 1) \\ &\quad - x_{t-2}(v_1 - 1, v_2 + 1) - x_{t-2}(v_1 + 1, v_2) - x_{t-2}(v_1, v_2 - 1) + x_{t-3}(v_1, v_2). \end{aligned} \quad (10)$$

Now we establish the following lemma.

**Lemma 1** *Let  $(v_1, v_2) \in \hat{\mathcal{V}}$ .*

$$x_t(v_1, v_2) = m_{t-v_1}^1(v_2) + m_{t-K+v_1+v_2}^2(v_1) + m_{t-v_2}^3(K - v_1 - v_2). \quad (11)$$

The proof of this lemma is similar to the proof for Lemma 2 in [8], since the coding scheme in (10) is similar to the one used in [8], which considers symbols in  $\mathbb{F}_2$  instead of  $\mathbb{F}_q$ . Thus, we omit the proof of this lemma here to save space.

Now we prove that in each round, a source symbol is decoded at each destination, which validate the scheme. Since the network and our coding schemes are symmetric, w.l.o.g. we consider only the sessions  $m^1(i)$  from left to right.

**Lemma 2** *For the session  $m^1(v_2)$  and its destination  $v = (K - v_2, v_2) \in \mathcal{V}_i$ , the symbol  $m_{t-v_1}^1(v_2)$  can be decoded at the end of round  $t - 1$  by*

$$y_{t-1}^{i \oplus 2}(v) - y_{t-2}^{i \oplus 1}(v) + x_{t-3}^i(v) + \tilde{y}_{t-1}(v), \quad (12)$$

for Scheme 1.

*Proof:* W.l.o.g. we assume  $v \in \mathcal{V}_0$ . By the definition of the categories, for the four neighbors of node  $v$ , we have nodes  $(v_1 - 1, v_2 + 1), (v_1, v_2 - 1) \in \mathcal{V}_1$  and nodes  $(v_1 - 1, v_2), (v_1 + 1, v_2 - 1) \in \mathcal{V}_2$ . Thus we have  $y_{t-1}^2(v_1, v_2) = x_{t-1}(v_1 - 1, v_2) + x_{t-1}(v_1 + 1, v_2 - 1)$ ,  $y_{t-2}^1(v_1, v_2) = x_{t-2}(v_1 - 1, v_2 + 1) + x_{t-2}(v_1, v_2 - 1)$  and  $\tilde{y}_{t-1}(v_1, v_2) = \tilde{x}_{t-1}(v_1 + 1, v_2 - 1)$ .

Thus, by (4), (5) and Lemma 1 we have (12) equal to

$$\begin{aligned} & x_{t-1}(v_1 - 1, v_2) + x_{t-1}(v_1 + 1, v_2 - 1) + x_{t-2}(v_1 - 1, v_2 + 1) \\ & + x_{t-2}(v_1, v_2 - 1) + x_{t-3}(v_1, v_2) + \tilde{x}_{t-1}(v_1 + 1, v_2 - 1) \end{aligned} \quad (13)$$

$$\begin{aligned} = & m_{t-v_1}^1(v_2) + m_{t-2}^2(v_1 - 1) + m_{t-v_2-1}^3(1) + m_{t-1}^2(v_1 + 1) - m_{t-2}^2(v_1 - 1) \\ & - m_{t-v_1-2}^1(v_2 - 1) - m_{t-3}^2(v_1) - m_{t-v_2-1}^3(1) + m_{t-3}^2(v_1) \\ & + m_{t-v_1-2}^1(v_2 - 1) - m_{t-1}^2(v_1 + 1) \end{aligned} \quad (14)$$

$$= m_{t-v_1}^1(v_2). \quad (15)$$

■

The proof for the validity of Scheme 2 is similar to Scheme 1 since the two schemes are dual. For Scheme 2, observe that for  $v \in \mathcal{V}_i$ ,  $x_{t+1}^{i\oplus 1}(v) = -x_{t+2}^{i\oplus 2}(v) = y_t^i(v) + x_{t-2}^{i\oplus 1}(v) = -y_t^i(v) - x_{t-1}^{i\oplus 2}(v)$ . We then define  $x_t(v) = x_{t+1}^{i\oplus 1}(v) = -x_{t+2}^{i\oplus 2}(v)$ , and the symbol  $m_{t-v_1}^1(v_2)$  can be decoded by  $y_t^i(v) + x_{t-2}^{i\oplus 1}(v) + \tilde{y}_{t-1}(v)$  follows the same steps as the (13)-(15). The validity of Scheme 2 is thus proved.

## 4 Energy Benefit

In this section, we compare our schemes to some existing schemes, in particular, the traditional routing based scheme, and the network coding based scheme proposed in [8]. Here, we consider the energy consumption for the schemes, which is defined as the average energy required by all nodes for each destination to retrieve one source symbol. Here, we ignore the energy consumption in an initial startup phase and consider only the steady-state behavior. The throughput of the network, e.g., the rate of the sessions, is not of our concern in this paper.

Firstly, we consider the scheme based on traditional routing strategy. With traditional routing, clearly, the optimal scheme is that all sessions go along their shortest paths. Since each interior node in the network is on the shortest paths of 3 sessions heading to 3 different directions, 3 transmissions and receptions are needed to relay one symbol for each session. Meanwhile, for the nodes on the borders, which are the sources and the destinations, they only need to transmit and receive the symbols for their corresponding sessions. Since the network and sessions are determined by  $K$  and the pair of transmit and receive energy  $e_t$  and  $e_r$ , the energy consumption is thus a function of  $K$ ,  $e_t$  and  $e_r$ . Hence, we have the energy consumption for traditional routing  $E^{\text{TR}}(K, e_t, e_r)$  as

$$E^{\text{TR}}(K, e_t, e_r) = 3(K - 1)(e_t + e_r) + (K - 1)(K - 2)(3e_t + 3e_r)/2. \quad (16)$$

In [8], a network coding scheme is proposed, in which the interior nodes broadcast the linear sums of the symbols heading different directions, instead of transmit them separately. In each round, which is defined similarly to the round in our schemes, each interior node needs to transmit only once but receive 6 times, and each boundary node needs to transmit twice and receive 4 times. We can thus calculate the energy consumption of this scheme, denoted as  $E^{\text{NC}}(K, e_t, e_r)$ , and

$$E^{\text{NC}}(K, e_t, e_r) = 3(K - 1)(2e_t + 4e_r) + (K - 1)(K - 2)(e_t + 6e_r)/2. \quad (17)$$

Here, we define the *energy benefit* of a certain scheme as the ratio of  $E^{\text{TR}}(K, e_t, e_r)$  to the energy consumption of that scheme when  $K$  tends to infinity. Thus, we directly have the energy benefit of the network coding based scheme proposed in [8] by (16) and (17)

$$B^{\text{NC}}(e_t, e_r) = \lim_{K \rightarrow \infty} \frac{E^{\text{TR}}(K, e_t, e_r)}{E^{\text{NC}}(K, e_t, e_r)} = \frac{3e_t + 3e_r}{e_t + 6e_r}, \quad (18)$$

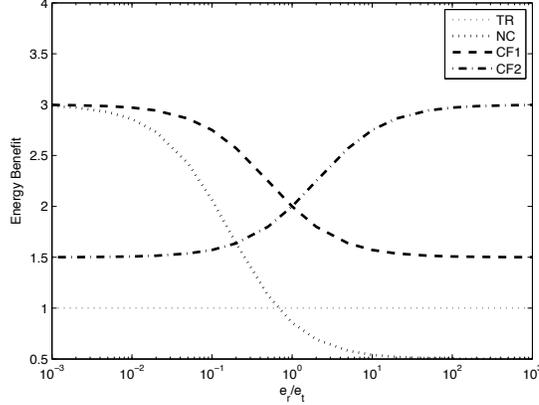


Figure 3: The energy benefit comparison between schemes as a function of  $e_r/e_t$ . Here, TR stands for traditional routing, NC stands for network coding based scheme in [8], and CF1 and CF2 stand for Scheme 1 and 2 in Section 3, respectively.

which has also been presented in [9]. From (18), it is clear that this scheme reduces the energy consumption of the network by a factor of 3 when  $e_r$  is negligible comparing to  $e_t$ . However, for the case that  $e_r$  is comparable to  $e_t$ , the performance boost is very limited or completely gone.

Now we consider the schemes in Section 3. According to the schemes, we have the energy consumption of Scheme 1 and 2, denoted as  $E^{\text{CF1}}(K, e_t, e_r)$  and  $E^{\text{CF2}}(K, e_t, e_r)$ , respectively, and

$$E^{\text{CF1}}(K, e_t, e_r) = 3(K-1)(2e_t + 3e_r) + (K-1)(K-2)(e_t + 2e_r)/2, \quad (19)$$

$$E^{\text{CF2}}(K, e_t, e_r) = 3(K-1)(3e_t + 2e_r) + (K-1)(K-2)(2e_t + e_r)/2. \quad (20)$$

By the definition of energy benefit and (16), we have the energy benefit of the schemes

$$B^{\text{CF1}}(e_t, e_r) = \lim_{K \rightarrow \infty} \frac{E^{\text{TR}}(K, e_t, e_r)}{E^{\text{CF1}}(K, e_t, e_r)} = \frac{3e_t + 3e_r}{e_t + 2e_r}, \quad (21)$$

$$B^{\text{CF2}}(e_t, e_r) = \lim_{K \rightarrow \infty} \frac{E^{\text{TR}}(K, e_t, e_r)}{E^{\text{CF2}}(K, e_t, e_r)} = \frac{3e_t + 3e_r}{2e_t + e_r}. \quad (22)$$

In Figure 3, we compare the energy benefit of the schemes for different  $e_r/e_t$ . It shows that we significantly decrease the energy consumption of the network and achieve a higher energy benefit. Compared to the traditional routing based strategy, compute-and-forward will save the energy of the network by a factor between 1.5 and 3, (both schemes have a energy benefit of 2 when  $e_r = e_t$ ), depending on the ratio between transmit energy and receive energy. In other words, applying compute-and-forward in this network is always beneficial for energy saving. This is essentially different from the energy benefit of plain network coding of [9]. Since when  $e_r$  is large, the plain network coding based scheme even consumes more energy than traditional routing approach. Furthermore, Scheme 1 also outperforms the plain network coding based scheme at all configurations when  $e_r > 0$ .

## 5 Conclusion

In this paper, we have proposed two compute-and-forward based schemes that achieve energy benefit between 1.5 to 3, depending on the transmit and receive energy of the

nodes in the network, which indicates that the energy consumption in the network can be saved by at least a factor of 1.5 compared to traditional routing. Moreover, Scheme 1 also outperforms the plain network coding based scheme for any  $e_r > 0$ . These results show the superiority of compute-and-forward based schemes used for an energy saving purpose in networks where the receive energy is not negligible, since they are capable to reduce the number of receptions. The energy benefit of compute-and-forward based schemes in other networks is a very interesting problem for further study.

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