International Conference on Particle-Based Methods
Particles 2009
E. Oñate and D.R.J. Owen (Eds)

NODAL INTEGRATION OF MESHLESS METHODS

W. Quak*, **A.H. van den Boogaard***

*Department of Mechanical Engineering
University of Twente
P.O. Box 217, 7500 AE Enschede, The Netherlands
e-mail: w.quak@ctw.utwente.nl, web page: http://www.tm.ctw.utwente.nl

Key words: nodal integration, SCNI, meshless methods, forming

Abstract. Meshless methods offer interesting properties for the simulation of bulk forming processes. This research concerns the investigation of the stabilized conforming nodal integration scheme (SCNI) for use in metal-forming processes. Two tests are carried out. Firstly, the performance of SCNI is compared to a standard integration scheme. The performance seems problem specific. Secondly the footing of a piece of nearly incompressible material is used for testing the locking behavior of the method. No volumetric locking was found.

1 INTRODUCTION

Finite element simulations of large-deformation bulk forming processes in a Lagrangian formulation can be problematic. When simulating for instance an extrusion or forging process, many re-meshing steps need to be taken to avoid too much mesh distortion. After a re-meshing step, the material data and the state variables have to be mapped from the old mesh to the new mesh. This mapping can cause inaccuracies.

It is expected that meshless methods avoid these mesh-related problems since their shape functions are not defined on a mesh and their nodal connectivity is constantly re-evaluated during the simulation.

Secondly, the particle character of meshless methods might contribute to avoid the mapping of state variables. If material points and nodes coincide, the convection of state variables becomes superfluous. Especially for history dependent material models this will be a beneficial property. However, having material points and nodes a the same position will require a nodal integration scheme. This type of integration is usually unstable. Therefore the stabilized conforming nodal integration scheme (SCNI) is examined in this study. Furthermore, this integration scheme has another interesting property for metal forming. The phenomena of volumetric locking is absent.

The objective of this research is to examine the behavior of this nodal integration scheme. Therefore it is compared to a standard Gauss type of integration technique as used in finite elements. Secondly the scheme is tested in incompressibility by simulating a footing problem.

2 NUMERICAL SCHEME

The SCNI method was introduced by Chen *et al.* [1]. The essence of the SCNI method is that the strain at a node is determined by averaging the strain over a domain accompanying that particular node. This domain is usually a Voronoi cell. So in formula form, the strain for small deformations is calculated as:

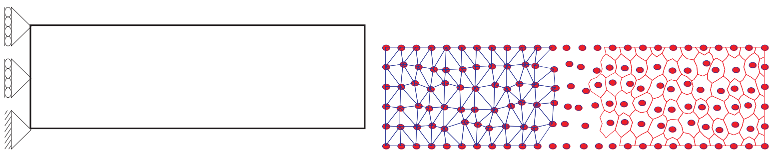
$$\tilde{\varepsilon}_{ij}(\mathbf{x}_k) = \frac{1}{\Omega_k} \int_{\Omega_k} \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \mathrm{d}\Omega \tag{1}$$

where $\mathbf{x}_k$ is the location of a node and Ω_k the accompanying area or volume for 2d or 3d respectively. This modified strain definition is used in a Galerkin weak form. For the following tests, the displacement field is parameterized by local maximum-entropy shape functions as introduced by Arroyo and Ortiz [2]. The shape functions are calculated by means of a Newton-Rhapson procedure. The tolerance to stop the iterations is the machine accuracy. The shapes posses first order reproducibility. The shape functions are defined independently of the tesselation, which is only used for averaging the strain. Finally, the boundary conditions are applied by using the method of Lagrangian multipliers. For now, the geometry is assumed linear.

3 RESULTS

3.1 a comparison of two integration schemes

In this test, the performance of the SCNI scheme is compared to a Gauss type integration scheme. This integration scheme is constructed on the Delaunay triangulation of a cloud of nodes. In each Delaunay triangle, a three point integration rule is used. The SCNI integration employs strain averaging cells based directly on the Delaunay triangles. Instead of calculating the Voronoi diagram of the Delaunay triangulation, a triangle is divided in three parts and each of the parts is used for the averaging procedure of the corresponding node. The construction of Voronoi cells on the boundary is avoided this way. Constructing Voronoi cells at the boundary of a domain is not straightforward. By applying the divergence theorem on Equation (1) a boundary integral is obtained which is approximated with a three point integration rule on each of the cell sides.



(a) beam model (b) triangulation and tesselation

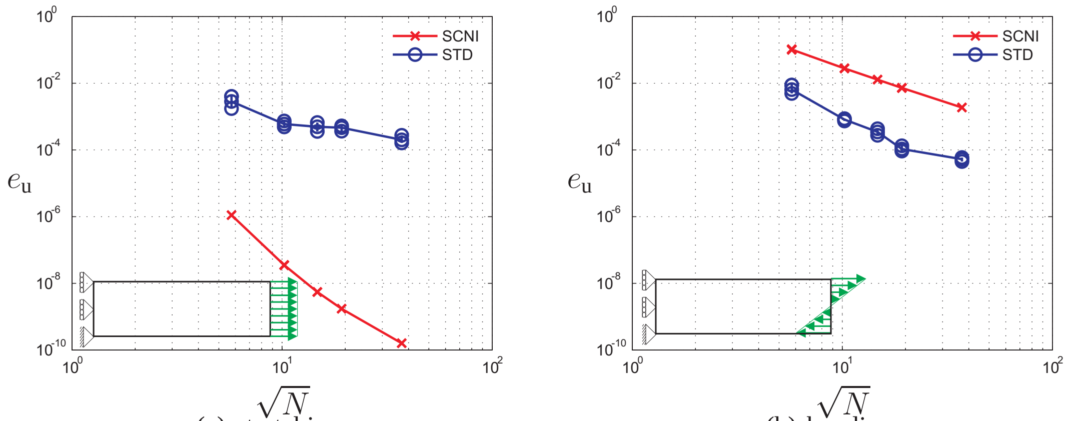
Figure 1: problem and discretization

Figure 1(a) shows the geometry of the problem. A cloud of nodes in the shape of the beam is generated. The nodal locations are randomly perturbed to make an irregular grid. In Figure

1(b) the computational geometrical discretization can be seen. At the left-hand side a part of the triangulation is displayed. The right-hand side shows the tesselation partly. Two load cases were analyzed. Firstly the stretching of a plane strain beam is analyzed. Secondly the bending of the beam is treated. A compressible linear elastic material model was used. Three runs were performed to examine the effect of the random perturbation on the accuracy. Figure 2 shows the results. STD and SCNI are the abbreviations of the standard integration scheme and the nodal integration scheme respectively. On the vertical axis a discrete error norm is plotted which is defined as follows:

$$e_{\mathrm{u}} = \frac{1}{N}\sqrt{\sum_{i=1}^{N} \|\mathbf{u}_{\mathrm{h}}\left(\mathbf{x}_i\right) - \mathbf{u}_{\mathrm{exact}}\left(\mathbf{x}_i\right)\|^2} \quad (2)$$

The nodal locations are given by $\mathbf{x}_i$, the approximated solution is $\mathbf{u}_{\mathrm{h}}$, and N is the number of nodes.



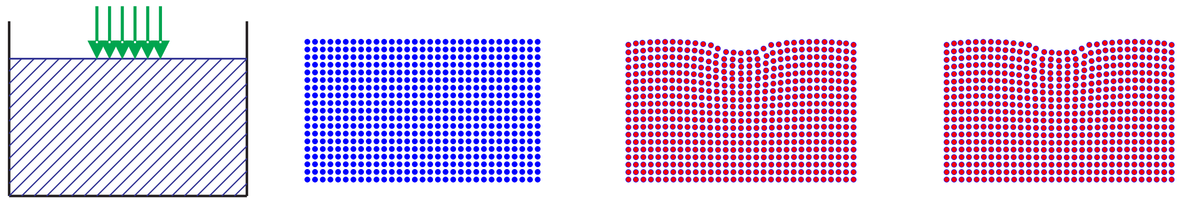
(a) stretching (b) bending

Figure 2: error e_{u} for the two problems

It can be seen that the behavior of the two schemes is different. SCNI is more accurate for the first problem where the opposite is the case for the second problem. On the contrary, the standard integration scheme gives similar performance for the two problems. Secondly, it can be seen that for the nodal integration, the error is not affected by the perturbation of the nodes. The error for the three simulations based on differently perturbed node grids is nearly equal. For integrating non-polynomial shapes the SCNI method seems suitable, though a drawback is the less accurate behavior in pure bending.

4 footing of incompressible material

In this test the performance of the method in incompressibility is examined. For this purpose a square block of nearly incompressible material is supported on three sides and indented at the free side. Figure 3(a) shows the geometry. The displacements of the left, bottom and right side of the block are completely suppressed (stick). At the top side a pressure is prescribed to indent the material. The test is performed with an elastic plane strain material model. Two cases are analyzed, once with a Poisson's ratio ν of 0.499 and once with $\nu = 0.49999$. If there is locking, the results between the two cases would differ. The displacements with $\nu = 0.49999$ would be a lot smaller than the solution obtained with $\nu = 0.499$.



(a) footing model (b) undeformed (c) $\nu = 0.499$ (d) $\nu = 0.49999$

Figure 3: undeformed and deformed shapes of the footing problem

Figure 3(b) shows the nodes of the undeformed geometry. Figures 3(c) and 3(d) show the deformed configuration for a Poisson's ratio of $\nu = 0.499$ and $\nu = 0.49999$ respectively. It can be seen that there is no spurious stiffening of the results. To check the absence of volumetric locking for all circumstances a more rigorous test is required, though for now locking has not been found.

5 CONCLUSION

The comparison of the SCNI integration scheme with a standard integration scheme showed differences between the two. For the stretching of the beam, the SCNI scheme is more accurate than the standard integration scheme whereas the pure bending of the beam showed the opposite. A footing test showed the absence of volumetric locking for the SCNI method. No spurious stiffening was seen.

Currently, the scheme is being implemented geometrically non-linear by means of an implicit updated-Lagrangian formulation.

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