Wireless Communication Graphs

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Abstract

In this paper, we consider graph models which represent the communication in wireless ad-hoc networks. In such networks, each node equipped with a radio has a certain transmission range, and all surrounding nodes in this range can receive a transmission sent. In ideal settings, this transmission range creates a circular disk around each node, so that disk intersection graphs are common models. We review some properties of these disk graphs and introduce more realistic and sophisticated geometric graphs to model wireless communication.

These models are explored and exploited to obtain approximability results for two important problems: maximum independent sets and minimum dominating sets. Although these more realistic models have less structure , the same complexity status as for disk graphs can be achieved. Furthermore, we present simple, constant factor approximation algorithms for the problems that run locally in each node, and that do not rely on positional information, making them independent of localization.

1. Introduction

A wireless, ad-hoc network is created by the wireless communication links between a collection of radio transceivers. The characteristics coming from this setting give the resulting communication graph certain structures. In this paper, we look at these structures and present geometrically inspired graphs that model the resulting network.

The network we look at consists of a possibly large set of wireless nodes that are spread out in a certain area. This setting closely reflects wireless sensor networks (WSNs), whose purpose is physical environment monitoring, and providing this information to the user (application). Each node is equipped with one or more sensors, whose readings are transported via intermediate nodes towards a data sink. In WSNs, there are additional challenges with respect to the wireless communications due to the lack or shortage of resources, especially energy.

The graph models we present are geometric intersection and containment graphs of objects that represent the area where a transmission of a certain node can be received by others. In the most basic (and ideal) case, this area is a disk centered at the node's position. Objects, fading, reflections etc. influence this area. Thus, assuming circular areas may not reflect the reality of wireless communication close enough. We therefore extend these disk graphs towards more general objects. So-called Coverage Area Graphs need no specific shape, only a minimum volume of the area where communication is possible.

In the context of efficient wireless networking, certain subgraphs have a prominent role for several communication strategies. In this paper, we focus on independent and dominating sets. A subset of the nodes is called independent if no two nodes from this set are able to communicate with one another directly. A subset is called dominating if all nodes of the network are in reach of at least one node of this subset. We are interested in the problem of finding independent sets of maximum cardinality, and dominating sets of minimum size.

The graph models allow for strong results on the performance of algorithms solving the above problems. They admit a (centralized) polynomial-time approximation schemes (PTAS) with a $(1+\varepsilon)$ performance ratio. However, especially in large-scale, low-resource wireless networks, like WSNs, the lack of a fixed networking infrastructure calls for decentralized, or localized, distributed algorithms. We present constant factor approximations by a locally executed algorithm for the problems.

Localization plays an important role in these networks. However, for example after the initial employment of the sensor nodes, positional information may not (yet) be available at all the nodes. Nevertheless, some structures need to be set up and maintained during all phases of a network's lifetime to ensure efficient operation. Reliable localization is a non-trivial task [24].

For the most basic model, where all nodes have the same circular transmission range, the problem of computing a feasible location for each of the nodes is an NP-hard problem [4], similar results are known for other graph models [15]. So, on a practical level, algorithms that work without relying on positional information are preferable, and we therefore put a focus on algorithms creating independent and dominating sets which do not rely on localization algorithms.

The remainder of this paper is organized as follows. In the next section, we introduce the maximum independent set problem and the minimum dominating set problem in more detail and give some applications where these problems are used for efficient network operations. In Section 3, we propose four geometrically inspired graph models for wireless communication. Then, in Section 4, we present strategies to solve the considered optimization problems and explore their performance with respect to the wireless graph models. The paper concludes with a short summary of the results obtained.

2. Problems and Applications

There is a great number of basic structures and resulting optimization problems in graph theory that are important when dealing with efficient wireless communication strategies. Two of the most prominent structures are *independent* and *dominating* sets, which we introduce next, and focus on throughout the remainder of this paper. In this section, we then continue to give a short overview of some of the applications where these structures are a vital part.

Generally speaking, communication in networks is modeled by a directed graph $G = (V, A)$. In our case, the set V represents the nodes, each equipped with a radio for wireless communication. The edges then represent the possible recipients of messages, i.e. there is an edge $(u, v) \in A$ if and only if a transmission from node $u \in V$ can be received by a neighbor $v \in V$. We denote by $N(v) \subset V$ the set of all neighbors of a node $v \in V$. By nature of the wireless medium, a message broadcast by a node can be received by all neighbors.

The problems we are considering involve the creation of subsets of the nodes with certain properties:

- A subset $I \subset V$ is called *independent* if no two nodes in I are connected by an edge in G, and
- $D \subset V$ is a *dominating* set if every node is either contained in D or there exists an edge connecting a node from D to this node.

Nodes in an independent set do not interfere each other during simultaneous transmissions, and nodes in a dominating set can be used to efficiently reach the entire network by broadcasts from only these nodes. If, additionally, a dominating set consists of only one connected component, it is referred to as *connected* dominating set.

An independent set is called *maximal* if no other node can be added to it without violating the independence property. It is easy to see that any maximal independent set is also a dominating set in the graph. In fact, any non-dominated node could be added to the independent set while keeping the property.

We are now interested in the problems of finding an independent set of large cardinality, the Maximum Independent Set (MIS) problem, and finding a dominating set of small size, the Minimum Dominating Set (MDS) problem.

Both problems are known to be NP-hard, and for a general graph, there are known lower bounds on the best possible approximation (unless P=NP). A maximum independent set is not approximable within a factor of $|V|^{1/2-\varepsilon}$ for any $\varepsilon > 0$ [10], and there is an $O(|V|/(\log |V|)^2)$-approximation given in [3]. A minimum dominating set cannot be approximated within $c \log |V|$, for $c > 0$ [22], but there is an algorithm with a performance guarantee of $1 + \log |V|$ [13]. This means that, especially in large networks, an efficient approximation is almost not possible.

Besides the introduced basic versions of the considered problems, there also exist weighted versions: each node is given a weight, and the goal is to find a structure of minimum/maximum weight, which is computed by summing up the weights of the nodes in the respective subsets. A weight may correspond to a node's capability to perform additional duties. It can be determined taking into consideration aspects like the residual energy of a node, its memory and processing capabilities, the number of neighbors, or mobility indicators. Usually, these weights are computed locally in each node, and may depend on the application the structure is used for.

The MIS and MDS problems are used in a variety of different applications, especially at the lower levels of the network protocol stack. Some of the applications directly dealing with the topology of the wireless, ad-hoc network are given next.

A. Clustering

Cluster-based control structures allow for more efficient use of resources. A hierarchical view of the network created through clustering decreases the complexity of the underlying network, especially in sensor networks which are expected to consist of large amounts of individual nodes. Also, a clustered structure can make a highly mobile topology appear more static and thus alleviate the effects of mobility.

On a topology level, clustering is usually done by grouping nodes inside a certain area, which are then controlled by a designated node. For the lower layers of the networking protocol stack, clusters are created based on proximity, i.e. based on nodes that are in each other's radio range.

Many proposed clustering schemes work by identifying control nodes that form a (maximal) independent set in the network, and which then perform certain specific tasks like allocation of bandwidth and channel [2], [5], [9].

B. Flooding and Broadcasting

Many on-demand routing algorithms designed for ad-hoc networks rely on efficient flooding strategies, including DSR [14] and AODV [21]. Basic, network-wide flooding causes the so-called broadcast storm problem [18], resulting in excessive contention and collisions, i.e. a large protocol overhead. Using a (connected) dominating set of small size as a virtual backbone to propagate flooding messages, overcomes this problem by greatly reducing the number of messages [1], [7].

A virtual backbone also supports efficient broad- and multicast traffic, e.g. the propagation of link quality information in QoS-routing [23].

C. Sleeping Patterns

Wireless sensor networks are expected to be in operation for long periods of time. Running on batteries this is a challenging task. Sensor networks, for example in event detection, have long periods of inactivity. These networks are also expected to be rather dense so that the given redundancy can be exploited and allow several nodes to turn off their radio and other parts to conserve energy. However, for the network to remain operational, connectivity needs to be maintained in such a way that if a node with inactive radio senses something of

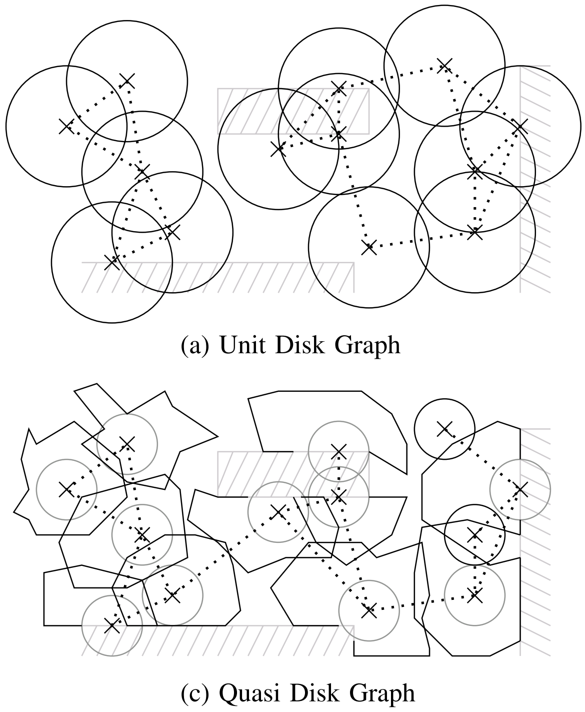

Fig. 1: Example of different geometric intersection graphs.

importance, it can become active and successfully route this information to a central user.

An example of a connected dominating set based solution is given in [12], where the nodes locally decide on their participation in the network.

3. Graph Models for Wireless Communication

As seen in the previous section, assuming the communication graph $G = (V, A)$ to be a general graph does not allow for efficient algorithms to create independent or dominating sets, especially in large-scale networks. However, in general, the nature of wireless transmissions does not lead to an arbitrary graph, but to a graph with some structure. In this section, we therefore propose geometrically inspired models for wireless communication.

For these models, we may suppose that the wireless nodes are placed in the 2-dimensional euclidean plane, i.e. for the nodes, there exists a mapping $f : V \to \mathbb{R}^2$ which gives each node $v \in V$ its location $f(v) \in \mathbb{R}^2$ in the plane. Furthermore, each node $v \in V$ has a certain area which is covered by the node's radio. This area is represented by $A_v \subset \mathbb{R}^2$. As a consequence another node $u \in V$ can receive a transmission sent by v if $f(u) \in A_v$ holds.

The localization function $f : V \to \mathbb{R}^2$, together with the coverage area A_v of each node $v \in V$, is called a *geometric representation* of V. Communication graphs are then defined using the geometric objects that present the coverage area of a node's radio.

There are two ways of defining the communication graph, i.e. the set set of edges of the graph. The first way is the *containment model*, which is characterized by

$$(u, v) \in A \iff f(u) \in A_v.$$

This model gives the possible direct communication between the nodes, it is thus a directed communication graph.

If we only look at the coverage areas of the nodes, we can also define the *intersection model* as follows:

$$(u, v) \in A \iff A_u \cap A_v \neq \varnothing.$$

With this symmetric model, interference during simultaneous transmissions can be explored. If two nodes transmit at the same time, a third node in the nonempty intersection receives both transmissions and may thus not be able to reconstruct the messages (hidden terminal problem).

We now define a *Coverage Area Graph* (CAG) as follows.

Definition 1: Consider a set V of nodes, and for each $v \in V$ a coverage area A_v. Then, the resulting geometric intersection (containment) graph is called an intersection (containment) *Coverage Area Graph* (CAG).−

The intersection graph is undirected. If we consider each undirected edge to be a two-way edge, we can state the following relationship between the two models:

Lemma 1: Let $A_v \subset \mathbb{R}^2, v \in V$, be a collection of geometric objects. Furthermore, let $G = (V, A)$ and $G' = (V, A')$ be the resulting geometric intersection and containment graph respectively. Then G' is completely contained in G.−

Proof: From the definitions, clearly $A' \subset A$ holds. □

We now look at specially structured coverage areas A_v, which then result in specially structured geometric graphs for the wireless communication.

In ideal settings, e.g. open field without obstacles, the transmission area is a circle centered around the position of the node. The most basic model used for wireless communication is the *Unit Disk Graph* (UDG). Suppose that all nodes send with the same signal strength and thus have the same circular coverage area with radius $c > 0$. The set of edges then satisfies the following simple characterization:

$$(u, v) \in A \iff \|f(u) - f(v)\| \leq c.$$

Scaling the model, we may assume the transmission range to be of unit length. Note that for the communication graph of unit disks, containment and intersection graph are basically the same, and all communication links are bidirectional.

When the nodes are able to adjust the transmission power, different circular coverage areas emerge. Suppose that the maximum transmission range of a node is given by $C > 0$. Furthermore, in order to achieve any communication, a minimum signal strength is needed, which we assume to create a coverage area of radius $c > 0$. This yields a set of disks of different radii, whose resulting graph is referred to as *Bounded Disk Graph* (BDG). In this case, and the following models, intersection and containment model result in different graphs. For a BDG, we define $d_{\mathrm{BDG}} := c/C$ to be a parameter which gives the ratio between smallest and largest disks. Also, this model may be scaled such that $C = 1$.

While the above two graph models are widely used to obtain strong theoretic results for graph algorithms, one might argue that they are not that realistic since they assume too ideal settings for radio propagation. Next, we go on to present more realistic geometric models for wireless communication graphs.

Refining the idea behind Bounded Disk Graphs by no longer limiting the reasons of different radii to signal power adjustment, but also to environmental reasons like objects, we can define a *Quasi Disk Graph* (QDG,[16]). In such a graph, there are two circles of radius $c > 0$ and $C > c$ which are placed around each node $u \in V$ with the following characteristics for communication: Any node $v \in V$ with $\|f(u) - f(v)\| \leq c$ receives transmissions sent by u, and a node $\bar{v} \in V$ with $\|f(u) - f(\bar{v})\| \geq C$ cannot receive such a transmission from u. In the area between this minimum and maximum range, c and C, the existence of a communication edge is not specified. The coverage area, within these limits, may thus take any shape. Again, let $d_{\mathrm{QDG}} := c/C$ denote the parameter of a QDG giving information about the relation between the smallest and largest possible transmission range.

For some nodes, e.g. mounted on concrete walls, it may not be possible to give a positive radius on the transmission range where there is coverage. Completely leaving the idea of transmission radii, we come back to the CAG. In contrast to the previous three models, we no longer demand the transmission area to be centered around (or even intersecting with) the actual location of a node. For the CAG model, let $C > 0$ denote the minimum radius around $f(v)$, such that the disk with this radius covers the area A_v. In other words, any node $\bar{v} \in V$ with $\|f(v) - f(\bar{v})\| \geq C$ cannot receive a transmission from v for all nodes $v \in V$. Further, we define $a > 0$ to be the minimum area of all transmission areas, i.e. $a := \min_{v \in V}\{\mathrm{vol}(A_v)\}$. Then, a parameter $d^2_{\mathrm{CAG}} := C^2/a$ gives the relation between maximum transmission area and minimum coverage area. Note that in this case, we compare the areas of coverage. Therefore, d can be seen as an indicator of the respective ranges, but especially $\sqrt{a}$ needs not to give the actual value for the smallest transmission range like in the previous three models.

Note that the latter two models, QDG and CAG, not only allow for modeling the influence of objects on the transmission, but also for fading and resulting unreliable transmissions. All that is needed to apply these models is an area where there is certain reception (inside area) and another area where there is no reception (outside area). Unreliable transmissions, fading etc. may occur in the area that lies in between these two, as radio propagation details in this area need not to be specified.

A graphic overview of the different geometric communication graph models is presented in Figure 1, effects of fading have been neglected in (c) and (d).

Denote by $\mathcal{G}_{\mathrm{UDG}}$ the set of all Unit Disk Graphs, by $\mathcal{G}_{\mathrm{BDG}}$ the set of all BDGs, etc. It is easy to see the following relationship between the above graph models:

Lemma 2:

$$\mathcal{G}_{\mathrm{UDG}} \subset \mathcal{G}_{\mathrm{BDG}} \subset \mathcal{G}_{\mathrm{QDG}} \subset \mathcal{G}_{\mathrm{CAG}}. -$$

For the remainder of this paper, for all the models, we assume the parameters d for the communication graphs to be fixed (or bounded). For given wireless nodes, the values for d are usually determined experimentally. Note that an extension of the concepts and graphs to geometric spaces of higher dimensions is straightforward.

4. Algorithms and Performance

In this section, we present simple algorithms to solve both the MIS and MDS problems on communication graphs. In a general, undirected graph, algorithmic performance bounds depend on the size of the graph. In contrast to this, the wireless transmission graphs presented in Section 3 allow for size-independent, and thus better bounds.

In this section, we do not differentiate between containment and intersection graph model, the obtained results hold for both models. Since the problems of Section 2 are defined for undirected graphs, we assume all arcs resulting from the geometric models to be undirected as well. This assumption follows the applications, as bidirectionality is usually assumed for wireless communications. It can be achieved by eliminating unidirectional edges.

Both problems, MIS and MDS, admit a polynomial-time approximation scheme (PTAS) on all the geometric graphs presented earlier [19], [20]. For any $\varepsilon > 0$, a $(1 + \varepsilon)$-approximation can be computed in polynomial time, and the respective algorithms do not need a geometric representation for this.

In the context of ad-hoc, wireless networks, a centrally executed algorithm may not be feasible, and topology changes may force many recalculations. In the following we therefore

```
INPUT: Graph G = (V, A)
I := ∅;
WHILE V ≠ ∅ DO
    Choose v ∈ V;
    I := I ∪ v;
    V := V \ N(v);
END;
```

Fig. 2: Generic Greedy Algorithm.

present a simple, distributed strategy that gives a constant factor approximation for the graph models. The performance guarantees come from a forbidden induced subgraph property. This property is satisfied by the graph models, and it allows for bounding the worst-case performance of the algorithms presented.

A graph is said to be $K_{1,p}$-free if it contains no induced subgraph that is isomorphic to a $(1, p)$-bipartite graph. We state two results about $K_{1,p}$-free graphs from [11] and [17].

Lemma 3: Let $G = (V, A)$ be a $K_{1,p}$-free graph. Then

- any maximal independent set is a $(p-1)$-approximation of a maximum independent set in G, and
- the cardinality of any maximal independent set is at most $p-1$ times that of a minimum dominating set.−

Probably the easiest way to create a maximal independent set in any graph is a greedy strategy: Adding a node to the (partial) subset and then eliminate this node and all neighbors from the graph. This generic greedy approach is presented in Figure 2.

It is easy to modify the centrally executed greedy algorithm to run locally, where each node only needs to interact with its direct neighbors. Such an algorithm is better suited for large-scale networks, as it runs (almost) independent of the network size. Suppose that each node has a unique weight assigned to it, e.g. an *id*-number. The local version of the algorithm then works as follows:

- all nodes start out in an undecided state,
- an undecided node with largest weight in its neighborhood, which does not yet contain a master-node, declares itself as master-node, and
- nodes that learn about a neighboring master node declare themselves as slave-nodes.

It can be shown that this algorithm is equivalent to the greedy strategy. Moreover, it is efficient in the sense that for the algorithm to complete, only one fixed-size message per node is sufficient to create the desired structure (master-nodes). Furthermore, the local algorithm can be extended by maintenance routines so that the structure is kept up during topology changes due to mobility, node failures etc., e.g. [2].

For the weighted version of the MIS problem, a small adjustment has to be made to the greedy algorithm: we always need to add a node with largest weight in its neighborhood to the partial subset in order to obtain the $(p-1)$-approximation guarantee [17].

Next, we show for each of the geometric graph models that there is a bound on p, making each of the resulting graphs

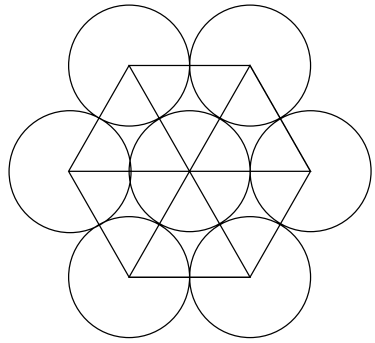

Fig. 3: A UDG is $K_{1,6}$-free.

$K_{1,p}$-free. Thus, by Lemma 3, we know that a greedy strategy leads to a $(p-1)$-approximation. Especially if p is a constant, we obtain a constant factor approximation.

It is easy to see that a UDG is $K_{1,6}$-free, as it is impossible to arrange 6 non-touching disks that are still intersecting with a central disk (Figure 3).

For a Bounded Disk Graph, we look at the largest disk of radius C surrounding a node $v \in V$. Inside this disk with area πC^2, we can place no more than $\frac{\pi(C+c)^2}{\pi c^2} = O(\frac{1}{d^2_{\mathrm{BDG}}})$ centerpoints of disks with smallest radius $c < C$ in such a way that they do not intersect with each other. Thus, a BDG is $K_{1,p}$-free for $p = O(\frac{1}{d^2})$, and the greedy strategy yields an $O(\frac{1}{d^2})$-approximation for the problems.

A similar argument as for BDGs, looking at the minimum and maximum transmission range of nodes, gives the same $O(\frac{1}{d^2_{\mathrm{QDG}}})$ approximation guarantee for Quasi Disk Graphs.

For Coverage Area Graphs, we again consider the number of nodes we can place independently inside a disk of radius $C > 0$. Obviously, in the extreme case, each of these nodes' coverage areas is minimal, i.e. $\mathrm{vol}(A_v) = a$. However, the coverage area may stretch up to C away from a node. We therefore look at placing the nodes inside a disk of radius C, where the coverage area is allowed to be as far as C away from the node's position. Overall, we can thus place no more than $\frac{(2C)^2}{a} = O(\frac{1}{d^2_{\mathrm{CAG}}})$ of the nodes.

In the unweighted case, the above results are independent of the greedy choice of the node that will be added to the respective subset. For the MIS problem, a more sophisticated choice of the node may yield a better approximation guarantee. If the graph is a UDG, always choosing the "leftmost" node will yield a 3-approximation. Furthermore, this approach can be adjusted to the case when no geometric representation is available [17]. For BDGs, a maximum independent set can be approximated within a factor of 5 by always choosing a node with smallest remaining transmission range. This node, together with its neighborhood, is then locally $K_{1,6}$-free.

So far, we have only stated results for the general case of the Maximum Independent and Minimum Dominating Set problem. If a connected dominating set is required, extending the greedy strategy to connect the dominating nodes also yields a constant-factor approximation. This is a consequence of the following, well-known lemma which states that any

Maximum (Weight) Independent Set

	$\mathcal{G}$	$\mathcal{G}_{\text{UDG}}$	$\mathcal{G}_{\text{BDG}}$	$\mathcal{G}_{\text{QDG}}$	$\mathcal{G}_{\text{CAG}}$
bound	$\lvert V\rvert^{1/2-\varepsilon}$				
central	$O(\frac{\lvert V\rvert}{(\log\lvert V\rvert)^2})$	$1+\varepsilon$	$1+\varepsilon$	$1+\varepsilon$	$1+\varepsilon$
local	$O(\lvert V\rvert)$	$3^1[5^2]$	$5^3[O(\frac{1}{d^2})]$	$O(\frac{1}{d^2})$	$O(\frac{1}{d^2})$

[1] unweighted [2] weighted [3] with representation

Minimum Dominating Set

	$\mathcal{G}$	$\mathcal{G}_{\text{UDG}}$	$\mathcal{G}_{\text{BDG}}$	$\mathcal{G}_{\text{QDG}}$	$\mathcal{G}_{\text{CAG}}$
bound	$c\log\lvert V\rvert$				
central	$1+log\lvert V\rvert$	$1+\varepsilon$	$1+\varepsilon$	$1+\varepsilon$	$1+\varepsilon$
local	$O(\lvert V\rvert)$	5	$O(\frac{1}{d^2})$	$O(\frac{1}{d^2})$	$O(\frac{1}{d^2})$

TABLE 1: PERFORMANCE GUARANTEES.

dominating set is a factor of less than 3 away from a connected dominating set.

Lemma 4: For any dominating set D in a connected graph G, it is (greedily) possible to find at most $2|D|-2$ nodes to connect D.–

Summarizing, for the graph models proposed for wireless communication graphs, Table 1 gives the performance guarantees of centralized and local, polynomial-time algorithms for the MIS and MDS problem. Here, $\mathcal{G}$ denotes the set of all graphs. Unless otherwise stated, the results refer to the weighted version of the MIS, and the respective algorithms do not assume a geometric representation given as part of the input, i.e. work without localization.

5. CONCLUSIONS

In this paper, we review and propose geometric containment and intersection graph models for communication in ad-hoc, wireless networks. These range from ideal assumptions, resulting in a Unit Disk Graph, to more realistic approaches, resulting in Coverage Area Graphs. All these graphs allow more efficient algorithms of optimization problems common in many networking issues:
A simple greedy-strategy already yields a constant-factor approximation of the maximum (weight) independent set and minimum (connected) dominating set problems. Moreover, these algorithms can run locally and distributed in each node in a very efficient manner. For centrally executed algorithms, a $(1+\varepsilon)$-approximation is possible.

With respect to to geometric representations, it is possible to achieve the same approximation results without relying on the information given by a localization algorithm, but by connectivity information alone. Also, areas of unreliable transmissions can easily be incorporated into the model.

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