A NEW METHOD TO ACCOUNT FOR LARGE-SCALE MIXING PROCESSES IN SPATIALLY-AVERAGED FLOW MODELS

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Abstract. A new method is proposed to calculate the cross-sectional flow field in compound channels for 1D flow models. The proposed method involves a new parameterization of the interface stress between adjacent compartments, typically between the main channel and floodplain of a two-stage channel. This expression, proportional to the difference in squared velocities allows for a simple calculation of the average flow velocities in different compartments. For two-stage channels good agreement is found between the model results and data from literature. Also, it is shown that the description of compartment-averaged velocities may be applied to flows in channel cross-sections with arbitrary geometry, thus providing a potentially useful improvement to 1D river flow models.

1. Introduction

In 1D river flow models, the Divided Channel Method (DCM) is the standard methodology to account for lateral flow variability within the channel cross-section. 1D river flow software packages as SOBEK, MIKE11 and HEC-RAS are all based on the DCM. However, an important flow process in compound channels that is not captured by the DCM is momentum exchange between lateral flow compartments due to large-scale mixing patterns [1]. Several studies have been devoted to quantifying these exchanges. Some of these approaches are based on parameterization for an apparent shear stress between defined flow compartments [2][3]. Also, continuous lateral descriptions were proposed [4][5]. Nevertheless, none of these approaches have been widely incorporated in standard 1D engineering software, mainly because proposed methods were either too complex, or the range of applicability is limited and ill-defined. Here we show that lateral momentum exchange can be relatively easy incorporated in 1D flow models, using a simple new parameterization for the apparent interface shear stress.

2. Apparent shear stress between adjacent flow compartments

We consider a cross-sectional flow compartment (j) that is characterized by a cross-sectional flow area A_j, wetted perimeter P_j and average flow velocity U_j. The bed roughness along the wetted perimeter in this flow compartment is represented by friction function f_j. The flow compartment is surrounded by two other flow compartments, each having different flow conditions. Due to different flow velocities in the neighboring compartments, apparent shear stresses (τ) are experienced along the vertical interfaces that mark the division between compartments. Consequently, the force balance for the flow compartment under consideration may be expressed as

$$\rho g A_j s = \rho f_j P_j U_j^2 + h_{j-1/2} \tau_{j-1/2} + h_{j+1/2} \tau_{j+1/2}, \tag{1}$$

where h is the vertical height of the interface[a], s is the streamwise channel slope and ρ the density of water. The friction function f_j can be described in terms of a Manning coefficient n_j as

$$f_j = g n_j^2 R_j^{-1/3}, \tag{2}$$

where $R_j = A_j/P_j$ is the hydraulic radius of compartment j.

The standard shear stress parameterization for a mixing layer is proportional to the lateral velocity gradient squared [6], which in discrete form may be expressed as

EDM: $$\tau_{j+1/2} = \Psi \rho \left(U_{j+1} - U_j\right) \left|U_{j+1} - U_j\right|. \tag{3}$$

This is the parameterization used in the Exchange Discharge Method (EDM) [2], having a proportionality coefficient $\psi \approx 0.020$ (in a straight compound channel setting [7]). The parameterization given in eq. (3) is similarly defined for interface 'j-1/2', and yields a negative stress contribution if the flow velocity in a neighboring compartment is smaller than U_j and a positive contribution if larger than U_j.

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[a] Subscript 'j-1/2' refers to the interface between compartments '*j*-1' and '*j*', and '*j*+1/2' between '*j*' and '*j*+1'

For each flow compartment in the channel cross-section, a force balance as in eq. (1) is set up, which are all inter-related through the interface stresses, as in eq. (3). Consequently, a closed set of equations in the compartment-averaged flow velocities is established. Solving this set of equations is rather cumbersome, as the equations are non-linear in the compartment-averaged flow velocities (e.g. [2]). Therefore, we propose an alternative parameterization of the interface stress

$$\tau_{j+1/2} = \gamma\rho\left(U_{j+1} - U_j\right)U_{j+1/2}, \tag{4}$$

where $U_{j+1/2}$ is the depth-averaged flow velocity *at* the interface between compartments '*j*' and '*j*+1'. With this parameterization it is implicitly assumed that bed generated turbulence is partly responsible for the shear stress over the vertical interface. Therefore, the proposed parameterization is, by definition, only valid in flow situations where bed turbulence affects flow in the entire water column. Next, we estimate $U_{j+1/2}$ by taking the average of U_j and U_{j+1}, which gives a new shear stress parameterization, proportional to velocities squared

IDCM:
$$\tau_{j+1/2} = \frac{1}{2}\gamma\rho\left(U_{j+1}^2 - U_j^2\right). \tag{5}$$

Here, γ is a dimensionless proportionality constant, to be determined from experiments. In the remainder, we refer to this parameterization as belonging to the Interacting Divided Channel Method (IDCM), in contrast to the parameterization of the Exchange Discharge Method (EDM), given by eq. (3). Using the newly proposed interface stress, eq. (5), in combination with eq. (1) yields a set of equations linear in U^2, and is therefore easier to solve.

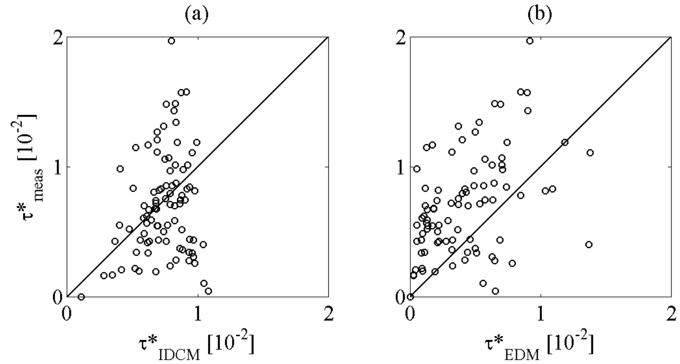

Figure 1. Comparison of two interface stress parameterizations with measurements in two-stage compound channels (see Table 1). Shear stresses are scaled to bankfull velocities as $\tau^* = \tau / \rho U_b^2$.

Table 1. Overview of laboratory flow experiments in two-stage channels with a main channel (mc) and one or two (N_{fp}) floodplains (fp). S is the side slope and W the width of the relevant flow compartment, U_b is the bankfull flow velocity in the main channel, having bank height h_b. Refs.: KD83 [8], FCF [9][10], AK02 [11].

Experiment	N_{fp} [-]	W_{mc} [m]	S_{mc} [-]	W_{fp} [m]	S_{fp} [-]	h_b [m]	s [10^{-3}]	n [$s/m^{1/3}$]	U_b [m/s]
KD83 A	2	0.152	v	0.456	v	0.076	0.966	0.01	0.35
KD83 B	2	0.152	v	0.304	v	0.076	0.966	0.01	0.35
KD83 C	2	0.152	v	0.152	v	0.076	0.966	0.01	0.35
FCF s1	2	1.5	1	4.1	v	0.15	1.027	0.01	0.82
FCF s2	2	1.5	1	2.25	1	0.15	1.027	0.01	0.82
FCF s3	2	1.5	1	0.75	v	0.15	1.027	0.01	0.82
FCF s6	1	1.5	1	2.25	1	0.15	1.027	0.01	0.82
FCF s8	2	1.5	v	2.25	1	0.15	1.027	0.01	0.8
FCF s10	2	1.5	2	2.25	1	0.15	1.027	0.01	0.82
AK02a	1	0.398	v	0.407	v	0.05	2.024	0.009	0.58
AK02s	2	0.398	v	0.407	v	0.05	2.024	0.009	0.58

3. Comparison of shear stress parameterization to flow experiments

From eleven laboratory flow experiments found in literature we determined a best fitting overall constant of proportionality $\gamma = 0.020$ for the IDCM (see Table 1 for characteristics of used experiments). Next, we compared the parameterizations given in equations (3) and (5) to the individual results from the flow experiments (Figure 1). It can be seen that their performance is comparable, but that neither of the shear stress parameterizations is very accurate. Nevertheless, of greater importance is the impact of the new interface stress parameterization on compartment-averaged flow velocities. In Figure 2 it can be seen that these predictions are quite promising, and provide a clear improvement to velocities that are estimated with the standard DCM approach.

4. Arbitrary cross-section geometry

It was shown that, compared to the widely used DCM, IDCM considerably improves estimated flow velocities in a two-stage channel. Assuming that multi-staged channels are similarly subjected to lateral momentum exchanges, here we demonstrate the impact of the proposed interface stress parameterization in an arbitrary cross-section. In Figure 3 a hypothetical, arbitrarily shaped channel cross-section is given, which may have laterally varying bed roughness characteristics. In the graphs below, vertical bars reflect flow velocities in the cross-section estimated using DCM. The connected dots give the corresponding velocities if momentum exchange according to eq (5) is taken into account (adopting $\gamma = 0.020$). As expected, lateral momentum exchange causes slow flowing regions to speed up due to adjacent faster flows, and, correspondingly, relatively fast flows are slowed down.

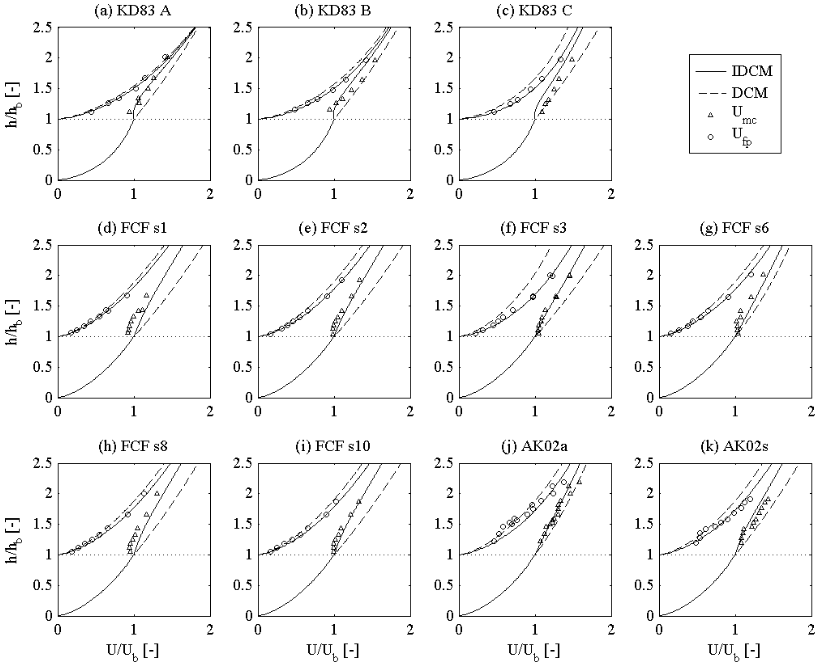

Figure 2. Comparison of IDCM results using $\gamma = 0.020$ with results from the standard DCM, for the experiments listed in Table 1 (U_{mc} is the flow in the main channel, U_{fp} in floodplain).

5. Conclusion

A new parameterization is proposed for the interface stress between flow compartments in straight compound channels, representing lateral momentum exchange. The model equations are linear in squared velocities and therefore allow a relatively simple calculation procedure. The proposed method can be incorporated in 1D river flow models to account for lateral exchange processes in compound channel settings. More investigations are necessary to understand characteristics of parameter γ.

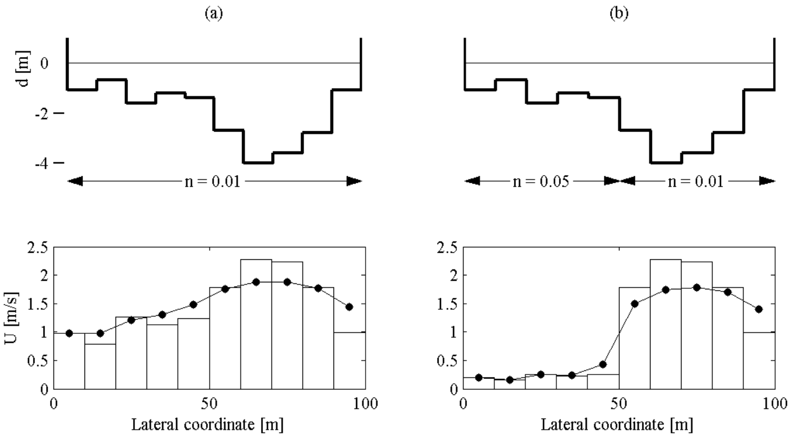

Figure 3. Flow velocities in an arbitrarily shaped channel cross-section (top row), with (connected dots) and without (vertical bars) taking lateral momentum exchange into account: (a) based on ten flow compartments with homogeneous bed roughness characteristics and (b) same cross-section but with two different resistance zones.

Acknowledgements

This research is supported by the Technology Foundation STW, applied science division of NWO and the technology programme of the Ministry of Economic Affairs (the Netherlands).

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