Undetected Error Probability for Data Services in a Terrestrial DAB Single Frequency Network

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Abstract

DAB (Digital Audio Broadcasting) is the European successor of FM radio. Besides audio services, other services such as traffic information can be provided. An important parameter for data services is the probability of non-recognized or undetected errors in the system. To derive this probability, we propose a bound for the undetected error probability in CRC codes. In addition, results from measurements of a Single Frequency Network (SFN) in Amsterdam were used, where the University of Twente conducted a DAB field trial. The proposed error bound is compared with other error bounds from literature and the results are validated by simulations. Although the proposed bound is less tight than existing bounds, it requires no additional information about the CRC code such as the weight distribution. Moreover, the DAB standard has been extended last year by an Enhanced Packet Mode (EPM) which provides extra protection for data services. An undetected error probability for this mode is also derived. In a realistic user scenario of 10 million users, a 8 kbit/s EPM sub channel can be considered as a system without any undetected errors ($P_{ud} = 6 \cdot 10^{-40}$). On the other hand, in a normal data sub channel, only 110 packets with undetected errors are received on average each year in the whole system ($P_{ud} = 5 \cdot 10^{-13}$).

1 Introduction

Reliable communication is one of the most important aspects in wireless systems. In this paper, the non-recognizable or undetected error probability is derived for the DAB (Digital Audio Broadcasting) system. DAB was standardized in the late nineties [1] and is the European successor of current FM radio broadcasting.

DAB is not only designed for audio services, it can also transmit video or data. Data services include traffic information, electronic programming guide etc. To guarantee error-free reception, DAB uses several techniques, explained in section 2.1, to protect the transmitted data bits against errors. However, there will always be a (non-zero) probability that an undetected error will occur. Such an error can cause systems, such as DAB car radios, to malfunction. An example being that incorrect traffic jam information is displayed.

This paper is an extension of [2], where the probability of undetected errors in the system was derived for a 8 kbit/s data sub channel using protection level EEP3-A. In addition, an undetected error bound was derived for CRC codes. Here, this bound is compared with other bounds from literature and the results are validated with simulations. In 2006, the DAB standard has been extended with EPM (Enhanced Packet Mode), which provides extra protection to data services by using a Reed-Solomon code. An undetected error probability is derived for this mode too.

The outline of this paper is as follows. First, the DAB system and the used error detection and correction techniques are discussed. Then, a bound for the undetected error probability for CRC codes is proposed and is compared with other bounds. After this, the undetected error probability for the Reed-Solomon code is presented and is followed by a realistic user scenario for a DAB network. This paper ends with conclusions.

2 Digital Audio Broadcasting

The physical layer of DAB [1] include an OFDM-based transmission with D-QPSK modulated sub carriers using RCPC (Rate Compatible Punctured Convolution Codes) for error correction. For data services, an optional Enhanced Packet Mode is available which uses a shortened Reed-Solomon (204,188) code for extra protection. Every DAB channel consists of a *multiplex* of 2304 kbit/s (including error correction) which contains multiple sub channels (e.g. radio stations). In a typical situation, the multiplex contains about 10 radio stations.

Terrestrial DAB has been designed to operate in a Single Frequency Network (SFN) [3] which means that all DAB transmitters broadcast on the same frequency. The network has been designed in such a way that the delay spread of the received paths of other transmitters are within the cyclic prefix duration, i.e. signals from other transmitters can be considered as extra received paths.

2.1 Error detection and correction techniques

DAB uses several techniques to protect the transmitted data bits against errors:

- Interleaving
- Forward Error Correction (FEC)
- Cyclic Redundancy Codes (CRC)

2.1.1 Interleaving

The first part is the interleaving function. In wireless communication errors occur often in bursts and the purpose of the interleaver is to convert these burst errors into independent errors. This is required as the FEC decoding function requires independent bit errors. To meet this goal, the interleaver consists of two randomize functions, both in frequency and time.

2.1.2 Forward Error Correction

At the transmitter, extra information (Forward Error Correction (FEC) data), is added to the transmitted data, which allows the receiver to detect and correct errors in the received signal. DAB uses RCPC codes for this purpose. The most common mode for data services is protection level EEP3-A (Equal Error Protection). EEP3-A has a code rate of $\frac{1}{2}$ which means that for every information bit, two bits are transmitted. The service area of DAB can be defined as where the BER[1] (Bit-Error Rate) is lower than 10^{-4} [3, 4].

In a realistic situation, the user is not always at the border of a service area. For example, figure 1 depicts the measured BER with our measurement vehicle for protection level EEP3-A of a pilot network in Amsterdam. This field trial was carried out by the University of Twente and it was commissioned by the Dutch Ministry of Economic Affairs. More information about the pilot can be found in [5]. From this figure one can deduce that a realistic scenario would be that a user is only 5% of the time at the border of the service area. At the border of the service area, the user experiences a BER of 10^{-4}. Within the service area, the $\frac{E_s}{N_0}$ is much higher and therefore the BER is several magnitudes lower. For this reason, the border (5%) will dominate the probability of undetected errors in the DAB system.

In 2006, the DAB standard has been extended to provide extra protection to data services. This mode is called Enhanced Packet Mode. To facilitate this, the error correction and detection system of DAB has been extended with an optional shortened Reed-Solomon (204,188) outer code.

2.1.3 Cyclic Redundancy Check

Although the BER is largely reduced by the FEC decoder, still bit errors can occur. To detect these errors, the system uses a Cyclic Redundancy Check (CRC) [6] that are added to each transmitted packet. In DAB, the CRC-CCITT generator is used with polynomial: $x^{16} + x^{12} + x^5 + 1$.

To each packet a CRC checksum is added. The CRC checksum is calculated by dividing the packet data by the CRC generator i.e. `1 0001 0000 0010 0001`. The remainder of this division is the CRC checksum. At the receiver side, the division is repeated and both checksums are compared. If they are unequal, the packet contains errors and is invalid.

The CRC-CCITT checksum can detect bit errors as long as the combination of the errors is not a multiple of the CRC generator, because in this case the remainder (i.e. checksum) of the division remains the same. The code contains 4 elements, which means that this code can detect any one, two or three bit errors and any odd number of errors. Most combinations of 4 errors in the packet are detected but not all. Combinations with errors (e.g. 6 bit or 8 bit) are less likely to occur, especially for low BER values.

For example[2], the probability that 6 random bit errors occur in a packet of length 192 with BER 10^{-4} is $6 \cdot 10^{-14}$ and for 4 errors this probability is $5 \cdot 10^{-9}$. As the latter is already magnitudes smaller, the undetected error probability is dominated by the 4 bit error case.

[1] After FEC decoding.

[2] See also Section 4.

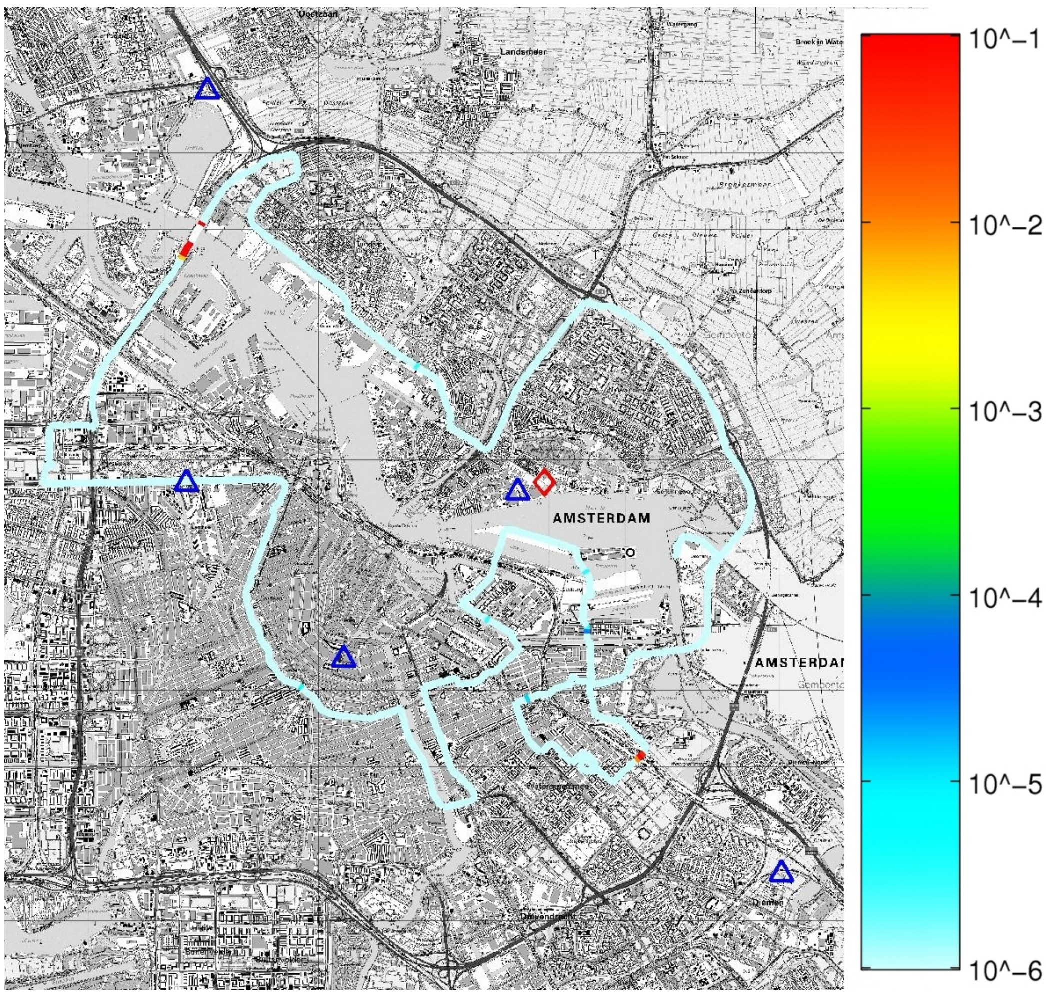

Figure 1: Typical performance for Protection level EEP3-A in Amsterdam (the color is an indication for the BER), blue triangles are transmitter locations and the red diamond is a DAB transmitter of an adjacent channel.

3 Undetected error probability

3.1 CRC codes

In this section an error bound is presented which has been deduced by analyzing the implementation of the CRC check. To calculate the CRC checksum, the packet is divided by the CRC code. For a (N, K) code with $N-K$ equal to 16, this means in binary calculations that at N positions a XOR operation of $N-K+1$ bits long has to be performed[3]. If the bit errors are a multiple of the CRC code they are undetected. So, if in one of the stages of the division, errors occur at the positions of the CRC-code elements, it is undetected. The probability for this to happen is for a single stage in the division is [2]:

$$P_{ud,single} = \frac{n_1!}{\frac{L!}{(L-n_1)!n_1!}} \cdot \frac{L!}{(L-n_1)!n_1!} \cdot \epsilon^{n_1}(1-\epsilon)^{L-n_1} \quad (1)$$

$P_{ud,single}$	the undetected error probability for a single XOR stage
ϵ	the bit error rate
n_1	the number of 'ones' in the CRC code
L	the length of the CRC generator i.e. $N-K+1$
N	the packet length
!	the factorial function

The last part of the equation is the binomial distribution function [7] i.e. the probability that n_1 bit errors occur in one XOR operation. However, only bit errors at non-zero elements of the generator are undetected and there are only $n_1!$ combinations out of the total number of possibilities $(\frac{L!}{(L-n_1)!n_1!})$ for n_1 errors which are undetected (i.e. first part of the equation). In each CRC checksum calculation at most N XOR operations are performed. So the undetected error probability becomes:

$$P_{ud} = N n_1! \epsilon^{n_1}(1-\epsilon)^{L-n_1} \quad (2)$$

An important advantage of the undetected error probability bound (equation 2) is that it does not require additional information about the used CRC generator. Only the packet length N, the CRC code length L, the number of non-zero elements in the CRC generator n_1 and the BER are required.

3.2 Other bounds

In order to calculate the exact undetected error probability of a linear code, the weight distribution has to be known. For most practical situation this is not available and therefore several algorithms exists which approximate the weight distribution [8]. The first method approximates the weight distribution by an binomial distribution:

[3] A XOR operation is only carried out if the first bit equals 1, otherwise it is skipped. In addition, in DAB the CRC calculation is initialized by setting the shift register to all 'ones' [1].

$$P_{ud} \approx \sum_{i=d_{min}}^{d_{max}} \frac{\binom{N}{i}}{2^{L-1}}(1-\epsilon)^{N-i}\epsilon^{i} \quad (3)$$

P_{ud} the undetected packet error probability

ϵ the bit error rate

d_{min} the minimum number of non-zero elements in any nonzero code word

d_{max} the maximum number of non-zero elements in any nonzero code word

N the packet length

L the length of the CRC generator

The second approximation is based on assessing the fraction (F) of bit sequences of lenght N with d_{min} non-zero elements that have also a valid CRC code. The undetected error probability can be assessed by multiplying the fraction F by the probability on d_{min} errors:

$$P_{ud} \approx F\binom{N}{n_e}\epsilon^{n_e}(1-\epsilon)^{N-n_e} \quad (4)$$

with n_e the number of bit errors.

3.3 Reed-Solomon codes

For Reed-Solomon codes an upper bound for the number of code words (A_i) with weight n_e is presented [9]:

$$A_i \leq \binom{N}{n_e}(q-1)^{n_e-d_{min}+1} \quad (5)$$

with q the number of Galois field elements and N the packet length[4] i.e. 255. For a RS (204,188) code q is 256. Moreover, the code is able to correct up to 8 symbols, so $d_{min} = 9$.

The undetected error probability for Reed-Solomon codes is [8]:

$$P_{ud} = \sum_{i=1}^{n} A_i \left(\frac{\epsilon}{q-1}\right)^{i} (1-\epsilon)^{N-i} \approx A_{d_{min}} \left(\frac{\epsilon}{q-1}\right)^{d_{min}} (1-\epsilon)^{N-d_{min}} \quad (6)$$

4 User scenario

In the previous sections, the undetected error probability has been given both for CRC and for Reed-Solomon codes. In this section, these bounds are applied to a realistic situation for a DAB network. The probability is calculated for a 8 kbit/s data service with protection level EEP3-A when it is used every day for 8 hours by 10 million users[5]. In a sub channel with protection level EEP3-A, every packet contains 192 information bits [1] and a packet has a duration of 24 ms.

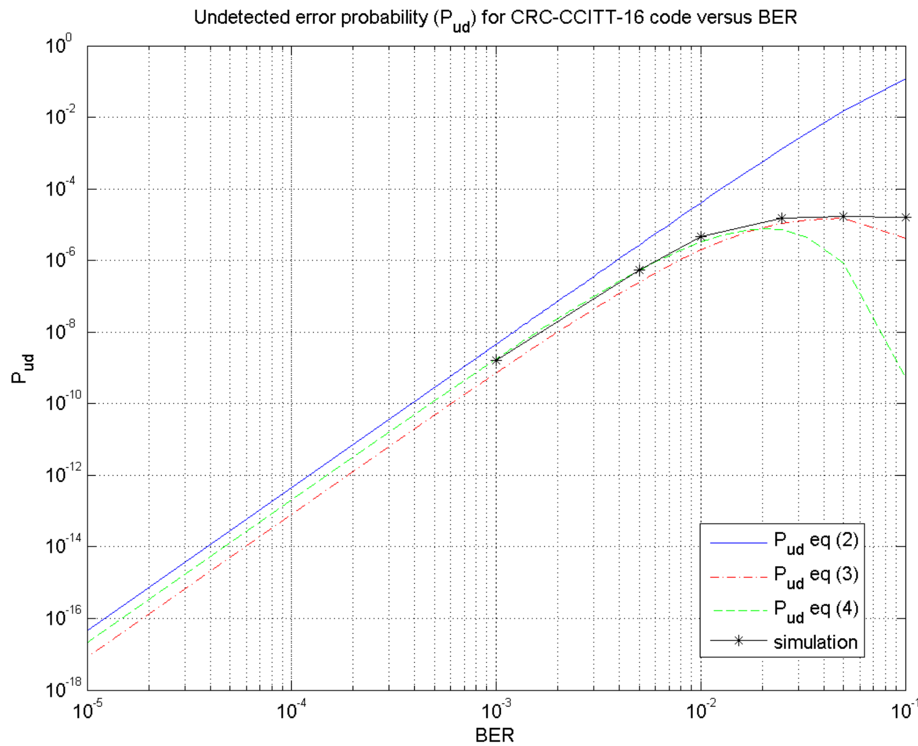

Figure 2: Undetected error probability for the CRC-CCITT-16 code

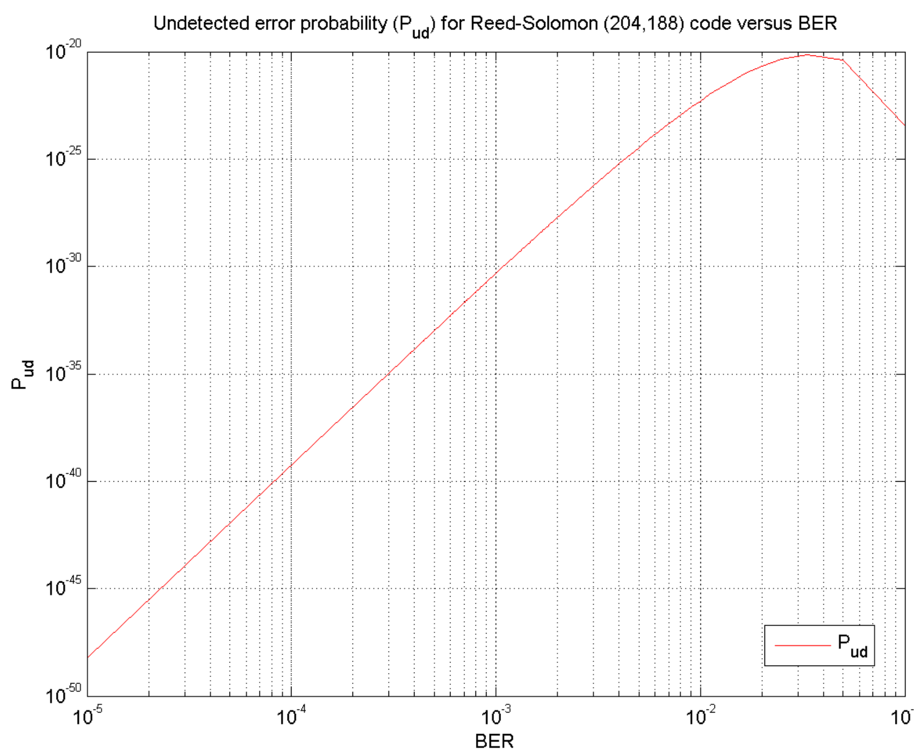

Figure 3: Undetected error probability for the Reed-Solomon (204,188) code

Figure 2 depicts the undetected error probability for the CRC-CCITT-16 code versus the (input) BER. Four lines are shown: three approximations for the undetected error probability P_{ud} (equation 2, 3 and 4) and the P_{ud} found by simulation (implemented in C++ using the IT++ library [10])[6]. At high BER values, the proposed undetected error bound (equation 2) overestimates the error. For low BER values ($< 10^{-2}$) the bound is a good upper bound for the undetected error probability, although equation 4 is more tight. In fact, equation 4 is a good estimate of the undetected error probability for low BER values and equals the simulation results[7]. The difference between equation 2 and 4 is a factor 2 for low BER values[8]. However, the proposed bound does not require additional information about the code, such as the weight distribution and is for that reason much easier to calculate.

In figure 3, the performance of the Reed-Solomon (204, 188) code is shown. The detection capabilities of this code is much stronger compared to the CRC code.

At the border of the service area, the BER is 10^{-4}. Equation 2 gives an undetected error probability of $5 \cdot 10^{-13}$ for the CCITT CRC code. This value equals the one found in [11]. For an EPM sub channel the probability[9] is $6 \cdot 10^{-40}$.

In section 2.1.2, it has been derived that it is likely that only 5% of the time this probability will occur (border of the service area). Therefore, the total undetected packet error probability for this user scenario is:

$$N_{ud,total} = 0.05 N_{packets} P_{ud,total} = 0.05 \cdot (365 \cdot 8 \cdot 3600 \cdot \frac{1}{0.024} \cdot 10^7) \cdot P_{ud} \qquad (7)$$

with $N_{packets}$ the total number of packets received by all users in one year.

For a data service of 8 kbit/s, on average 110 packets with undetected errors will occur every year in the whole system and for a EPM sub channel this value is $2 \cdot 10^{-25}$ packets. Considering 10 million users, both values are very small. The EPM sub channel can be considered as a system without undetected errors.

5 Conclusions

In this paper, a new undetected error probability bound for CRC codes is proposed. Other bounds give a tighter bound, but the proposed formula does not require additional information about the used CRC generator and is therefore easy to calculate. Only the packet length N, the CRC code length L, the number of non-zero elements in the CRC generator n_1 and the BER are required. Moreover, the (upper) bound is sufficiently accurate for the purpose at hand:

[4] The Reed-Solomon (204,188) code is a shortened code of the RS (255,239) code.

[5] For example, the number of cars in the Netherlands (2005) is about 7 million.

[6] For each point at least 10 undetected errors were simulated.

[7] Equation 4 only takes into account the weight of the minimal distance. So, for high BER values ($> 10^{-2}$), the difference with the simulated values becomes larger. In this region, also errors occur which have a larger distance than the minimum distance.

[8] A maximum number of XOR operations (worst case) is assumed in the proposed bound (equation 2). So, if it is assumed that on average only 50% of the XOR operation are executed, the proposed bound provides for low BER values equal results as the bound of equation 4.

[9] This is de P_{ud} after Reed-Solomon decoding and before the CRC comparison. As the BER after Reed-Solomon is not known, it is assumed that the CRC check is always correct.

for low BER values, the bound differs a factor 2 compared with the simulated undetected error probability.

Moreover, the undetected error probability is calculated for DAB data sub channel using a realistic user scenario of 10 million users. This value is calculated both for a normal data sub channel ($P_{ud} = 5 \cdot 10^{-13}$) and for an Enhanced Packet Mode (EPM) data sub channel ($P_{ud} = 6 \cdot 10^{-40}$) which uses a Reed-Solomon code for extra protection. The latter system can be considered as a system without undetected errors and in the normal data sub channel only 110 packets with undetected errors are received on average each year in a user scenario of 10 million users.

More research is needed to investigate if the proposed bound for CRC codes can also be used for other CRC codes.

Acknowledgements

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