Force Limited Vibration Testing: An Evaluation of the Computation of C^2 for Real Load and Probabilistic Source

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Abstract

To prevent over-testing of the test-item during random vibration testing Scharton proposed and discussed the *force limited random vibration testing* (FLVT) in a number of publications. Besides the random vibration specification, the total mass and the turn-over frequency of the load(test item), C^2 is a very important parameter for FLVT. A number of computational methods to estimate C^2 are described in the literature, i.e. the simple and the complex two degrees of freedom system, STDFS and CTDFS, respectively. Both the STDFS and the CTDFS describe in a very reduced (simplified) manner the load and the source (adjacent structure to test item transferring the excitation forces, i.e. spacecraft supporting an instrument). The motivation of this work is to evaluate the method for the computation of a realistic value of C^2 to perform a representative random vibration test based on force limitation, when the adjacent structure (source) description is more or less unknown. Marchand discussed the formal description of getting C^2, using the maximum PSD of the acceleration and maximum PSD of the force, both at the interface between load and source, in combination with the apparent mass and total mass of the the load. This method is very convenient to compute the factor C^2. However, finite element models are needed to compute the spectra of the PSD of both the acceleration and force at the interface between load and source. Stevens presented the *coupled systems modal approach* (CSMA), where simplified asparagus patch models (parallel-oscillator representation) of load and source are connected, consisting of modal effective masses and the spring stiffnesses associated with the natural frequencies. When the random acceleration vibration specification is given the CMSA method is suitable to compute the value of the parameter C^2. When no mathematical model of the source can be made available, estimations of the value C^2 can be find in literature. In this paper a probabilistic mathematical representation of the unknown source is proposed, such that the asparagus patch model of the source can be approximated. The chosen probabilistic design parameters have a uniform distribution. The computation of the value C^2 can be done in conjunction with the CMSA method, knowing the apparent mass of the load and the random acceleration specification at the interface between load and source, respectively. Data of two cases available from literature has been analyzed and discussed to get more knowledge about the applicability of the probabilistic method

keywords: Random vibration, Force limited vibration testing (FLVT), Coupled systems modal approach (CSMA), Probabilistic system.

1 Introduction

In spacecraft design the force limits are established to prevent over-testing of the test-article (load) , because its dynamic behavior on the shaker table is different from its dynamic behavior when placed on the actual supporting structure (source).

In [22] the history, the actual status and application guidelines of the FLVT are discussed and 41 interesting references regarding the FLVT are provided.

During the FLVT both the random acceleration as well as the random force limits are specified, however, the random acceleration specification may be overruled by the random force limits.

The well-known semi-empirical method (SEM) of the force-limit approach is a method to establish force-limits at the interface between the load and the source , [10, 21, 22].

$$\begin{aligned} W_{FF}(f) &= C^2 M_o^2 W_{AA}(f) \quad f \leq f_0, \\ W_{FF}(f) &= C^2 M_o^2 W_{AA}(f) \left(\frac{f_0}{f}\right)^n \quad f > f_0, \end{aligned} \tag{1}$$

where $W_{FF}(f)$ is the force spectral density, $W_{AA}(f)$ is the acceleration spectral density, M_o is the total mass of the test item and C^2 is a dimensionless constant which depends on the configuration. f(Hz) is the frequency and f_0 is the natural frequency of the primary mode with a significant modal effective mass. The factor n can be estimated from the apparent mass of the load, in general, $n = 2$. C^2 should not be selected without adequate justification [19] .

Scharton et al revisited the force limiting vibration testing in a presentation [19] and reviewed the methods of estimation of C^2 using the *simple two degrees of freedom system* (STDFS). The factor n can be estimated from the apparent mass of the load.

Dharanipathi main conclusions in [8] are that the range of values of C^2 is between 2 and 5, however, there are several cases where $C^2 = 10 \cdots 17$, and that C^2 does not depend on the damping in the structure.

In [25] Soucy et al recommend values for C^2, however, based on limited number of measured (flight) data. It has been observed that in normal conditions $C^2 = 2$ might be chosen for complete spacecraft or strut mounted heavier equipment. $C^2 = 5$ might be considered for directly mounted lightweight test items.

Based on the frequency shift of a two degrees of freedom system [20] Scharton developed two methods to establish the value C^2; the simple two degrees of freedom system (STDFS) [21] and the *complex two degrees of freedom system* (CTDFS) [6].

In [13] Gordon proposed a conservative analytical value of $C^2 = 9$, which is based on the STDFS when the load/source ratio is 0.16. This conservative estimation of C^2 will cover model uncertainties. The test configuration remain relatively simple because no force measurement devices are used during the random vibration test.

Stevens presented a paper [26], to compute the force limits, based on the *coupled system modal approach* (CSMA). The coupled asparagus patch models of both source and load are needed. These models can be extracted from finite element analysis models or apparent mass measurements. This CMSA method forms the core of this paper.

In general, the mathematical model (FEM, modal effective masses, $\cdots$) of the load is available, because the random vibration test will be conducted under the responsibility of contractor/subcontractor which is responsible for the design of the load as well. To apply the methods to obtain the value C^2 the dynamical properties of the source need to be known, however, if the mathematical description of the supporting structure (source) of the load is lacking a probabilistic source is necessary.

In [28] the replacement of the source by a probabilistic-source is discussed. The mathematical modeling of the probabilistic source will be an asparagus patch model, consisting of a number of parallel placed lightly damped SDOF systems, with the modal effective masses [12, 17] as the discrete mass and the spring stiffnesses representing the undamped natural frequencies. The CMSA method [26] is applied to compute maximum random accelerations and forces at the interface between load and source.

The Rosenblueth *point estimated moments* (PEM) will be applied [16, 18] to minimize the number of samples (analysis cases) describing the probabilistic design parameters. The probability density functions of the probabilistic design parameters is assumed to be uniform.

The method proposed in [28] has been further investigated, using available data from literature [7, 10], to study the applicability of the probabilistic approach.

2 Force Limits Analysis Method

The semi-empirical force-limit vibration test (FLVT) approach has been established to prevent over–testing of a flexible test item when placed on the shaker table with a very high impedance compared to the impedance of the supporting structure of the test item. This (FLVT) test philosophy or method is described in [22]. The simple equations to compute the PSD of the force limits W_{FF} from the PSD of the random acceleration test specification W_{AA} are already given in equation (1).

Marchand provides in [15] an equation to compute the value of C^2 in the interface between the source and the load, both consisting of MDOF systems . Considering that the maximum PSD of the interface force $W_{FF_{max}}$ and the maximum PSD of the interface acceleration $W_{AA_{max}}$, which need not to occur at the same frequency, the value of C^2 can be defined as

$$C^2 = \frac{W_{FF_{max}}}{M_o^2 W_{AA_{max}}}, \tag{2}$$

where M_o is the total mass of the load.

3 Coupled System Modal Approach Method (CMSA)

The CMSA method, proposed by Stevens in [26], is the selected method to compute the force limits for the random vibration testing of the load. The dynamic or apparent mass of the load [9], as well as the random acceleration test specification are required. The acceleration at the interface between load and source is illustrated in Fig. 1. The reduced asparagus patch models of both source and load are shown in Fig. 1. The

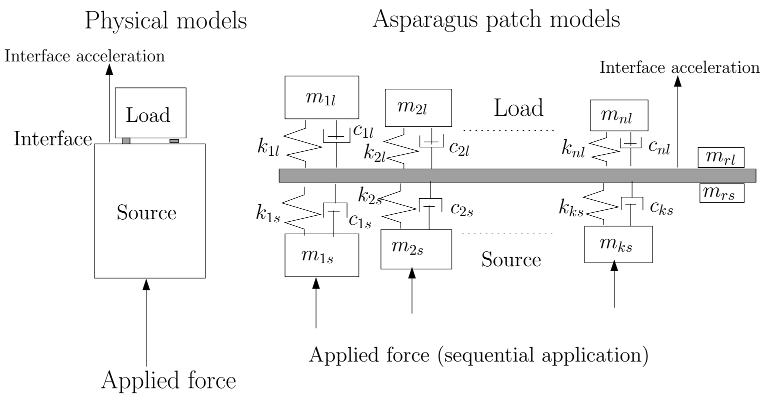


Figure 1: Coupled system in parallel-oscillator representation

spring stiffnesses and damper values are, respectively, given by $k_{il} = \omega_{il}^2 m_{il}$ and $c_{il} = 2\zeta_i \omega_{il} m_{il}$, where

$\omega_{il},\ i = 1, 2, \cdots, n$ are the natural frequency of the load. ζ_i is the modal damping ratio of mode i. The notations for the source are similar.

The random acceleration vibration specification $W_{AA}(f)$ at the interface between the source and the load is provided (specified). In general, this specification is an envelope that is based on data "smooths over" of peaks and valleys. The load is very responsive at the anti-resonance frequencies and acts as a dynamic absorber to reduce the input.

To compute the parameter C^2 in (1), equation (2) is applied. Therefore we need to compute the random acceleration spectrum at the interface between the load and the source. That random acceleration spectrum is multiplied by the apparent mass of the load to obtain the random force spectrum at the interface. The mathematical models (parallel oscillators, Fig. 1) of the source and the load are represented by their modal effective masses and associated spring stiffness and damping and are coupled. The modal effective masses can be either calculated by a modal analysis with a fixed-free finite element model [27], or extracted from a measured apparent mass of the load, i.e. on a shaker table performing sinusoidal base-excitation [11, 23].

To calculate the maximum random force spectrum at the interface between source and load the following procedure is followed:

- Generate the mathematical models (Asparagus patch models) of both the source and load (Fig. 1).
- Compute the apparent mass (dynamic mass) of the load, fixed at the interface between source and load
- The random acceleration vibration specification to be applied to the load is specified
- Define the random load spectrum $W_F(f)$ to be applied subsequently at every oscillator of the source. This may be a unitary band-limited white noise spectrum or a unitary scaled random vibration spectrum.
- Perform for every subsequent loaded oscillator of the source a random acceleration response analysis and scale to the spectra such that the maximum acceleration at a certain excitation frequency is equal to the specified acceleration spectrum at that frequency. Multiply these scaled random acceleration spectra by the squared absolute value of the apparent mass spectra of the load. The composite random load spectrum $W_{FF}(f)$ then represent the upper bound. This upper bound is divided by the square absolute value of the apparent mass spectra of the load to compute the associated upper bound interface random acceleration $W_{AA}(f)$. The following simple spectra are taken; $W_{AA}(f) = 0.01$ g^2/Hz and $W_F(f) = 1$ N^2/Hz, both between 20-2000Hz.
- Apply equation (2) to compute C^2. M_o is the rigid body mass of the load.

4 Definition (Availability) of Source and Load

To perform a random vibration test of the load the test conductor needs the availability of a hardware (H/W) model of the load, i.e. the item to be tested on a shaker table. When the FLVT [22] is planned the value of C^2 (1) shall be obtained either by experience (data base) [22] or applying the simple two degrees of freedom (STDFS) system and or the complex two degrees of freedom (CTDFS) system as described in [24]. When modal characteristics of both source and load can be made available from FEA/FEM or measurements equation (2) can be used [15]. Simplified computations may be done when the CSMA method will be applied as illustrated in Fig. 1.

4.1 Load

4.1.1 Mathematical Model

We assume the availability of a mathematical description (finite element model) of the load. An estimation of the modal damping ratio shall be done, in general, based on past experiences or measurements. The finite element model degrees of freedom at the interface between the load and source shall be fixed. The following modal data of the load is needed to build the asparagus patch model for the CMSA method:

- The total mass of the load M_l (kg)
- The undamped natural frequencies $f_i,\ i = 1, 2, \cdots, n$ (Hz) assuming clamped conditions at the interface load/source
- The associated modal effective masses $m_{il},\ i = 1, 2, \cdots, n$ (kg) and the residual mass m_{rl}, in the three translational directions, respectively. The cross-coupling is not considered.
- The estimated or measured modal damping ratios $\zeta_i,\ i = 1, 2, \cdots, n$
- The apparent mass $\mathbf{M}_l(f)$ (kg) of the load in the three translational directions with respect to the interface

4.2 Source

Coté stated in his paper [5] that the asparagus patch model of the source (common to the load); modal effective masses, natural frequencies, can be extracted from a finite element model, experiment or from experience. However, in this subsection we assume that the finite element model or experimental results cannot be made available, so the simplified model will be constructed using engineering design rules (i.e. ECSS).

The dynamic characteristics (design parameters) of the source with respect to the interface between the load and the source are considered to be probabilistic related to the modal properties of the load.

The probabilistic design parameters are discussed in detail in [28] and are common to the modal data of the load. The probabilistic design parameters of the source are described in the following section.

5 Virtual Building of Asparagus Patch Model of the Source

The design parameters of the source are related to the mass and modal properties of the load and are discussed in [28].

5.1 Total mass

The total rigid body mass of the source M_s shall be provided (i.e. by the prime contractor). If the M_s can't be made available the following total mass, with uniform distribution, of the source is assumed

$$M_s = 0.1 \cdots 10 M_l \tag{3}$$

When the mass of the source M_s is known, the mean of the source mass is $\mu = M_s$ and the standard deviation $\sigma = 0$.

5.2 Natural frequencies

When the lowest undamped natural frequency of the load is f_l, the interface source/load fixed, the assumed undamped natural frequency of the source will vary between

$$f_{1s} = \frac{f_l}{2} \cdots \frac{f_l}{\sqrt{2}}. \tag{4}$$

This range is based on the design practice that the dynamic interference between load and source is minimized.

This undamped natural frequency of the source is associated with a high modal effective mass m_{1s}. The probability density function of the first natural frequency f_{1s} is uniform.

The following (first guess) distribution of natural frequencies, with substantial modal effective mass, is defined by:

$$\begin{aligned} f_{2s} &= 2f_{1s}, \\ f_{3s} &= 4f_{1s}, \\ f_{4s} &= 6f_{1s}. \end{aligned} \tag{5}$$

Force limits typically cover only the first three modes [14]. Therefore, it is usually adequate to specify the force limits only in the frequency regime encompassing a few modes in each axis, which might be out to approximately 100 Hz for a large spacecraft, 500 Hz for an instrument, or 2000 Hz for a small component [22].

5.3 Modal Effective Masses

The first undamped natural frequency f_{1s} will be associated with the first significant modal effective mass m_{1s}. The fundamental modal effective masses of simple systems is assumed to be a first approximation of modal effective mass of the source. This modal effective mass will be assumed in the following mass range with a uniform probability distribution

$$m_{1s} = 0.4 \cdots 0.6 M_s. \tag{6}$$

This range may be confirmed by the calculation of the modal effective mass of simple structures [28]. The residual mass is the sum of the modal effective masses excited outside the frequency range of interest and the residual mass m_{rs} will be assumed to be 5% of the total mass of the source, such that

$$m_{rs} = 0.05 M_s. \tag{7}$$

The summed modal effective masses of the computed modes shall be about 95% of the total mass of the source M_s.

Further Δm is the sum of the missing distribution of the modal effective mass and is defined by

$$\Delta m = M_s - (m_{1s} + m_{rs}). \tag{8}$$

The deterministic distribution (best guess) of the modal effective $m_{ks}(f_{ks}),\ k = 2, \cdots, 4$ will be descending and the effective masses of the remaining modes are distributed according to the following scheme:

$$\begin{aligned} m_{2s} &= 0.5\Delta m, \\ m_{3s} &= 0.3\Delta m, \\ m_{4s} &= 0.2\Delta m. \end{aligned} \tag{9}$$

5.4 Modal Damping Ratio

We will assume a uniform distribution of the modal damping ratio $\zeta = 0.01 \cdots 0.1$.

5.5 Summary of Mean and Standard Deviation of Stochastic Variables

The probability density function of the stochastic variables M_s, f_{1s}, m_{1s} and ζ are assumed to be uniform.

The summary of mean and standard deviation of the selected probabilistic variables, with a uniform distribution [1] is presented in Table 1.

Table 1: Mean and standard deviation stochastic variables, [18]

Description	Symbol	Mean μ	Standard deviation σ
Mass (kg)	M_s	$5.0500M_l$	$2.8579M_l$
Natural frequency (Hz)	f_{1s}	$0.6036f_l$	$0.0598f_l$
Modal effective mass (kg)	m_{1s}	$0.5000M_s$	$0.0577M_s$
Modal damping ratio (-)	ζ	0.055	0.0260

5.6 Probabilistic Analysis by the Rosenblueth 2k+1 PEM & CMSA

The Rosenblueth Point Estimates Method (PEM) for probability moments [16, 18], computes the mean and the variance of the value C^2 in combination with the CMSA. If the number of design variables is k, $2k+1$ samples (analysis cases) are to be computed.

The Y_0 value is computed by substituting the mean values for all k design variables, Y_{nm} is computed by substituting for the n^{th} design variable the value $\mu_n - \sigma_n$ and for the other design variables the mean values and Y_{np} is computed by substituting for the n^{th} design variable the value $\mu_n + \sigma_n$ and for the other design variables the mean values, respectively.

The mean of two point estimates Y_{np}, Y_{nm} is given by

$$Y_n = |Y_{np} + Y_{nm}|/2, \; n = 1, \cdots, k, \tag{10}$$

and the variance is V_n can be obtained by

$$V_n = \left| \frac{Y_{np} - Y_{nm}}{Y_{knp} + Y_{nm}} \right|, \; n = 1, \cdots, k. \tag{11}$$

When the stochastic variables are statistically independent the following approximation of the mean $\bar{Y} = \mu_Y$ and the variance $V_Y = \sigma_y/\mu_Y$ can be made [18]

$$\frac{\bar{Y}}{Y_0} = \prod_{n=1}^{2k+1} \frac{Y_n}{Y_0}, \tag{12}$$

and

$$1 + V_Y^2 = \prod_{n=1}^{2k+1} (1 + V_n^2). \tag{13}$$

[1] $f(x) = 1/(b-a), \; a \le x \le b$, $f(x) = 0$, otherwise, $\mu = (a+b)/2$, $\sigma = |b-a|/(2\sqrt{3})$, [1]

6 Test Cases

6.1 Introduction

The probabilistic description of the asparagus patch model of the source has been investigated using two cases taken from literature:

- ESA study: "IFLV-Improvement of Force Limites Vibration Testing Methods for Equipment Instrument Unit Mechanical Verification", [7].
- The Linear Drive Unit (LDU), which is an Orbital Replacement Unit (ORU) of the International Space Station (ISS) program, [10].

6.2 ESA IFLV Study

This real life example is taken from the ESA study: "IFLV-Improvement of Force Limites Vibration Testing Methods for Equipment Instrument Unit Mechanical Verification" presented by Destefanis in [7]. The IFLV study facilitated a full test campaign (both sine and random) on a test system composed of an honeycomb panel (source) which supported an optical unit (MIRI) (load) and an electronic box (EBOX) (not considered) (see Fig. 2). Force measurement devices (FMDs) were installed at the mechanical interfaces between units and honeycomb plate. The test runs were performed both on the system and on the units (MIRI, EBOX) standalone, therefore collecting experimental evidence of the difference (in terms of mechanical interface forces) between soft mounted and hard mounted configurations.

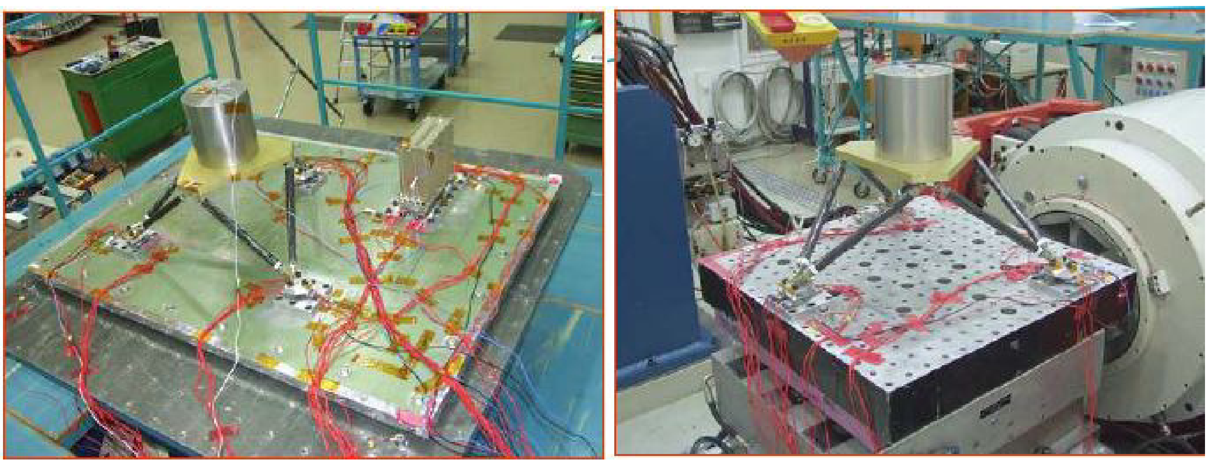


Figure 2: IFLV total and MIRI test setup on shaker slip table, courtesy [7]

6.2.1 Mass Properties of IFLV System

The individual mass properties of the test setup are taken from [7]. These mass properties were extracted from the very detailed finite element models of the EBOX, MIRI, panel and *Force Measurement Devices* (FMD) and are presented in Table 3, however, the EBOX is further not considered in this paper. The fourth column represents the mass properties of the hard-mounted MIRI and FMD's (FMD's between the MIRI and shaker (slip) table).

Table 2: Mass properties of individual items

Mass item	(kg)	(kg)	(kg)
Optical Unit MIRI	27.945	27.945	27.945
Electronic box EBOX	1.257	1.257	
Sandwich honeycomb panel	3.166	3.166	
Force Measurements Devices & plates		4.266	1.693
Total mass	32.268	36.634	29.639

6.2.2 Dynamic Properties of IFLV System & Individual Parts

Modal analysis were done on the total test setup (with and without FMD's), the EBOX, the MIRI and the Honeycomb panel hard-mounted, respectively. The classical results are: the undamped natural frequencies and associated modal effective masses. The modal effective masses are associated to the Z-axis, that is perpendicular to the sandwich panel. The results of the modal analyses are given in Table 3.

Table 3: Mass & Modal properties [7]

Mass item	M (kg)	f_1(Hz)	m_1 (kg)
Optical Unit MIRI	27.945	104.71	27.47
Sandwich honeycom panel	3.166	287.74	1.50

6.2.3 C^2 Interface MIRI/Panel

The values of C^2 are applicable in the Z-direction, thus perpendicular to the panel, and in particular between the sandwich panel and het MIRI instrument. The C^2 values, computed by the STDFS and Ceresetti methods are taken from [7]. Applying the CMSA method the dynamic properties of the panel are computed with respect to the interface between panel and MIRI instrument. The dimensions of the panel are not completely known, but a natural frequency of a panel supported at the midpoint, [2], is approximately 50 Hz. The corresponding modal effective mass varies between 1.50-3.0 kg. The CMSA method gives C^2 values in line with the other methods. The computational results of C^2 are presented in Table 4.

Table 4: Values of C^2 (Z-dir) , $Q = 10$ [7]

load/Source	m_2 (kg)	m_1(kg)	C^2	Remark
MIRI/Panel	27.47	1.50	1.10	STDFS [22]
	27.47	1.50	2.56	CTDFS [22]
	C	1.50	1.10	Ceresetti [3]
	27.47 (105Hz)	1.50-3.0 (50Hz)	1.70-1.74	CMSA [26]
	Experience gained		2-5	Chang [4]

6.2.4 Probabilistic Computation of C^2

The deterministic asparagus patch model of the load (MIRI) is derived from the dynamic properties with respect to the interface between the load and the source (sandwich panel) taken from Table 4 and presented in Table 5. The residual mass is augmented with an artificial high natural frequency outside the frequency range of 20-2000Hz. The sum of the modal effective and residual masses is equal to the total mass of the MIRI, $M_l = 27.945$kg. The damping is probabilistic and applicable to both the load and the source.

To start the probabilistic computation of C^2, with the Rosenblueth $2k + 1$ point estimation method, the uniform distributions of the design variables of the source; the total mass M_s, the first fundamental natural

Table 5: Asparagus patch model MIRI, Z-dir.

Modal effective mass (kg)	$m_{1l} = 27.47$	$m_{rl} = 0.475$
Natural frequency (Hz)	$f_{1l} = 104.71$	$f_{rl} = 2500$
Modal damping ratio ζ (-)	0.01-0.1	

frequency f_{1s}, the first primary modal effective mass m_{1s} and modal damping ζ, presented in Table 1, are used.

The results of the probabilistic computations, the mean and the standard deviation of C^2 and additional variations of the distributions of the total M_s, the modal effective mass m_{1s} and the fundamental natural frequency f_{1s} are presented in Table 6.

Table 6: Probabilistic computations of C^2, Z-dir., $M_l = 27.945$kg (ref. means reference values)

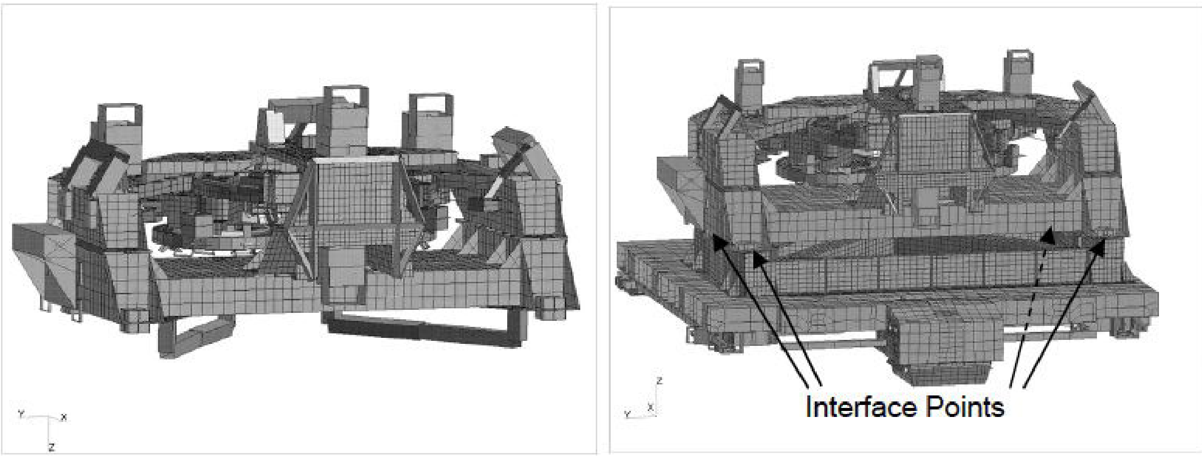
Design Variable	Distribution	Mean μ	Standard deviation σ	C^2_μ	C^2_σ
M_s (ref.)	$0.1 \cdots 10M_l$	5.05 M_l	$2.858M_l$	6.26	1.24
M_s	$0.1 \cdots 1M_l$	0.505 M_l	$0.260M_l$	3.63	0.69
M_s	$0.05 \cdots 0.15M_l$	$0.1M_l$	$0.029M_l$	2.64	0.59
M_s	$0.113M_l$	$0.113M_l$	$0.0M_l$	2.68	0.62
M_s (ref.)	$0.05 \cdots 0.15M_l$	$0.1M_l$	$0.029M_l$		
m_{1s} (ref.)	$0.4 \cdots 0.6M_s$	$0.5M_s$	$0.0577M_s$	2.64	0.59
m_{1s}	$0.6 \cdots 0.8M_s$	$0.7M_s$	$0.0577M_s$	2.59	0.60
M_s (ref.)	$0.05 \cdots 0.15M_l$	$0.1M_l$	$0.029M_l$		
m_{1s} (ref.)	$0.4 \cdots 0.6M_s$	$0.5M_s$	$0.0577M_s$		
f_{1s} (ref.)	$0.5 \cdots 0.707f_{1l}$	$0.6036f_{1l}$	$0.0598f_{1l}$	2.64	0.59
f_{1s}	$0.2 \cdots 0.5f_{1l}$	$0.35f_{1l}$	$0.0866f_{1l}$	2.26	0.39
f_{1s}	$0.707 \cdots 0.8f_{1l}$	$0.754f_{1l}$	$0.0268f_{1l}$	5.19	0.94
M_s (ref.)	$0.05 \cdots 0.15M_l$	$0.1M_l$	$0.029M_l$		
m_{1s} (ref.)	$0.4 \cdots 0.6M_s$	$0.5M_s$	$0.0577M_s$		
f_{1s} (ref.)	$0.5 \cdots 0.707f_{1l}$	$0.6036f_{1l}$	$0.0598f_{1l}$		
f_{2s}, f_{3s}, f_{4s} (ref.)	$2f_{1s}, 4f_{1s}, 6f_{1s}$			2.64	0.59
f_{2s}, f_{3s}, f_{4s}	$1.25f_{1s}, 1.5f_{1s}, 2f_{1s}$			3.30	0.13
f_{2s}, f_{3s}, f_{4s}	$1.5f_{1s}, 2f_{1s}, 3f_{1s}$			3.13	0.10

Compared to the estimated values of C^2, given in Table 4, it can be concluded from the probabilistic computed values of the mean and the standard deviation of C^2, that a good estimation of the total mass of the source is important to obtain more reliable figures of C^2. The distributions of the other design parameters were well chosen, however, the following observations can be made:

- If the stiffness of the source is too low ($f_{1s} \ll f_{1l}$), the load will act like a rigid body and no load anti-resonance effects may be expected.
- If the source is too stiff ($f_{1s} \leq f_{1l}$), the dynamic coupling between the load and the source will increase.
- Clustering the natural frequencies of the source will amplify the internal response between load an the source.

6.3 LDU/FSE/FRAM

The Linear Drive Unit (LDU) is an Orbital Replacement Unit (ORU) of the International Space Station (ISS) program. During the flight of the LDU to the ISS, it is connected to a Space Shuttle Orbiter by an adaptor plate and locking system. The LDU is connected to the adaptor plate by four points, which will be known as interface points. The configuration of the LDU, flight support equipment (FSE) adapter plate and active flight release attachment mechanism (FRAM) together forms the integrated model. The integrated model is attached to the Orbiter at seven points, which have various constraint directions. The models are shown in Figure 3. The mass of the LDU (load) is $M_l = 113.85$ kg and the remaining FSE and FRAM parts (source) make up $M_s = 187.33$ kg. The modal effective masses of the significant modes and the C^2 of the semi-empirical force limits equations (1) are given in the next sections. The dynamic properties of the FSE/FRAM



(a) LDU model (b) Integrated model

Figure 3: LDU-FSE-FRAM FEM in launch configuration [10]

are not presented in [10], hence unknown.

6.3.1 Dynamic Properties LDU and Value C^2

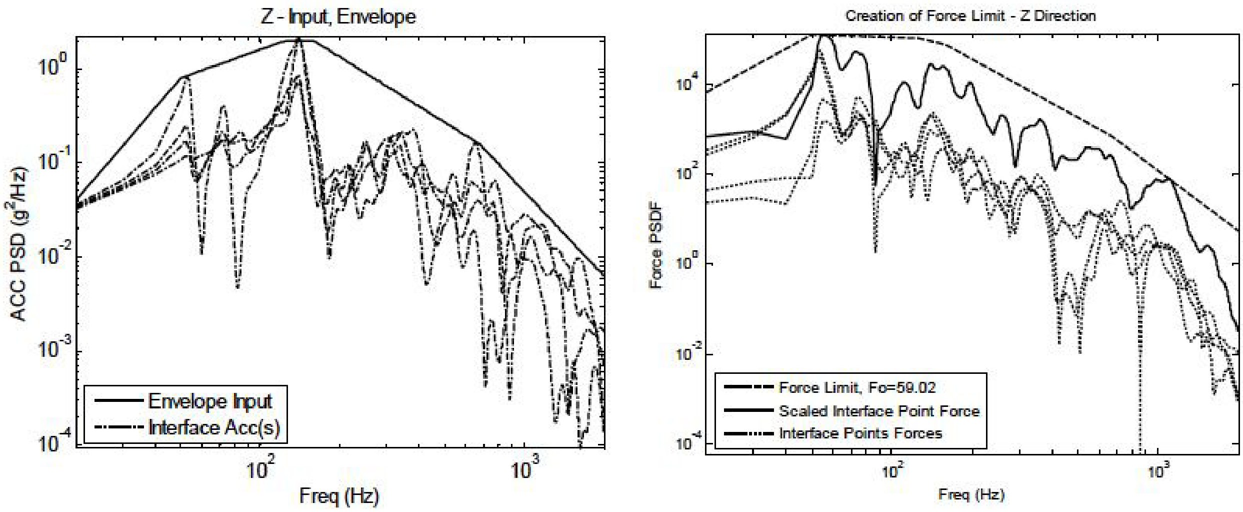
The natural frequencies and associated modal effective masses of the first three dominant modes of the LDU, fixed at the interface between LDU and FSE (see Fig. 3), are taken from the paper of Fitzpatrick [10] and presented in Table 7. The Z-dir is perpendicular to the mounting plane

Table 7: Dynamic properties LDU, courtesy [10]

		Modal effective mass		
Mode $\neq$	Frequency (Hz)	X-dir (kg)	Y-dir (kg)	Z-dir (kg)
1	59.0	0.4	0.0	36.0
4	75.0	0.1	49.9	0.1
7	92.7	27.9	0.1	18.2

6.3.2 C^2 from Literature

The value C^2 was derived from the STDFS equations and the scaled force power spectral density response at the interface between LDU/FSE and taken from [10] and given in Table 8. The scaled random interface force is computed from the enveloped random acceleration specification multiplied by the squared magnitude of the apparent mass of the LDU (load). The PSD acceleration at the four interface points between the LDU and FSE/FRAM are represented by the four dotted curves. The final random acceleration specification is the envelope of these four curves. This is illustrated in Fig. 4 a). The drawn curve in Fig. 4 b) represents the



(a) Envelope of acceleration (Z-dir)

(b) Force limits created from SEM (Z-dir)

Figure 4: Scaled random interface force procedure, courtesy [10]

PSD of the interface force and corresponds to the enveloped random acceleration. Applying equation (2) the value C^2 can be established.

Table 8: Values of C^2 and n, from STDFS and analytical data (FEA) , courtesy [10]

	STFDS (Q=10)	X-dir	Y-dir	Z-dir
C^2	3.48	2.5	4.8	2.7
n	2	2	2	1.5

6.3.3 Probabilistic Computation C^2

The asparagus patch model of the load (LDU) is derived from the dynamic properties with respect to the interface between the load and the source (FSE/FRAM) taken from Table 7 and presented in Table 9. The residual mass is augmented with an artificial high natural frequency outside the frequency range of 20-2000Hz. The sum of the modal effective and residual masses is equal to the total mass of the LDU, $M_l = 133.85$kg. The damping is probabilistic and applicable to both the load and the source.

Table 9: Asparagus patch model LDU, Z-dir., $M_l = 113.85$ kg

Modal effective mass (kg)	$m_{1l} = 36.0$	$m_{2l} = 18.2$	$m_{rl} = 59.65$
Natural frequency (Hz)	$f_{1l} = 59.0$	$f_{2l} = 92.7$	$f_{rl} = 2500$
Modal damping ratio ζ (-)	0.01-0.1		

To start the probabilistic computation of C^2, with the Rosenblueth $2k + 1$ point estimation method, the uniform distributions of the design variables of the source; the total mass M_s, the first fundamental natural frequency f_{1s}, the first primary modal effective mass m_{1s} and modal damping ζ as presented in Table 1, are used.

The results of the probabilistic computations, the mean and the standard deviation of C^2 and additional variations of the distributions of the total M_s, the modal effective mass m_{1s} and the fundamental natural frequency f_{1s} are presented in Table 10.

Table 10: Computation of C^2 for the LDU, Z-dir., $M_l = 113.85$kg (ref. means reference values)

Design Variable	Distribution	Mean μ	Standard deviation σ	C^2_μ	C^2_σ
M_s (ref.)	$0.1 \cdots 10 M_l$	5.05 M_l	$2.858 M_l$	2.18	0.54
M_s (ref.)	$1.0 \cdots 2.0 M_l$	1.5 M_l	$0.289 M_l$	1.83	0.33
M_s	$1.0 \cdots 2.0 M_l$	1.5 M_l	$0.289 M_l$		
m_{1s}	$0.4 \cdots 0.6 M_s$	$0.5 M_s$	$0.0577 M_s$	1.83	0.33
m_{1s}	$0.6 \cdots 0.8 M_s$	$0.7 M_s$	$0.0577 M_s$	2.12	0.47
M_s (ref.)	$1.0 \cdots 2.0 M_l$	1.5 M_l	$0.289 M_l$		
m_{1s} (ref.)	$0.4 \cdots 0.6 M_s$	$0.5 M_s$	$0.0577 M_s$		
f_{1s} (ref.)	$0.5 \cdots 0.707 f_{1l}$	$0.6036 f_{1l}$	$0.0598 f_{1l}$	1.83	0.33
f_{1s}	$0.2 \cdots 0.5 f_{1l}$	$0.35 f_{1l}$	$0.0866 f_{1l}$	3.35	1.97
f_{1s}	$0.707 \cdots 0.8 f_{1l}$	$0.754 f_{1l}$	$0.0268 f_{1l}$	3.32	0.60
M_s (ref.)	$1.0 \cdots 2.0 M_l$	1.5 M_l	$0.289 M_l$		
m_{1s} (ref.)	$0.4 \cdots 0.6 M_s$	$0.5 M_s$	$0.0577 M_s$		
f_{1s} (ref.)	$0.5 \cdots 0.707 f_{1l}$	$0.6036 f_{1l}$	$0.0598 f_{1l}$		
f_{2s}, f_{3s}, f_{4s} (ref.)	$2 f_{1s}, 4 f_{1s}, 6 f_{1s}$			1.83	0.33
f_{2s}, f_{3s}, f_{4s}	$1.25 f_{1s}, 1.5 f_{1s}, 2 f_{1s}$			4.26	1.47
f_{2s}, f_{3s}, f_{4s}	$1.5 f_{1s}, 2 f_{1s}, 3 f_{1s}$			3.90	1.44

If the total mass of the load and the source is in the same order the analytical computed values of C^2 given in Tables 8 and 10, do correlate very good. We may conclude that the distributions of all design parameters are well chosen, however, a good guess of the total mass of the source is beneficial. In addition the same observations as for the MIRI can be made.

7 Discussion/Conclusions

In [28] a probabilistic method was proposed to compute the value of C^2 of the semi-empirical equations (1), when only the dynamic properties of the deterministic load are known and the dynamic properties of a probabilistic source are represented by probabilistic design variables with a uniform distribution.

To verify the probabilistic model of the source two test cases were analyzed, which were taken from [7] (MIRI instrument) and [10] (LDU orbital replacement unit). The CMSA method was applied combining a deterministic asparagus patch model of the load and the probabilistic asparagus patch model of the source. The following observations and conclusions can be made:

- The MIRI instrument is the load and the sandwich panel is the source. The total mass of the load is $M_l = 27.945$ kg, and the total mass of the source is $M_s = 3.164$ kg. The ratio is $M_l/M_s = 8.8$ and applying the STDFS estimation method $C^2 = 1.1$, $Q = 10$.
  - The computation of C^2 starting with initial distributions of the design variables of the probabilistic source, shown in Table 1, give a too high mean $C^2_\mu = 6.26$. The distribution of M_s is too far from the actual value. Tuning the band-limited uniform distribution of $M_s = 0.1 \cdots 0.15 M_l$ gave much better result of $C^2_\mu = 2.64$. The other initial distributions of f_{1l}, m_{1s} and ζ are well chosen.
  - If it is expected that the mass of the source $M_s \ll M_l$, thus the ratio $M_l/M_s \gg 1$ one shall tune the distribution of M_s more in accordance to the estimated or provided mass of the source.
- The LDU is the load and the supporting structure FSE/FRAM is the source. The total mass of the load is $M_l = 113.85$ kg, and the total mass of the source is $M_s = 187.33$ kg. The ratio is $M_l/M_s = 0.6$. gives with the STDFS estimation method $C^2 = 3.48$, $Q = 10$.

- The computation of C^2 starting with initial distributions of the design variables of the probabilistic source, shown in Table 1, gave a good correlated mean value $C^2_\mu = 2.18$. The initial distributions of M_s, f_{1l}, m_{1s} and ζ are well chosen.
- If the expected mass of the source $M_s \approx M_l$ and the ratio $M_l/M_s \approx 1$, the initial distributions from Table 1 are very convenient.

- If the stiffness of the source is too low, the load will act like a rigid body and no load anti-resonance effects may be expected.
- If the source is to stiff the dynamic coupling between the load and the source will increase.
- Clustering the natural frequencies of the source will amplify the internal response between load an the source.
- It is recommended to achieve a good knowledge of the mass of the source M_s.
- The residual modal effective mass shall be incorporated into the asparagus patch model of the load.

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