

Modelling sediment pick-up and deposition in a dune model

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Abstract

Often river bed form modelling is done with an equilibrium bed load transport formula like that of Meyer-Peter & Müller (1948). However, a physically more correct way would be to model it with separate models for the sediment pick-up and deposition processes as described by Nakagawa & Tsujimoto (1980). Besides the physics of the sediment transport itself, using such a method allows for the modelling of higher-order processes as well like spatial lag in bed load transport.

As shown by Shimizu et al. (2009) applying the aforementioned pick-up and deposition model in a dune evolution model, makes it possible to model dunes well. Specifically it made it possible to determine a transition to upper stage plane beds, as well as capturing hysteresis well.

In this paper we will explore the effect of using different kinds of bed load models in a relatively simple dune evolution model. The Nakagawa & Tsujimoto (1980) bed load model, will be implemented in the dune evolution model of Paarlberg et al. (2009). Results of this model version will be compared with the original version (using the Meyer-Peter & Müller formula) and a later version that directly models spatial lag with a relaxation equation.

1. INTRODUCTION

Hydraulic roughness values play an important role in correctly determining water levels (Casas et al., 2006; Vidal et al., 2007; Morvan et al., 2008), which is critical for flood management purposes.

River dunes increase the hydraulic roughness significantly, because their shape causes form drag. Water level forecasts during a high river water discharge therefore depend on accurate predictions of the evolution of river dune dimensions.

In the past, many approaches have been used to model dune dimensions, varying from equilibrium dune height predictors (e.g. Yalin, 1964; Allen, 1978; Van Rijn, 1984) to different forms of stability analyses (e.g. Kennedy, 1963; Engelund, 1970; Fredsøe, 1974; Yamaguchi & Izumi, 2002). Recently, models have been developed that calculate the turbulent flow field over bedforms, in some cases in combination with morphological computations (e.g. Nelson et al., 2005; Tjerry & Fredsøe, 2005; Shimizu et al., 2009; Nabi et al., 2010). These models are valuable to study detailed hydrodynamic processes, but are computationally intensive.

To be able to efficiently predict dune dimensions over the time-scale of a flood wave Paarlberg et al. (2009) developed a model in which the flow and

sediment transport at the flow separation zone is parameterized instead of using complex hydrodynamic equations. This model is able to predict the evolution of dunes from small initial disturbances up to equilibrium dimensions with limited computational time. In addition, this model has been coupled with an existing hydraulic model to form a ‘dynamic roughness model’ (Paarlberg et al., 2010). Results are promising, as the coupled model clearly shows the expected hysteresis effects in dune roughness and water levels and different behaviour of sharp-peaked versus broad-peaked flood waves (Paarlberg et al., 2010).

Paarlberg et al. (2009) assume that equilibrium between shear stress and transport is present, so the formula devised by Meyer-Peter and Müller (1948) is used. As Nakagawa & Tsujimoto (1980) argue, a lag distance between flow properties (and thereby bed shear stress) and sediment transport is the principal cause of bed instability and thereby regime transitions. They further identify two sources of this lag distance. The first is the spatial distribution of bed shear stress, which is handled in the Paarlberg et al. (2009) model by applying the transport formula to the local bed shear stress. The second is the probability distribution of sediment particle step length, which is the distance travelled from dislodgement to rest according to Einstein (1950). This effect is not taken into account in the bed load formulation of the original model.

To be able to model the latter effect with bed load transport, the Paarlberg et al. (2009) model is first extended with a linear relaxation equation applied on the Meyer-Peter and Müller (1948) transport, and secondly with the pick-up and deposition model of Nakagawa & Tsujimoto (1980). This bed load formula is also used in the model of Shimizu et al. (2009), with good results. The pick-up is determined from local bed shear stress. The sediment is deposited from the pick-up point with a distribution function, which uses a mean step length, exponentially decreasing with distance. By handling the transport like this a lag distance between shear stress and sediment transport is introduced.

The effects on bed morphologies and development characteristics of using the non-equilibrium transport relations versus the previous equilibrium transport relation will be explored. Different values of step length are used to see how it influences the results. It is expected that the dune shape will differ significantly between versions of the model due to the introduction of spatial lag with the two new model versions. This should improve the predictions of the model for future applications, as this lag is one of the causes of bed instabilities, and thereby controls transitions between bed form regimes.

2. MODEL SET-UP

1.1 General set-up

The basis of the present model is the dune evolution model developed by Paarlberg et al. (2009). Paarlberg et al. (2009) extended the process-based morphodynamic sand wave model of Németh et al. (2006), which is based on the numerical model of Hulscher (1996), with a parameterization of flow separation, to enable simulation of finite amplitude river dune evolution.



Figure 1. Schematization of a dune (flow left to right)

Flow separation is forced in the model when the leeside slope exceeds 10° . The form of the flow separation zone (see Figure 1) behind the dune and the effect it has on flow, bed shear stress distribution and the sediment transport is included

in a parameterized way using experimental data of turbulent flow over two-dimensional subaqueous bedforms (Paarlberg et al. 2007). In the flow separation zone the bed shear stress is assumed to be zero and all the sand transport that reaches the crest of the dune is avalanched under the angle of repose on the leeside of the dune (Paarlberg et al., 2009). This enables the model to predict river dunes with their characteristic shape and realistic dimensions without resolving the complex recirculating flow in the flow separation zone and remaining computationally cheap.

The model consists of a flow module, a sediment transport module and a bed evolution module which operate in a decoupled way. The model simulates a single dune which is assumed to be in an infinite train of identical dunes. Therefore periodic boundary conditions are used. The domain length and thereby dune length is forced by either using the simple relation Van Rijn (1984) found or using a numerical stability analysis as the original model by Paarlberg et al. (2009) does. In the first case the dune length is seven times the water depth, a reasonable approximation of the values Julien & Klaassen (1995) find, namely 7.3 and 2π times the water depth. In the latter case the length of the fastest growing disturbance is determined during simulation. Only the first approach will be used in this paper.

1.2 Flow model

In general the flow is forced by the difference in water level across the domain. Though the water depth at the start and end of domain are the same due to the periodic boundary conditions, the water level differs because the domain is sloped. The average bed level is taken as zero but has a slope (this average bed slope is an input parameter for the model). By solving the flow equations with a certain average water depth a discharge is found. The average water depth is adjusted until this discharge matches the discharge given as input.

1.2.1 Governing equations

The flow in the model of Paarlberg et al. (2009) is described by the two-dimensional shallow water equations in a vertical plane (2-DV), assuming hydrostatic pressure conditions. For small Froude numbers the momentum equation in vertical direction reduces to the hydrostatic pressure

condition, and that the time variations in the horizontal momentum equation can be dropped. The governing model equations that result are shown in equations (1) and (2).

$$u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -g \frac{\partial \zeta}{\partial x} + A_v \frac{\partial^2 u}{\partial z^2} + gi \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (2)$$

The velocities in the x and z directions are u and w, respectively. The water surface elevation is denoted by ζ , i is the average channel slope, and g and A_v denote the acceleration due to gravity and the vertical eddy viscosity respectively.

1.2.2 Boundary conditions

The boundary conditions are defined at the water surface ($z=h$) and at the bed ($z=z_b$). The boundary conditions at the water surface are (3) no flow through the surface and (4) no shear stress at the surface. The kinematic boundary condition at the bed is (5) that there is no flow through the bed.

$$\left. \frac{\partial u}{\partial z} \right|_{z=h} = 0 \quad (3)$$

$$u \left. \frac{\partial \zeta}{\partial x} \right|_{z=h} = w \quad (4)$$

$$u \left. \frac{\partial z_b}{\partial x} \right|_{z=z_b} = w \quad (5)$$

As basic turbulence closure, a time- and depth-independent eddy viscosity is assumed, leading to a parabolic velocity profile. In order to represent the bed shear stress correctly for a constant eddy viscosity, a partial slip condition at the bed (6) is necessary.

$$\tau_b = A_v \left. \frac{\partial u}{\partial z} \right|_{z=z_b} = S u_b \quad (6)$$

In this equation τ_b (m^2/s^2) is the volumetric bed shear stress and the resistance parameter S (m/s) controls the resistance at the bed. For more details about the model equations and numerical solution

procedure, reference is made to Paarlberg et al. (2009), Van den Berg and Van Damme (2005), and Van den Berg (2007).

1.3 Bed load transport model

For this work we compare three versions of the bed load model : the original, a later version with spatial lag via a relaxation equation, and a new version with the Nakagawa & Tsujimoto (1980) pick-up and deposition model. These three versions are explained in the next paragraphs.

1.3.1 Equilibrium transport model

In the original dune evolution model equilibrium bed load transport is taken into account. This is calculated by applying the formula of Meyer-Peter and Müller (1948) including gravitational bed slope effects. Below this formula is given in dimensional form (as volumetric bed load transport per unit width, m^2/s):

$$q_{b,e} = \begin{cases} \beta (\tau_b(x) - \tau_c(x))^n \left(1 + \eta \frac{\partial z_b}{\partial x} \right)^{-1} & \text{if } \tau > \tau_c \quad (7) \\ 0 & \text{if } \tau \leq \tau_c \end{cases}$$

where $\tau_c(x)$ is the local critical (volumetric) bed shear stress (m^2/s^2), $n=3/2$ and $\eta=\tan(\varphi)^{-1}$ with the angle of repose $\varphi=30^\circ$. The proportionality constant β (s^2/m) describes how efficiently the sand particles are transported by the bed shear stress (Van Rijn, 1993) and its value can be estimated with

$$\beta = \frac{m}{\Delta g} \quad (8)$$

where $\Delta=\rho_s/\rho-1=1.65$ (ρ_s/ρ is the specific grain density), and m is an empirical coefficient which is set to 4 by Paarlberg et al. (2009) based on analysis done by Wong and Parker (2006). The local, critical bed shear stress $\tau_c(x)$, corrected for bed slope effects, is given by the following equation:

$$\tau_c(x) = \tau_{c0} \frac{1 + \frac{\partial z_b}{\partial x}}{\sqrt{1 + \left(\frac{\partial z_b}{\partial x} \right)^2}} \quad (9)$$

with τ_{c0} the critical bed shear stress for flat bed, defined by equation (10). In this equation θ_{c0} is the critical Shields parameter and D_{50} is the median grain size.

$$\tau_{c0} = \theta_{c0} g \Delta D_{50} \quad (10)$$

1.3.2 Linear relaxation of transport

Here the model differs from the model presented by Paarlberg et al. (2009). Instead of calculating the equilibrium transport (see previous paragraph) and taking that as the actual transport, the following relation is applied:

$$\frac{dq_b}{dx} = \frac{q_{b,e} - q_b}{\Lambda} \quad (11)$$

where q_b is the actual sediment transport and Λ is the mean step length. This is determined by:

$$\Lambda = \alpha D_{50} \quad (12)$$

where α is the non-dimensional step length (as used by Nakagawa & Tsujimoto, 1980). It should be noted that equation 11 needs a boundary condition (at $x=0$) whereas only a periodic boundary condition is defined. Therefore a value is guessed for $x=0$ and the rest of the values are determined using equation 11 and a backwards Euler scheme. The value at the end of the domain should be the same as the value at $x=0$, if this is not the case a new guess is made. This process is repeated until a satisfactory result is found (i.e. when the periodic boundary condition is met).

1.3.3 Pick-up and deposition model

The pick-up and deposition model of Nakagawa & Tsujimoto (1980) uses the following formulae to determine bed load transport. Pick-up of sediment (probability of a particle being picked up in s^{-1}) is determined by

$$p_s(x) = F_0 \sqrt{\frac{\Delta g}{D_{50}}} \tau_*(x) \left(1 - \frac{\tau_{*c}}{\tau_*(x)} \right)^3 \quad (13)$$

where $F_0=0.03$. Deposition at a location is determined by summing the sediment that arrives at that location. So, to determine the deposition at

a certain location x the distribution of picked up sediment from upstream locations is needed. The determination of deposition is done by applying the following formula:

$$p_d(x) = \int_0^{\infty} p_s(x-s) f(s) ds \quad (14)$$

where the distribution $f(s)$ determines the fraction of sediment that is deposited a distance s away from the pick-up point ($x-s$). The distribution function is defined as follows:

$$f(s) = \frac{1}{\Lambda} \exp\left(\frac{-s}{\Lambda}\right) \quad (15)$$

where λ is the step length. By using this function, all the sediment that has been picked up at certain location is deposited between that location and 5 times the step length in downstream direction. Finally the transport gradient is determined as follows:

$$\frac{dq_b(x)}{dx} = D_{50} [p_s(x) - p_d(x)] \quad (16)$$

1.4 Step length

Francis (1973), Fernandez Luque & Van Beek (1976) and Sekine & Kikkawa (1984) have done experiments to determine the dependence of particle velocity on various parameters under flat bed conditions. The latter authors have used this data to verify a numerical model of saltation of particles (Sekine & Kikkawa, 1992). All computed values are no more than two times larger or smaller than the observed values.

Their model further shows that the mean step length can vary between near zero and about 350 times the particle diameter, mostly dependent on friction velocity (positively) and settling velocity (negatively). The data shows a range of approximately 40 to 240 times the particle diameter. For this paper the step length will therefore be varied between 25 and 300 times the particle diameter, to get an idea of how sensitive the results are to this parameter.

1.5 Bed evolution

The bed evolution is modelled using the Exner equation given by (17), where the sediment transport rate is calculated with one of the three options and $\varepsilon_p=0.4$ is the bed porosity.

$$(1-\varepsilon_p) \frac{\partial z_b}{\partial t} = -\frac{\partial q_b}{\partial x} \quad (17)$$

It should be noted that in the case of flow separation this equation is only applied outside the flow separation zone. In the separation zone the bed transport at the crest of the dune is deposited on the leeside of the slope under the angle of repose (i.e. avalanched). So, an integral form of equation is used for the lee slope of the dune.

3. RESULTS

The reference case used for this study is an experiment done by Venditti et al. (2005). The relevant parameters can be found in the table below.

h_i [m]	0.152
i [10^{-4}]	12
q [m^2/s]	0.077
D_{50} [mm]	0.5
l_e [m]	1.3172
Δ_e [m]	0.048
h_e [m]	0.17
$\theta_{c,0}$ [-]	0.050

Table 1. Used parameters

New parameters in this table are h_i (initial water depth), q (discharge per unit width), l_e (equilibrium dune length), Δ_e (equilibrium dune height) and h_e (equilibrium water depth).

1.6 Results with the original bed load model

Using the original bed load model, Meyer-Peter and Müller (1948), an equilibrium dune height of 0.064m, dune length of 1.33m and water depth of 0.19m are found. The dune length is predicted well (the experimental result was 1.32m), but the dune height is overestimated by about 25%. The resulting water depth is reasonably close to the experimental result of 0.17m. In figure 2 the evolution of the dune shape is shown.

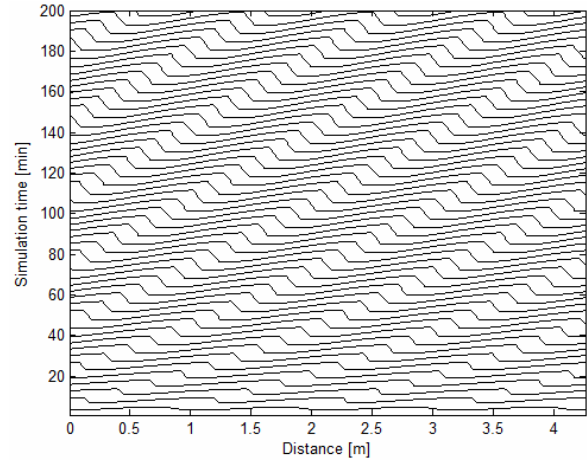


Figure 2. Dunes of the original model (flow left to right)

It should be noted that this figure is obtained by plotting the resulting single dune as a train of four identical dunes to make the results more clear.

1.7 Results with linear relaxation

Using the original bed load model, but with an additional forcing of spatial lag with a relaxation equation the following is found.

α [-]	Δ_e [m]	l_e [m]	h_e [m]
[original]	0.064	1.33	0.19
25	0.029	1.11	0.16
50	0.023	1.10	0.16
75	0.000	1.07	0.15
100	0.000	1.07	0.15

Table 2. Linear relaxation results.

As can be seen applying spatial lag in this way leads to a very strong suppression of the dune height and length. The first is because the spatial lag decreases the total transport and the lee side angle, and no more flow separation occurs. This severely limits the dune growth, leading to these very small dunes. The less steep dunes of limited height are shown in figure 3, presenting the bed morphology with a non-dimensional step length of 25.

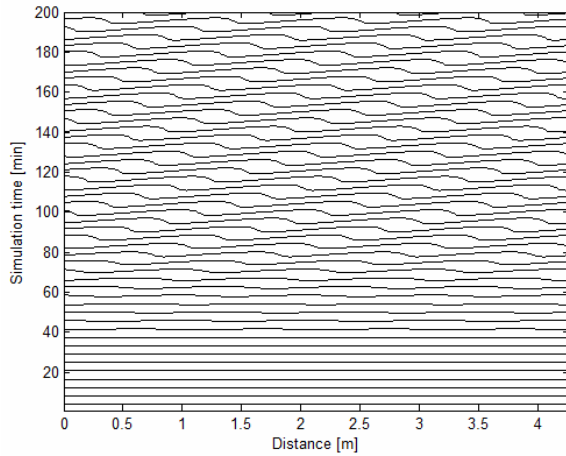


Figure 3. Dunes of the model with linear relaxation, $\alpha=25$ (flow left to right)

With a stronger lag (non-dimensional step length of 75 and greater) this ‘smearing’ effect is so strong that no more dune growth occurs at all. This is similar to what would occur when going towards an upper stage plane bed, where the bed washes out. Because the dune height is small, there is less hydraulic roughness so the water depth is limited as well. The dune length directly follows from this, so that remains small as well.

As presented at RCEM 2011, the authors found that this same analysis but then with the dune length selected by a stability analysis (see Paarlberg et al., 2009) led to different results. The dune height was still suppressed, but not so strong as presented here (Van Duin et al, 2011). For higher values of the non-dimensional step length dunes kept appearing as opposed to now. Interestingly the dune *length* was not suppressed at all, and greatly increased for larger values of the step length. During the selection for dune length, the transport with linear relaxation was used to determine which dune length lead to the strongest growth, and that method selected progressively longer dunes up until values of 200 for the non-dimensional step length before decreasing again. This interplay between the selected dune length and the introduced spatial lag is not fully understood (Van Duin et al, 2011), but should be taken into account in further model development.

1.8 Results with pick-up and deposition

Using the pick-up and deposition model of Nakagawa & Tsujimoto (1980) as the bed load model, the following is found.

α [-]	Δ_e [m]	l_e [m]	h_e [m]
25	0.067	1.33	0.19
50	0.066	1.33	0.19
75	0.064	1.32	0.19
100	0.067	1.33	0.19
150	0.069	1.34	0.19
200	0.070	1.35	0.19
250	0.076	1.39	0.20
300	0.079	1.41	0.20

Table 3. Pick-up and deposition results.

Against expectation, the water depth and thereby dune length are very similar to the experimental and original model results. The dune height is still too high compared to the experimental results, but very near the original model. This at least shows that the new bed load formula still performs reasonably well.

With the linear relaxation method dune height was suppressed strongly, and now it is not. This is because with linear relaxation the transport was greatly reduced, while now it is still about as high as with the original model. Flow separation still occurs, and so all in all the dune is able to grow like it did with the original model. Even larger values for the non-dimensional step length don’t lead to decreasing dune growth as it did with linear relaxation but actually *increasing* dune growth. With increasing step length in this model version, sediment is spread over a larger distance, so more sediment actually reaches the crest. Because flow separation still occurs, all the sediment that reaches the crest is avalanched there instead of being spread out over the lee side and trough as happened with linear relaxation.

The final resulting dune shape with a non-dimensional step length of 25 can be seen in figure 4.

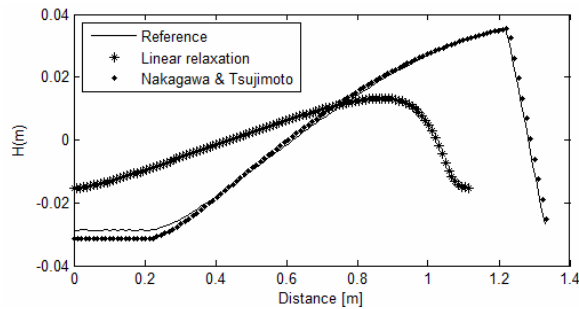


Figure 4. Dune shapes of the three versions, with $\alpha=25$ for linear relaxation and pick-up and deposition (flow left to right)

It is clear that it is very similar to the result with the original model version but strongly differs from the linear relaxation result. While by itself it is promising that the model still performs well, it was expected that the dune shape would differ significantly from the original version. Had this been the case, it would signal that the model should be able to handle transitions to other regimes better because it allows for more different dune morphologies. Now, it is not known whether this is improved so further research is needed. It is likely that a non-constant step length will lead to more different results and a better prediction of transitions, as the model of Shimizu et al. (2009) has shown.

4. CONCLUSIONS

This paper has shown how a computationally cheap dune evolution model depends on the bed load transport formulation used. With the equilibrium transport formula of Meyer-Peter and Müller (1948) the results are reasonable, though the dune height is overestimated. Applying a linear relaxation equation introduces a spatial lag equal to the step length, and leads to a strong suppression of dune height and length. This is so strong that for a higher step length no more dunes form.

Using the Nakagawa & Tsujimoto (1980) pick-up and deposition model the results are very similar to the original version. By itself it is promising that the model still performs well, it was expected that the dune shape would differ significantly. It is likely that using a non-constant step length will lead to different and better results with regards to regime transitions.

5. FUTURE WORK

The model will be further refined by improving the relation between bed shear stress and the step length of transported material. For this the conceptual model of Shimizu et al. (2009), a step length model for flat bed (Sekine & Kikkawa, 1992) and a formulation that depends on the transport parameter by van Rijn (1984) will be tested. Also, experiments have been undertaken by the authors regarding step length. With this knowledge and the different step length models the model will be improved further.

6. ACKNOWLEDGMENT

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