Spring-forward in composite plate elements

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ABSTRACT: Spring-forward is a distortion of corner sections in continuous fibre reinforced composite products. The linear thermoelastic prediction for the spring-forward of single curved geometries is incorporated in a FE formulation for plate elements in order to simulate the spring-forward of doubly curved products. The proposed methodology is demonstrated with the simulation of the process-induced distortion of a wing leading edge stiffener.

Key words: composites, spring-forward, finite elements

1 INTRODUCTION

Continuous fibre reinforced polymers increasingly proof their good performance as structural materials in demanding applications. The forming of these materials into parts is not straightforward, though. The formed parts often show distortions of the geometry that are unacceptable in the narrow-toleranced applications in which they are used.

The distortions can be divided in warpage and spring-forward. Warpage is the curvature observed in initially flat parts. It is mainly caused by non-homogeneous consolidation and tool-part interaction. Spring-forward is the decrease of the enclosed angle of bend composite parts. It is primarily the result of the anisotropic nature of continuous fibre reinforced composites.

Numerical prediction of the product distortions can contribute to a ‘first time right’ design, saving costs involved with trial-and-error. In this paper, a numerically efficient method is proposed to predict spring-forward of doubly curved composite products.

2 THEORETICAL BACKGROUND

The classical laminate theory (CLT) is adapted to incorporate the spring-forward phenomenon. First, spring-forward is described in terms of the curvature change of an arc section. This formulation is then extended to doubly curved surfaces, after which it is incorporated in the CLT. Subsequently, a finite element (FE) formulation based on the modified CLT is proposed.

2.1 Spring-forward of a single curved part

Figure 1 shows the decrease of the enclosed angle of an L-shaped part, for example due to free thermal contraction.

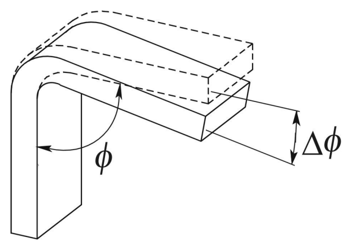

Fig. 1. Decrease of the enclosed angle of an L-shaped part

The linear thermoelastic prediction of the change of the enclosed angle ϕ is given as [1]:

$$\Delta\phi = \phi \frac{\varepsilon_r - \varepsilon_\theta}{1+\varepsilon_r} \quad (1)$$

and if only thermal strains are considered, equation (1) modifies to

$$\Delta\phi = \phi \frac{(\alpha_r - \alpha_\theta)\Delta T}{1+\alpha_r \Delta T} \quad (2)$$

where α_r and α_θ are the coefficients of thermal expansion (CTE) in radial and tangential direction,

respectively. The temperature change ΔT is the difference between the room temperature and the temperature at which the composite material is able to sustain elastic stress. Equation (2) was validated on woven fabric reinforced carbon/polyetherimide (carbon/PEI) curved specimens [2].
The relative change of the enclosed angle can be conveniently rewritten to a relative change of the curvature of the arc segment, assuming small deformations: $\Delta\phi/\phi = R\Delta\kappa$. The thermally induced spring-forward equation (2) becomes:

$$\Delta\kappa^{sf} = \frac{1}{R}\frac{(\alpha_r - \alpha_\theta)\Delta T}{1+\alpha_r \Delta T} \approx \frac{1}{R}(\alpha_r - \alpha_\theta)\Delta T \qquad (3)$$

where $\Delta\kappa^{sf}$ is the change of curvature caused by spring-forward and R is the radius of curvature.

2.2 Extension to doubly curved surfaces

A doubly curved surface can be described with two principal curvatures, which are oriented in the principal directions. It is convenient to describe the spring-forward of doubly curved surfaces in terms of the principal curvatures, since the twist curvature equals zero in the principle directions. The principle directions are rotated with respect to the axes of the global coordinate system by an angle θ_p, which is obtained by solving:

$$\tan(2\theta_p) = -\frac{2\kappa_{xy}}{\kappa_x - \kappa_y} \qquad (4)$$

where κ_x, κ_y and κ_{xy} are the curvatures in the global *x*, *y* and *xy* directions [2]. The change of curvature induced by spring-forward is given by

$$\begin{Bmatrix} \Delta\kappa_\xi^{sf} \\ \Delta\kappa_\eta^{sf} \\ 0 \end{Bmatrix} = \begin{Bmatrix} \frac{1}{R_\xi}(\alpha_r - \alpha_\xi) \\ \frac{1}{R_\eta}(\alpha_r - \alpha_\eta) \\ 0 \end{Bmatrix} \Delta T \qquad (5)$$

where subscripts ξ and η denote the two principal directions. The in-plane CTE's α_ξ and α_η are obtained by rotational transformation of the in-plane CTE's in global directions $\{\alpha_x, \alpha_y, \alpha_{xy}\}$. The change of curvature in the principal directions is subsequently transformed to a change of the curvatures in global coordinates $\{\Delta\kappa_x^{sf}, \Delta\kappa_y^{sf}, \Delta\kappa_{xy}^{sf}\}$.

2.3 Modification of CLT

The CLT relates the force and moment resultants ($\{N\}$ and $\{M\}$) to the midplane strains $\{\varepsilon^0\}$ and curvatures $\{\kappa\}$ of a plate according to

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \left(\begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} - \begin{Bmatrix} \alpha \\ \kappa^T \end{Bmatrix} \Delta T \right) \qquad (6)$$

where $\{\kappa^T\}$ represents the thermally induced curvatures. Equations (6) hold for shallow shells, although $\{\kappa\}$ and $\{\kappa^T\}$ then represent changes of the initial curvature [2]. The change of curvature induced by the spring-forward effect is incorporated according to

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \left(\begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} - \begin{Bmatrix} \alpha \\ \kappa^T \end{Bmatrix} \Delta T - \begin{Bmatrix} 0 \\ \Delta\kappa^{sf} \end{Bmatrix} \right) \qquad (6)$$

The in-plane CTE's $\{\alpha\}$ and the thermally induced curvatures $\{\kappa^T\}$ can be obtained from the CLT for uni-directional laminates or from CLT based woven fabric mechanics [3]. The through-thickness CTE α_r requires a three-dimensional (3D) analysis, which was proposed previously [4]. Kollár arrived at a similar formulation as (6) from thin shell theory in 1994.
The analytical prediction of spring-forward, equation (1), requires the membrane strain ε_θ. The coupling with the membrane strains is ignored here, but it can be included with an iterative scheme, for example.

2.4 Finite element formulation

The adapted CLT is applied in a FE formulation for discrete Kirchhoff triangle (DKT) plate elements. The FE system is derived from the principle of virtual work as:

$$[K]\begin{Bmatrix} u \\ \varphi \end{Bmatrix} = \{f\} \qquad (7)$$

where $\{u\}$ and $\{\varphi\}$ are the nodal displacements and rotations, respectively. The element stiffness matrix $[K]$ is elaborated as

$$[K] = \int_\Omega \begin{bmatrix} B^m & B^b \\ B^b & B^m \end{bmatrix}^T \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} B^m & B^b \\ B^b & B^m \end{bmatrix} d\Omega \qquad (8)$$

where $[B^m]$ and $[B^b]$ hold the first derivatives of the shape functions $[\Psi]$. The superscripts *m* and *b* denote membrane and bending, respectively, and Ω indicates the volume of the element. The force vector $\{f\}$ contains the nodal forces caused by tractions $\{t\}$ on the element boundary Γ and a temperature change ΔT:

$$\{f\} = \int_{\Gamma} [\Psi]\{t\} d\Gamma + \int_{\Omega} \begin{bmatrix} B^m & B^b \\ B^b & B^m \end{bmatrix}^T \begin{bmatrix} A & B \\ B & D \end{bmatrix} \left(\begin{Bmatrix} \alpha \\ \kappa^T \end{Bmatrix} + \begin{Bmatrix} 0 \\ \kappa^{sf} \end{Bmatrix} \right) d\Omega \quad (8)$$

3 IMPLEMENTATION

The spring-forward effect is included in the two-step modelling strategy that was proposed recently by Lamers [3,5]. The two steps consist of separate simulations of the draping and the succeeding thermoelastic cooling. The draping simulation, performed with efficient, multilayer membrane elements, provides the reoriented fibre directions and fibre stresses. Subsequently, the thermoelastic properties of the reoriented composite material are computed separately for each element, utilising a woven fabric micromechanics model. The CLT based stiffness matrix and thermal force and moment resultants are obtained and stored. The thermoelastic cooling step is executed with DKT elements, their integration point data overwritten with the separately calculated stiffnesses, forces and moments.

Only the module that computes the CLT based stiffnesses, forces and moments is adapted to include the spring-forward effect. Firstly, the through-thickness CTE α_r is calculated with a 3D woven fabric micromechanics model. Secondly, the principal curvatures and the principal directions are required.

The curvatures of the surface meshed with the current triangular elements is not directly known. Here, the surface is locally approximated by a quadratic function. The second derivatives with respect to the global coordinates x and y yield the curvatures κ_x, κ_y and κ_{xy}. The quadratic fit requires the relative position of the neighbouring elements, which is taken into account through the element normals. The resulting over-determined system of equations is solved by a least squares algorithm.

4 SIMULATIONS

The performance of the plate element is validated on a single curved strip. Subsequently, a wing leading edge stiffener rib is modelled.

4.1 Single curved arc section

A single curved strip was meshed with DKT elements. The 'draping' of such a shape does not result in fibre reorientation, hence the draping with membrane elements was omitted. The curved panel is shown in figure 3, meshed with different amounts of elements over the arc radius.

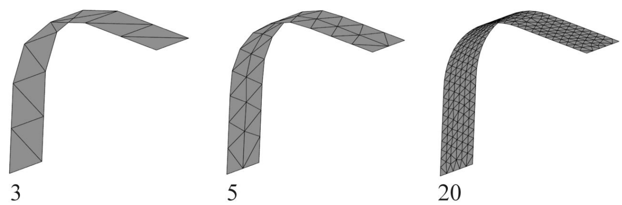

Fig. 2. L-shaped strip meshed with 3, 5 and 20 elements elements over the arc radius, respectively

The material represents 8H satin glass woven fabric reinforced polyphenylenesulphide, or glass/PPS, satcked in a crossply configuration. The material was oriented such that one yarn direction coincides with the axial direction. The other yarn direction follows the circumferential direction.

The CTE's of the glass/PPS composite are calculated as α_θ = 13.7·10^{-6} °C^{-1} and α_r = 39.1·10^{6} °C^{-1}. The spring-forward due to a temperature change of 1 °C is presented in table 1, simulated with the FE model and predicted with the analytical solution (2).

Table 1. Spring-forward of an L-shape caused by a ΔT of 1 °C

3 elements	5 elements	20 elements	analytical
2.39·10^{-3}	2.22·10^{-3}	2.24·10^{-3}	2.29·10^{-3}

The coarsest mesh, having three elements over the radius, deviates approximately 5% from the analytical solution, which is considered sufficiently accurate.

4.2 Stiffener of wing leading edge

Lamers [3] studied the effect of membrane stresses on the distortion of composite products. One of the parts considered was a stiffener of a wing leading edge, which is rubber press formed at Stork Fokker AESP. The simulation of the stiffener comprised

draping, trimming of scrap edges and thermoelastic cooling. The material consisted of 8H satin woven fabric glass/PPS, stacked according to a quasi-isotropic lay-up, or $(45°/90°)_s$.
It is assumed that the PPS matrix becomes elastic at 200 °C, somewhat below the crystallisation temperature (approximately 240 °C). It is assumed that no stress builds up above 200 °C. Figure 3 displays the stiffener with the vertical displacement indicated by the gradient fill. The deformations are induced by the thermally induced membrane stresses (ordinary CLT approach). Drape-induced fibre stresses are omitted.

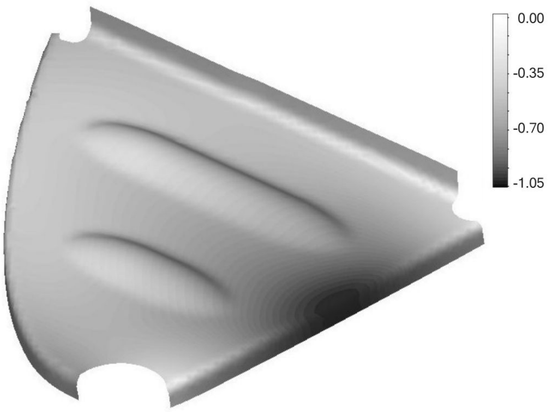

Fig. 3. Vertical displacements (mm) caused by thermal membrane stresses

Figure 4 shows the rib again. The deformations are solely induced by the spring-forward effect.

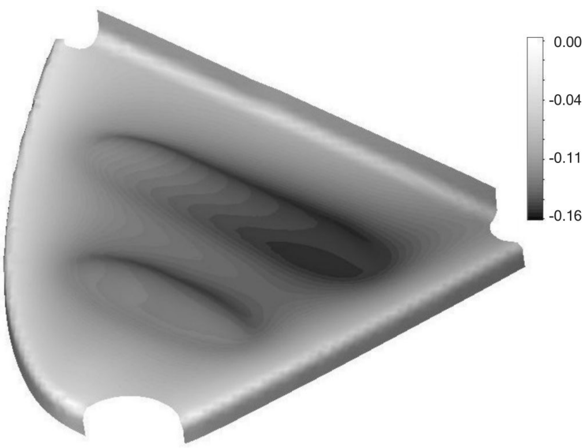

Fig. 4. Vertical displacements (mm) caused by spring-forward

Little warpage is observed due to the stiffness of the doubly curved geometry. The doubly curved flanges spring in, yet considerably less than the rear, standing edge. This edge shows more pronounced spring-forward, since it is only single curved. The deformations caused by the thermal membrane stresses (figure 3) are an order of magnitude larger than the deformations due to spring-forward.

5 DISCUSSION AND CONCLUSIONS

An analytical solution for spring-forward was implemented in a FE formulation for plates. The methodology serves as a first estimate for the linear thermoelastic spring-forward of doubly curved geometries. It is recognised that the coupling with the membrane strains is weak, a problem which can be solved by applying an iterative scheme.
A dominant interaction between the tools and glass/PPS composite panels during rubber pressing has been reported previously [7]. Unpublished experimental results indicate the same interaction in the case of spring-forward of glass/PPS parts. Future activities focus on including the observed tool-part interaction in an efficient numerical algorithm.

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