

# An Oversampled Filter Bank Multicarrier System for Cognitive Radio

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**Abstract**—Due to small sideband power leakage, filter bank multicarrier techniques are considered as interesting alternatives to traditional OFDMs for spectrum pooling Cognitive Radio. In this paper, we propose an oversampled filter bank multicarrier system for Cognitive Radio. The increased spacing between adjacent subcarriers in the oversampled filter bank multicarrier system largely reduce the intercarrier interference, the key limitation of the OFDM based Cognitive Radio. The proposed multicarrier system is compared with OFDM for BER performance and sideband power rejection. Design tradeoffs of the major parameters of the oversampled filter bank will be discussed. We also suggest a fast implementation of the proposed filter bank modulation based on generalized DFT filter bank model, followed by a computational complexity analysis.

## I. INTRODUCTION

Multimedia wireless applications have been increasing rapidly in recent years and this trend will continue in the future. The large demand for radio spectrum will make it no room to accommodate new wireless applications. However, recent studies have shown that most of the assigned radio spectrum is under-utilized. Cognitive Radio [1] is considered as a promising technology to address the paradox of spectrum scarcity and spectrum under-utilization. In Cognitive Radio, a spectrum sensing process locates the unused spectrum segments in a targeted spectrum pool. These segments will be used optimally without harmful interference to licensed users (users who have the legal license for the spectrum). This technology is called *spectrum pooling* [2]. In spectrum pooling, orthogonal frequency division multiplexing (OFDM) is used as the baseband transmission scheme. The cognition is realized by nullifying those subcarriers which cause interference to licensed users. The remaining frequency segments will be used optimally by Cognitive Radio. The additional benefit of OFDM is the reuse of the FFT module for spectrum sensing. However, due to the rectangular window in the time domain the OFDM system has large sidelobes which cause interference to adjacent bands. This fact has also been recognized in [2]. They proposed two methods to mitigate the interference to the licensed user: deactivating more subcarriers adjacent to the licensed system or applying non-rectangular windows to reduce the spectrum leakage. Both methods mitigate the interference at the cost of bandwidth efficiency. Moreover, the two methods didn't consider the system implementation issues. Therefore, the indication is that other multicarrier schemes could be interesting candidates for Cognitive Radio. This fact

has also been observed in a recent publication [3]. A filter bank modulation called filtered multitone (FMT) [4] has been applied to very high-speed digital subscriber line technology to achieve high-level spectral containment in subchannels. This is also the key characteristic expected for a spectrum pooling system, where sideband power leakage should be kept to a minimum. In this paper, we propose an oversampled filter bank multicarrier system for Cognitive Radio based on the idea of FMT.

The paper is organized as follows: Section II proposes the multicarrier system for Cognitive Radio based on the oversampled filter bank. In section III, we present an efficient implementation of the proposed system based on the generalized DFT filter bank model. The simulation results of the proposed system are shown in section IV and followed by some discussions. We will mention several interesting points for future work in section V. Finally, we draw some conclusions in section VI.

## II. CR BASED ON OVERSAMPLED FILTER BANK MULTICARRIER

The basic idea of multicarrier transmission is to divide a broadband channel into parallel subchannels and the high-rate data stream is split into low-rate streams and transmitted on each subchannel. This transmission scheme can be modelled as a filter bank system [5], shown in figure 1. At the transmitter,  $M$  complex symbols are upsampled by a factor of  $N$  and filtered by a base band *prototype* filter. The output of each  $M$  symbol stream will be properly shifted in frequency and added for transmission. The receiver demodulates the signal by a matched filter and downsampling by a factor of  $N$ . The transmitter and the receiver are in fact  $M$  band synthesis bank and analysis filter. When critical sampling applies  $N = M$  and the prototype filter is selected as a *sinc* shaped filter in frequency, the multicarrier system becomes an OFDM system. When  $N > M$ , the filter bank system is called an *oversampled filter bank* (OSFB). The oversampling will increase intercarrier spacing by a factor of  $\frac{N}{M}$  ( $N > M$ ), see figure 2. In such a way intercarrier interference (see the overlapping part between two subcarriers) is largely reduced, which is the basic idea of FMT [4]. One would argue the increased intercarrier spacing results in less subcarriers in a given bandwidth, thus losing the bandwidth efficiency. However, compared with OFDM systems where extra cyclic prefix always has to be introduced,

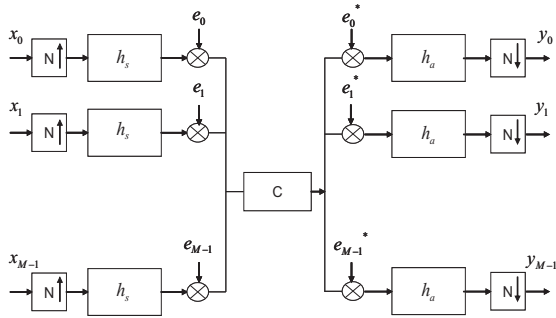


Fig. 1. A multicarrier system based on filter banks

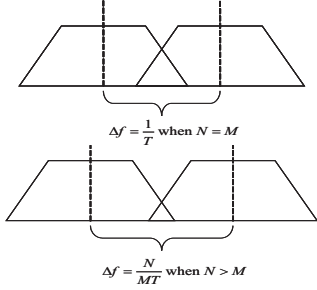


Fig. 2. Intercarrier spacing of the critically sampled and oversampled filter bank, where  $\frac{1}{T}$  denotes the symbol rate

OSFB is no worse than OFDM in terms of bandwidth efficiency. Therefore, we think the OSFB is a good candidate for the multicarrier based Cognitive Radio.

The basic idea of multicarrier based Cognitive Radio is to deactivate the subcarriers causing interference to licensed users and optimally use the remaining part of the targeted spectrum. The deactivation can be realized by loading zeros on the intended subcarriers while others are loaded with modulated complex symbols at the transmitter which is an  $M$  band oversampled synthesis filter bank. An  $M$  band oversampled analysis filter bank on the receiver reconstructs the signal and send only the symbols from those active subcarriers for demodulation. The deactivation information is sent to both the transmitter and the receiver through a control channel. The simplified OSFB based multicarrier Cognitive Radio system is shown in figure 3. The modulation mode for the active subcarriers is adaptive to the channel's SNR. The adaption can be done for all subcarriers as a whole or for each individual subcarrier based on subchannel's SNR. The adaptive bit loading for OFDM based Cognitive Radio in [6] can be applied to the latter case.

### III. EFFICIENT IMPLEMENTATION BASED ON GENERALIZED DFT FILTER BANK

The implementation of the OSFB is not as straightforward as the critically sampled filter bank. The authors in [4] indicate to implement periodically time-varying filters in the OSFB. However, this is difficult in practice. Therefore, we suggest an efficient implementation based on the generalized DFT filter bank (GDFT) model in [7].

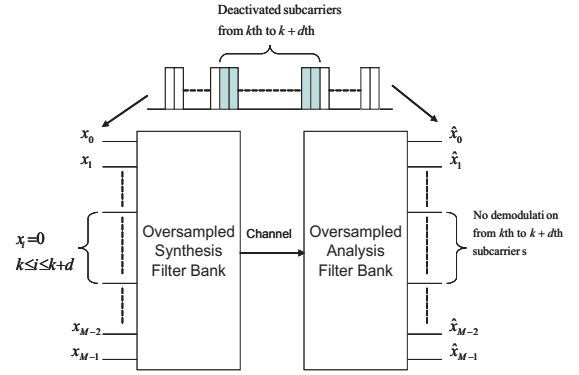


Fig. 3. An OSFB multicarrier system for Cognitive Radio

The transmitted analog signal  $s_a$  can be expressed as:

$$s_a(t) = \sum_{m=0}^{M-1} \sum_{n=-\infty}^{+\infty} x_m(nT)h_{s,m,a}(t-nT) \quad (1)$$

, where  $T$  denotes the symbol duration,  $x_m(nT)$  is the symbol on  $m$ th subcarrier at  $n$ th instance,  $h_{s,m,a}(t)$  is the analog synthesis prototype filter on the  $m$ th subband. Since the symbol rate is  $\frac{1}{T}$ , the sampling rate should be  $\frac{N}{T}$ . For each sampling instance  $k$ , the discrete signal  $s(k)$  can be written as:

$$s(k) = \sum_{m=0}^{M-1} \sum_{n=0}^{+\infty} x_m(n)h_{s,m}(k-nN) = \sum_{n=0}^{+\infty} s_n(k-nN) \quad (2)$$

, where  $x_m(n) = x_m(nT)$ ,  $h_{s,m}(k)$  denotes the digitized synthesis filter and only symbol instances for  $n \geq 0$  are considered. We define signal  $s_n(k)$  at each instance  $n$  as:

$$s_n(k) = \sum_{m=0}^{M-1} x_m(n)h_{s,m}(k) \quad (3)$$

The subband filter  $h_{s,m}(k)$  is derived from a real valued prototype filter  $p(k)$  by modulation as:

$$h_{s,m}(k) = p(k - \frac{L-1}{2})e^{j2\pi(m - \frac{M-1}{2})(k - \frac{L-1}{2})/M} \quad (4)$$

, where  $L$  is the filter length,  $\frac{M-1}{2}$  is set as carrier frequency and the delay of  $\frac{L-1}{2}$  is introduced to make a causal system. From eq. 3, we can see  $s_n(k)$  is the summation the multiplications of  $M$  band symbols with  $L$  filter coefficients. Thus the length of  $s_n(k)$  is  $L$ . We can write eq. 3 in a matrix form as:

$$s_n = H_s^T x_n \quad (5)$$

, where  $x_n$  is the symbol vector and  $H_s$  is an  $M \times L$  matrix:

$$H_s = [p(k - \frac{L-1}{2})e^{j2\pi(m - \frac{M-1}{2})(k - \frac{L-1}{2})/M}]_{M \times L} \quad (6)$$

A matrix multiplication of  $H_s^T$  and  $x_n$  can be done to produce signal  $s_n$ , however it costs  $M \times L$  complex multiplications. To reduce the computational complexity, we reconstruct  $H_s$  from

the  $M \times M$  generalized DFT matrix  $T$  [7] and a diagonal matrix  $\Lambda_p$  where the diagonal holds  $L$  coefficients as:

$$H_s = T \times [I_M \ (-1)^{M-1} I_M] \times [I_{2M} \ I_{2M} \dots \ I_{2M} \ \hat{I}_{2M,u}] \times \Lambda_p \quad (7)$$

The generalized DFT matrix  $T$  is expressed as:

$$T = \Lambda_1 W_M^* \Lambda_2 \quad (8)$$

, where  $W_M$  denotes an  $M$  point DFT matrix.  $\Lambda_1$  and  $\Lambda_2$  are diagonal matrices where the  $i$ th diagonal elements for  $\Lambda_1$  and  $\Lambda_2$  are  $e^{-j\pi(i-\frac{M-1}{2})(L-1)/M}$  and  $e^{-j\pi i(M-1)/M}$  respectively. In eq. 7,  $I_M$  and  $I_{2M}$  denote the  $M \times M$  and  $2M \times 2M$  identity matrices respectively and  $\hat{I}_{2M,u}$  is the first  $u$  column submatrix of  $I_{2M}$ , where  $u$  is  $L$  modulo  $2M$ . From eq. 5, eq. 7 and eq. 8, we have:

$$s_n = \Lambda_p \times [I_{2M} \ I_{2M} \dots \ I_{2M} \ \hat{I}_{2M,u}]^T \times [I_M \ (-1)^{M-1} I_M]^T \times \Lambda_2 W_M^* \Lambda_1 x_n \quad (9)$$

Similarly at the receiver, the recovered symbol  $\hat{x}_n$  can be written in matrix form:

$$\hat{x}_n = H_a r_n^T \quad (10)$$

, where  $\hat{x}_n$  denotes the symbols from  $M$  bands and  $r_n$  is the received signal with length  $L$ . In order to satisfy the perfect reconstruction condition, the analysis filter matrix  $H_a = H_s^*$  [5]. The recovered symbol  $\hat{x}_n$  can be expressed as:

$$\hat{x}_n = \Lambda_1^* W_M \Lambda_2^* \times [I_M \ (-1)^{M-1} I_M] [I_{2M} \ I_{2M} \dots \ I_{2M} \ \hat{I}_{2M,u}] \times \Lambda_p r_n \quad (11)$$

The generalized DFT implementation is based on eq. 9 and eq. 11, where the filter coefficient matrices consist of periodically varying GDFT matrices. Unlike the implementation in [4] where the coefficients are time varying, we can incorporate the periodicity into filter inputs. Figure 4 and 5 show the implementations of the GDFT filter bank transmitter and receiver respectively.

At the transmitter,  $M$  symbols are first transformed by  $T$ , which can be implemented as  $2M$  phase shifts of complex number and  $M$  point IFFT (here we consider that  $M$  is a power-of-two integer) based on eq. 8. The  $M$  transformed symbols  $X_i (i = 0, 1, \dots, M-1)$  are used to make sequence  $\bar{X}_{2M} = [X_{i=0,1,\dots,M-1} \ -X_{i=0,1,\dots,M-1}]$ . By repeating sequence  $\bar{X}_{2M} \lfloor L/M \rfloor$  ( $\lfloor \cdot \rfloor$  denotes integer division) times and appending the first  $L_{mod2M}$  elements in  $\bar{X}_{2M}$  at the end, an  $L$ -element sequence is produced to be multiplied with  $L$  filter coefficients. The multiplication results are accumulated to a length  $L$  shift register  $D$  which is set to be zeros at the initialization. After the accumulation, the first  $N$  samples in  $D$  are shifted out as transmitted symbols and all other samples are shifted  $N$  positions ahead with  $N$  zeros shifted in.

At the receiver,  $L$  received symbols in a shift register are multiplied with  $L$  filter coefficients. The  $i_{mod2M} (i = 0, 1, \dots, L-1)$  multiplication results are combined to form a  $2M$  sequence  $R$ . Then the second half of  $R$  is negated

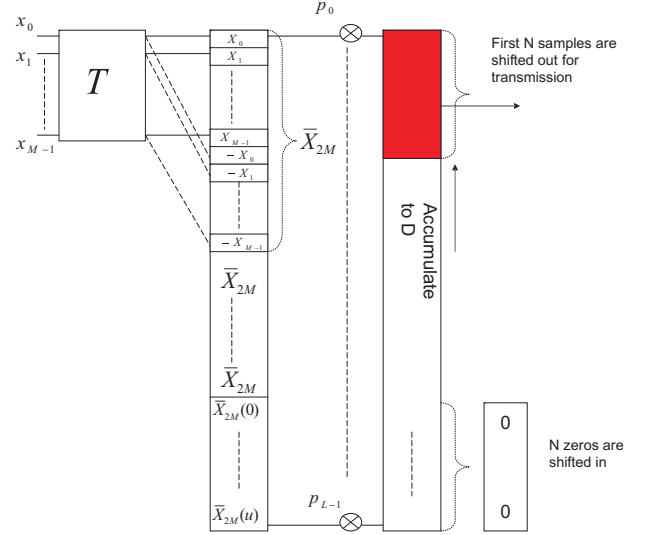


Fig. 4. The GDFT filter bank transmitter implementation

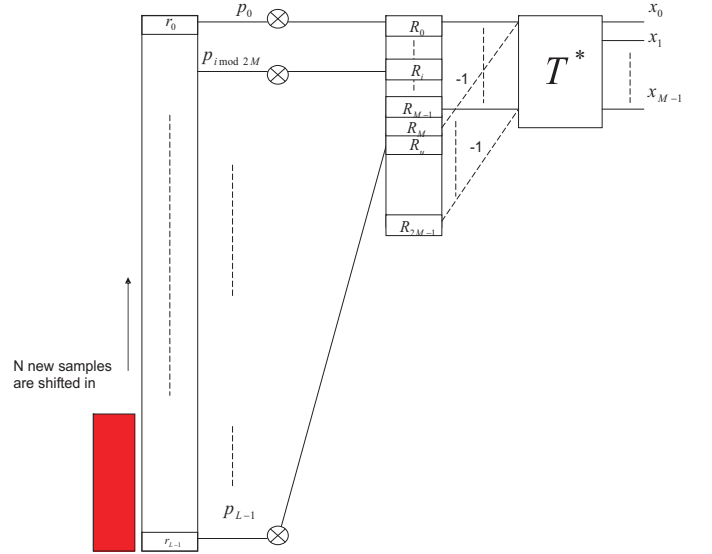


Fig. 5. The GDFT filter bank receiver implementation

and combined with the first half to produce  $M$  symbols to be transformed by  $T^*$  which is the conjugate of  $T$ . After the transform,  $M$  recovered symbols are obtained and  $N$  new symbols will be shifted in.

Based on the implementation, we made an computational complexity analysis by counting the number of complex multiplications. To transmit and receive each  $M$  symbols, we need  $2L$  complex multiplications with filter coefficients,  $4M$  for the phase shift, an  $M$  point IFFT and an  $M$  point FFT. The total computational complexity of the OSFB  $C_{OSFB}$  can be expressed as:

$$C_{OSFB} = 2L + 4M + 2 \times \left( \frac{M}{2} \log_2 M \right) \quad (12)$$

The computational complexity of the OFDM  $C_{OFDM}$  is:

$$C_{OFDM} = 2 \times \left( \frac{M}{2} \log_2 M \right) \quad (13)$$

From eq. 12 and eq. 13, the OSFB is more computational complex than the OFDM due to the extra filtering. Especially if the length of prototype filter  $L$  is large, the computational complexity increases enormously. This complexity raises a question on how to make efficient hardware implementations. The regular computational structures in figure 4 and figure 5 are good indications for parallel processing.

#### IV. RESULTS AND DISCUSSIONS

##### A. Simulation setup

In the simulation, we consider a CR with 32 subcarriers where 8 are deactivated to avoid the interference to licensed users. Both the OFDM based and the OSFB based multicarrier systems are considered. To simplify the evaluation of the system performance, the channel is assumed to be Additive White Gaussian Noise (AWGN) channel with no distortion for both systems. The modulation scheme is considered to be the same for every subcarrier. In the OSFB, the oversampling ratio is chosen as  $N = 36$ . The square root raised cosine filter with rolloff factor 0.15 is assumed to be the prototype filter. The Matlab function *rcosine* is used to generate the required prototype filter. We set the parameter group delay  $K$  as 9, thus the filter length  $L = 2KN + 1 = 649$ .

##### B. Comparison with OFDM

Figure 6 shows the BER performance comparison of the OFDM and OSFB for the given scenario. For fair comparison, we assume no interference to both systems from licensed users. QPSK and 16QAM modulation schemes are considered. The BER performance of the OSFB is a little better than the OFDM due to its less intercarrier interference. Figure 7 shows the transmitted spectrum for both systems. We can see the sideband power rejection of the OSFB is much better than the OFDM. The noise floor of the nullified spectrum is below -30dB for OSFB, however it is much higher in case of OFDM therefore causing significant interference to licensed users. Even if more adjacent subcarriers are turned off in the OFDM as indicated in [2], the interference to licensed bands is still high due to big sidelobes. Therefore, the traditional OFDM system without any modification is difficult to be applied in spectrum pooling system. The alternatives such as the OSFB are more promising in the context of spectrum pooling.

##### C. The design tradeoffs in the OSFB

There are a number of tradeoffs in designing an OSFB multicarrier system such as choices of different prototype filters, the length of filters and oversampling ratio. These tradeoffs have impacts on BER performance, sideband power rejection, bandwidth efficiency and computational complexity. Here we discuss some tradeoffs based on our system in the simulation.

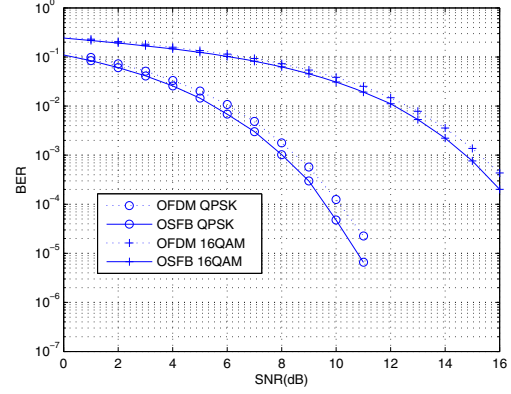


Fig. 6. BER performance on AWGN

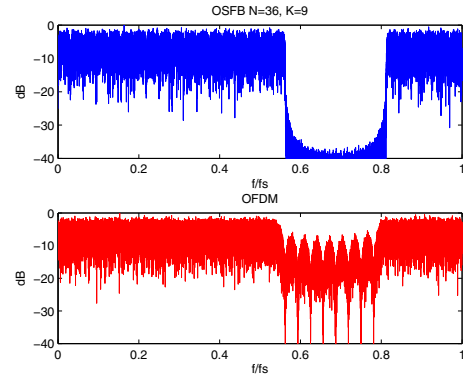


Fig. 7. Transmitted spectrum with null subcarriers

*The choice of group delay  $K$ :* In figure 8 and figure 9, we show the BER performance and the transmitted spectrum of the OSFB by increasing the group delay  $K$  of the prototype filter from 4 to 9 while keeping the rolloff factor and the oversampling ratio unchanged. QPSK modulation is used and the AWGN channel is assumed. Increasing  $K$  from 4 to 9 will increase the prototype filter length  $L$  from 289 to 649. We can see an improvement in the BER performance and especially the sideband rejection by increasing  $K$ . This is largely due to the factor that the high order prototype filter gives better frequency response and sideband rejection. However, the computational complexity increases by 90% according to eq. 12. Therefore, the length of the prototype filter is an important tradeoff between the system performance and the computational complexity.

*The choice of oversampling ratio  $N$ :* Intuitively, increasing the oversampling ratio will result in larger intercarrier spacing. The increased intercarrier spacing tends to reduce the power leakage to adjacent subcarriers, therefore better BER performance and less interference to licensed bands are expected. If we consider only one side of the spectrum leakage power  $P_{leak}$  generated by one subcarrier, it can be defined as:

$$P_{leak} = \int_{\frac{\Delta f}{2}}^{+\infty} \Phi(f) df \quad (14)$$

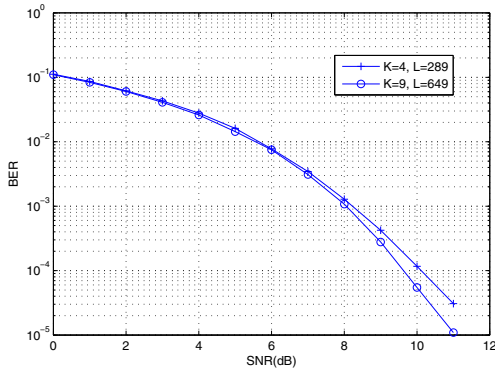


Fig. 8. BER performance of the OSFB with different  $K$  and QPSK modulation in AWGN

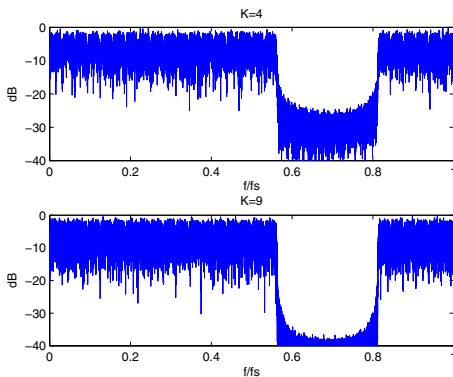


Fig. 9. Transmitted spectrum of the OSFB with different  $K$

, where  $\Phi(f)$  denotes the power density spectrum of subcarrier and  $\Delta f$  is the subcarrier spacing. If an ideal raised cosine filter is used as prototype filter in the OSFB, eq. 14 can be expressed as:

$$P_{leak} = \frac{1}{T} \int_{\frac{N}{2MT}}^{\frac{1}{T}} |H_{rc}(f)|^2 df \quad (15)$$

, where  $T$  is the symbol duration and  $H_{rc}(f)$  is frequency response of raised cosine filter. Ideally from eq. 15, the power leakage is reduced to zero when the sampling ratio  $N > 2M$  regardless of the choice of rolloff factor. However, in practice the ideal raised cosine filter can never be realized therefore the spectrum power leakage will always exist. Nevertheless eq. 15 can still serve as rough guidance to make tradeoffs between the oversampling ratio and the sideband power leakage in the OSFB based on the raised cosine prototype filter. However, increasing oversampling ratio obviously loses the bandwidth efficiency. Furthermore, the increased oversampling ratio will also result in an increase of computational complexity.

*The type of prototype filter:* In our discussion, only the raised cosine filter is considered. However, there is freedom for prototype filter design if the reconstruction error is on an acceptable level. Much research has been done for the prototype filter design for filter bank systems. For example in [8], a small side-lobe prototype filter may be an interesting

option for filter bank multicarrier for Cognitive Radio.

## V. FUTURE WORK

Although the OSFB is a promising option in the context of Cognitive Radio, there are still a number of challenges for the practical system. Compared with OFDM, filter bank based multicarrier is more subject to distortion and delay from channel. Therefore, channel equalization in filter bank based multicarrier systems is an important issue to be considered. The other drawback of filter bank based multicarrier systems is the high computational complexity. Our future work will also focus on how to make efficient hardware implementations for parallel processing. Besides, other filter bank based multicarrier systems for Cognitive Radio are also worth studying such as cosine modulated filter bank in [9] and wavelet multicarrier based on nonuniform filter bank in [10].

## VI. CONCLUSION

In this paper, we proposed an oversampled filter bank multicarrier for Cognitive Radio. It can achieve better BER performance and less sideband power leakage compared with OFDM. Therefore, the OSFB is a good candidate for multicarrier based Cognitive Radio. We suggested an efficient implementation based on the generalized DFT model followed by a computational complexity analysis. This implementation can be further exploited for efficient parallel processing. We also discussed some design tradeoffs for the proposed OSFB multicarrier system.

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