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Model order reduction of non-linear flexible multibody models

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In high precision equipment the use of compliant mechanisms is favourable as elastic joints offer the advantages of no friction and no backlash. For the conceptual design of such mechanisms there is no need for very detailed and complex models that are time-consuming to analyse. Nevertheless the models should capture the dominant system behaviour which must include relevant three-dimensional motion and geometric non-linearities, in particular when the system undergoes large def ections.

In [1] we discuss a modelling approach for this purpose where an entire multibody system is modelled as the assembly of non-linear f nite elements. The elements' nodal coordinates and so-called deformation mode coordinates are expressed as functions of the independent (or generalised) coordinates $\boldsymbol{q}$. With these expressions the system's equations of motion are derived as a set of second order ordinary differential equations in terms of the kinematic degrees of freedom $\boldsymbol{q}$, see e.g. [2] and the references therein:

$$\bar{\boldsymbol{M}}(\boldsymbol{q})\ddot{\boldsymbol{q}} = \mathrm{D}_q\boldsymbol{\mathcal{F}}^{(x)T}\left(\boldsymbol{f} - \boldsymbol{M}\mathrm{D}_q^2\boldsymbol{\mathcal{F}}^{(x)}\dot{\boldsymbol{q}}\dot{\boldsymbol{q}}\right) - \mathrm{D}_q\boldsymbol{\mathcal{F}}^{(e)T}\boldsymbol{\sigma}, \tag{1}$$

where $\bar{\boldsymbol{M}}$ is the system mass matrix computed from the global mass matrix $\boldsymbol{M}$. The notations $\mathrm{D}_q\boldsymbol{\mathcal{F}}$ and $\mathrm{D}_q^2\boldsymbol{\mathcal{F}}$ denote so-called f rst and second order geometric transfer functions. The vector $\boldsymbol{f}$ are the nodal forces. Generalised stress resultants $\boldsymbol{\sigma}$ represent the loading state of each element. The sound inclusion of the non-linear effects at the element level appears to be very advantageous [2]. Only a rather small number of elastic beam elements is needed to model e.g. wire f exures and leaf springs accurately. Still it appeared that for a more complex compliant mechanism a rather large number of degrees of freedom is needed for an accurate model in the relevant frequency range [1].

Model order reduction techniques have been studied by several authors as these techniques offer a method to reduce the number of degrees of freedom while an accurate description of the dominant dynamic behaviour may be preserved. In the present paper we propose to describe the vibrational motion as a perturbation of a nominal rigid link motion. For order reduction a modal reduction technique is applied by expressing the perturbations of the degrees of freedom $\delta\boldsymbol{q}$ as

$$\delta\boldsymbol{q} = \boldsymbol{V}\boldsymbol{\eta}, \tag{2}$$

where the elements of the vector $\boldsymbol{\eta}$ are the so-called principal coordinates and $\boldsymbol{V}$ is the modal matrix which is in general conf guration dependent. Applying modal reduction the number of principal coordinates $\boldsymbol{\eta}$ is reduced representing only a rather small number of low frequency modes. Although the non-linear equations (1) still need to be integrated, we expect a gain in computational eff ciency as large time steps can be applied in the absence of high frequent dynamic behaviour.

Consider the two-link f exible manipulator shown in Figure 1. This manipulator has been introduced as a benchmark by Schiehlen and Leister [3] and has been quoted in several papers. Some properties are given in the table next to the f gure. Joint angles $\phi_1(t)$ and $\phi_2(t)$ are prescribed with third order functions of time t moving from the initial to the f nal conf guration in 0.5 s. Different from the original benchmark, we don't include gravity in this paper.

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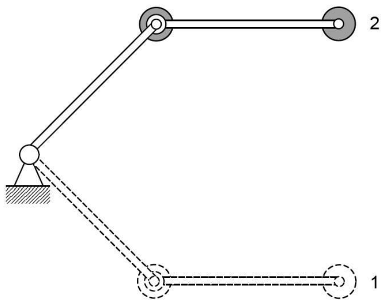

Property	Node 1	Node 2
Joint mass m	1.0 kg	3.0 kg

Property	Link 1	Link 2
Length l	0.545 m	0.675 m
Density ρ	2700 kg/m^3	
Young's modulus E	$7.3 \cdot 10^{10}$ N/m^2	
Cross-sectional area A	$9.0 \cdot 10^{-4}$ m^2	$4.0 \cdot 10^{-4}$ m^2
Cross-sectional area moment of inertia I	$1.69 \cdot 10^{-8}$ m^4	$3.33 \cdot 10^{-9}$ m^4

Figure 1: Planar two-link manipulator: Initial configuration (1) and final configuration (2) with some of the parameters (adapted from [3]).

The motion of this manipulator has been computed with a non-linear model in which three flexible beam elements are used for each link. Each beam allows two bending modes yielding twelve dynamic degrees of freedom in total. After the joint angles have reached their final values, a vibration of the elastic links is observed that is dominated by the lowest natural frequency of approximately 3 Hz.

Next this simulation has been repeated with only a small number of time invariant modes that are computed with a modal analysis in the initial manipulator configuration. The results in Fig. 2(a) show that with even only one mode the large scale motion at the tip is already described well. The detailed view near the upper extreme position reveals differences between the full order and reduced order simulations. Including the time invariant second mode improves the accuracy and only a negligible error remains. The computation time is reduced as no high frequency modes are present.

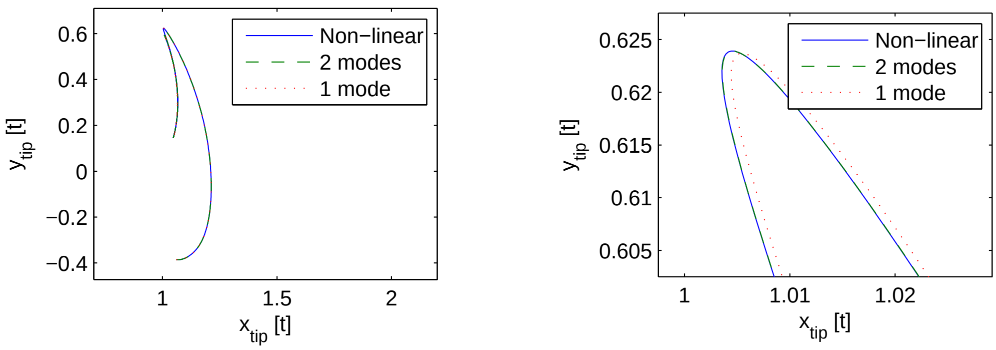

Figure 2: Motion of the manipulator tip during 0.7 s: Full view (left) and detailed view (right) near the upper extreme position.

The example illustrates the possibilities offered by the proposed order reduction according to Eq. (2) combined with the solution of the non-linear equation of motion (1). It should be noted that in this example the mode shapes do not vary much along the prescribed trajectory and the joint angles are prescribed. The application of the method to systems with controlled actuated joint angles and more significantly varying configurations is currently work in progress.

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