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EFFICIENT ANALYSES FOR THE MECHATRONIC DESIGN OF MECHANISMS WITH FLEXIBLE JOINTS UNDERGOING LARGE DEFORMATIONS

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Abstract. *The models used in the conceptual phase of the mechatronic design should on the one hand capture the dominant system behaviour while on the other hand complicated models should be avoided as these are too laborious to build and the analysis is too time-consuming. One or two dimensional lumped parameter models can be used for their simplicity. However, even for rather simple mechanical systems such models may ignore relevant three-dimensional or geometric non-linearities. Models obtained with standard linear finite element models often need many elements to achieve sufficient accuracy.*

In this paper analyses with a multibody approach based on non-linear finite elements are discussed. Due to the sound inclusion of the non-linear effects at the element level only a rather small number of elastic beam elements are needed to model typical components accurately. Configuration dependent linearised models can be generated for control system design. This approach is offered by the SPACAR *software package and its applicability is demonstrated with the analysis of a mechatronic device. The effects of e.g. non-linearities, non-co-located control in three dimensions and relevant natural frequencies are analysed easily and quickly with the low-dimensional and accurate models offered by the modelling approach.*

1 INTRODUCTION

The use of flexible joints in mechatronic designs of precision equipment is favourable as these joints offer the advantages of low friction and no backlash. Such mechanisms can even be used to realise large motions, as the joints can be designed to undergo relatively large elastic deformations. Then the analysis of such systems becomes more complicated as the dynamic properties like natural frequencies and buckling performance may vary considerably in between the extreme positions. In the conceptual phase of the mechatronic design process the designer would like to use models that on the one hand capture the dominant, possibly varying system behaviour while on the other hand complicated models should be avoided as these are too laborious to build and the analysis is too time-consuming. Lumped parameter models are relatively simple and can capture some of the dominant one or two dimensional behaviour, but three dimensional effects are not fully taken into account. Linear finite element analyses are better suited to reveal this more complicated behaviour, yet at the expense of a rather large number of degrees of freedom. Furthermore, although the analysis near the unloaded equilibrium position may be straightforward, non-linear effects have to be accounted for to analyse or simulate the system's behaviour accurately in the full operating range. Common linear finite element analyses often do not account correctly for the complete non-linear behaviour and/or are very time consuming. The need for accurate models of low complexity is e.g. recognised by Schiavo *et al.* [1].

In this paper we discuss the use of the SPACAR software, in particular applied for mechatronic design. It offers a multibody approach based on non-linear finite elements of which details have been published before, e.g. [2], and some recent achievements are submitted to this conference [3]. The sound inclusion of the non-linear effects at the element level appears to be very advantageous. Only a rather small number of elastic beam elements are needed to model e.g. a leaf spring accurately. The non-linear model can be linearised in a number of configurations throughout the complete operational range of the mechanism to obtain a series of locally linearised models, e.g. state space models for control system design of the linear time-varying system [4]. Numerically efficient models are obtained as the number of degrees of freedom is rather small. Consequently, the approach is particularly well suited during the early (mechatronic) design phase, where time consuming computations would severely hamper the design progress. The software has a user-friendly graphical user interface with which the models can be created easily [5]. For further processing and e.g. control system design the modelling results can be easily imported in MATLAB/SIMULINK.

The software is well capable of modelling complex system. In this paper the applicability will be illustrated considering a simple example with design goals as are outlined in the next section. Numerical results are presented in Sect. 3 and finally conclusions are drawn.

2 EXAMPLE SYSTEM

Application of the multibody system approach is illustrated with a model of the support mechanism with elastic leaf springs as shown in Fig. 1. It is supposed to have only one intentional degree of freedom (1-DOF) being the horizontal position of the bar on the top. The cylinder on the right represents a voice coil motor (VCM) that drives this horizontal motion of the system. The support consisting of two leaf springs should confine all other degrees of freedom. Both springs are fixed at the bottom (clamped support). Initially, the base is considered to be completely rigid, although elasticity of the support will be considered as well. One spring is cut partly to avoid an overconstrained design [6]. It should be noted that better solutions exist

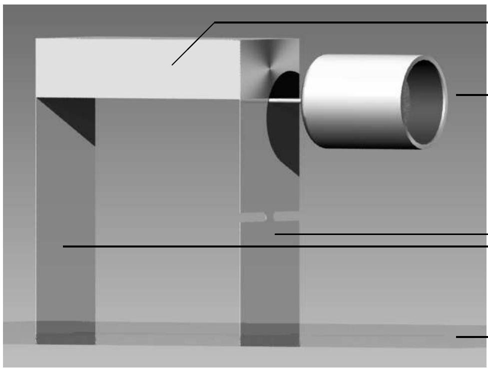


Figure 1: Example system with elastic leaf springs and driven by a voice coil motor.

to avoid an overconstrained system, but the presented system is discussed in this paper for its simplicity while it clearly demonstrates several design pitfalls.

For the mechatronic system to be designed it is assumed that the horizontal position x of the solid bar has to track a (low-frequent) reference position and the electric voltage U supplied to the VCM is computed by the control system. A straightforward one-dimensional analysis of the electromechanical system will reveal the nominal plant model is described by the transfer function

$$P_0(s) = \frac{x(s)}{U(s)} = \frac{\frac{k_m}{Rm}}{s^2 + \frac{k_m^2}{Rm}\, s + \frac{k}{m}}, \tag{1}$$

where the s denotes an analysis in the Laplace domain, m is the total (equivalent) moving mass, k the (equivalent) stiffness of the leaf springs, k_m the motor constant of the VCM and R the electrical resistance of the actuator coil. The system will be controlled with a PD-like controller that is tuned to obtain an open-loop cross-over frequency of 50 Hz. In the very simple one-dimensional 1-DOF model (1) neither high-frequent vibration modes of the system nor any three-dimension effects are taken into account. Both effects may significantly affect the system's performance and closed-loop stability.

A three-dimensional SPACAR model can be created very easily using only a few number of elements. Just a single flexible beam is needed to model each of the leaf springs. More elements can be used to obtain more accurate results, e.g. three beams can be combined to obtain a better model of the partly cut leaf spring. From a mechatronic point of view, the design goal is to obtain a lowest natural frequency that is well *below* the closed-loop bandwidth, or equivalently the open-loop cross-over frequency. In this way, the moving bar approximates a free moving mass and the best low frequent performance is obtained. On the other hand, the higher natural frequencies should be sufficiently far *above* the closed-loop bandwidth to avoid excitation of these modes, motion of the bar in unwanted directions, or even an unstable closed-loop system. Hence, an important property of the system is the ratio between the second and smallest natural frequencies, which should be sufficiently large.

SPACAR is well suited for this analysis, which is straightforward for a system near the unloaded equilibrium position, i.e. undeformed leaf springs. However, although the system is very simple at first sight, non-linear effects play a role as soon as large deformations of the leaf springs are considered. E.g. the stiffness in vertical direction and the buckling performance depend on the configuration. This will be addressed in the next section, in which the need to make a partial cut in one of the leaf springs is analysed first.

3 NUMERICAL RESULTS

3.1 Design considerations

The cut in one of the leaf springs is present to avoid an overconstrained design [6]. Without this cut, the solid bar will move as expected as long as both leaf springs are perfectly parallel. A small imperfection, e.g. during manufacturing, will result in a different motion as the horizontal motion of the bar may no longer involve pure bending of both leafs. This behaviour can be simulated by imposing a rotation around some axis at the base of one of the leaf spring. The system's natural frequencies can be computed as functions of the rotation angle. For the result in Fig. 2 two flexible beam elements are used for each leaf spring and the complete analysis takes only a few seconds. The figure shows that for a system with aligned leaf springs the ratio between the two lowest natural frequencies is about 25. This ratio drops significantly when the leaf springs are not perfectly parallel. Note that in this particular example the manufacturing tolerances will be sufficient to avoid this undesired behaviour. However, by investigating rotations around other axes as well as horizontal translations of the base, it can be motivated that erroneous behaviour can occur in an overconstrained system even due to a small misalignment.

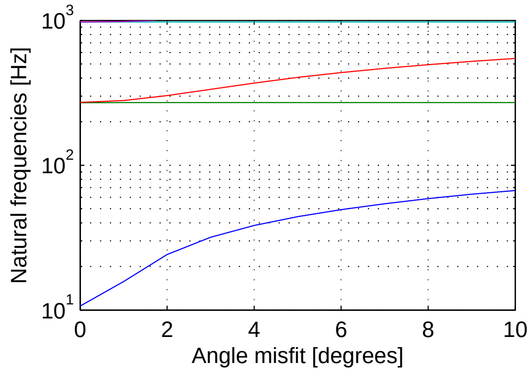


Figure 2: Natural frequencies of a system with two identical leaf springs with a manufacturing imperfection. The angle misfit is applied as a rotation around a vertical axis at the bottom of one of the leaf springs.

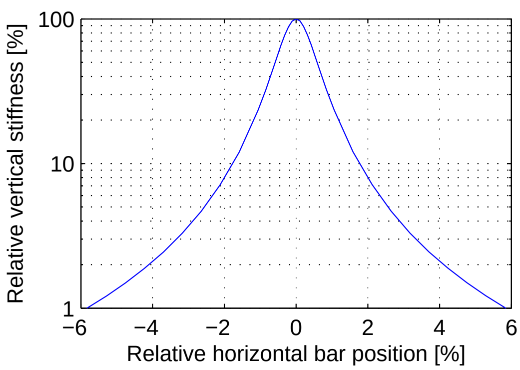


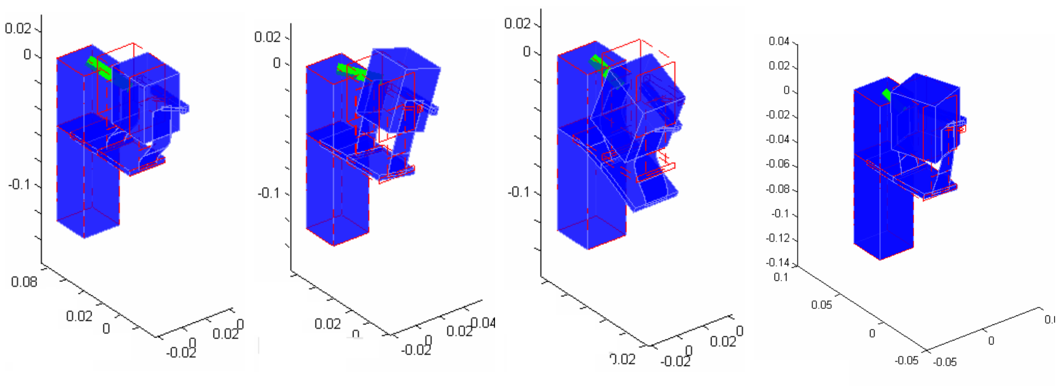
Figure 3: Vertical stiffness depending on the horizontal position of the bar. The vertical stiffness is relative to the maximum stiffness; the horizontal position is relative to the length of the leaf springs.

3.2 Non-linear behaviour

The non-linear behaviour of the mechanism is demonstrated by considering the vertical stiffness of the mechanism. This stiffness is defined as the ratio between an applied vertical force and the resulting deformation, where the force is applied on one side of the vertical bar just above the leaf spring. In the equilibrium position this stiffness shows a maximum, but it decreases rapidly when the solid bar moves horizontally. Figure 3 clearly illustrates this non-linear behaviour. Obviously, such dramatic change of the stiffness may have serious consequences for the usability of the considered design as it may influence the load carrying capabilities or the higher natural frequencies.

3.3 Mode shapes ideal system

The natural frequencies as mentioned in Sect. 3.1 will now be studied in more detail. For that purpose the elasticity of the support base is also modelled and this base is attached to a rigid support column. Figure 4 shows mode shapes associated with the lowest four natural frequencies. The lowest natural frequency and its mode match perfectly with the analysis of the simple lumped 1-DOF system. The higher natural frequencies differ from the results in Sect. 3.1 as the elastic behaviour of the base is modelled different.



1^{st} mode (11 Hz)	2^{nd} mode (119 Hz)	3^{rd} mode (207 Hz)	4^{th} mode (637 Hz)

Figure 4: Mode shapes of the system taking elasticity of the support into accont.

The second natural frequency appears to be a mode in which the solid bar moves perpendicular with respect to the driving direction. If the VCM actuates the system in the direction of the centre of mass, this mode is not excited by the VCM. Furthermore, if the sensor is mounted co-located and measures in the same direction, it will not measure this motion. This means that this natural frequency will not appear in the frequency response function of the system. A similar conclusion is obtained for the third frequency. Then the fourth natural frequency at 673 Hz is the first natural frequency that may affect the system performance. As this frequency is far above the cross-over frequency of 50 Hz, it is not expected to cause deterioration of the system performance.

This assumption can be verified by considering robust stability of the system. For robust stability the high-frequent Bode magnitude plot of the system $P(s)$ should remain below the inverse of the controller $C^{-1}(s)$. In Fig. 5 this criterium can be checked in the Bode magnitude plot where $P_1(s)$ is the transfer function for the system described above. Clearly it is robustly stable. Note that an appropriate level of internal damping of the applied material is taken into account (relative damping 0.3–0.5%).

It should be noted that this SPACAR analysis is carried out with a very small number of elements. Similar results for the natural frequencies and mode shapes are also found by using standard linear finite element models, but then in the order of 10^4 elements were used to model the elastic parts of this system. Furthermore, the analysis with SPACAR can be easily extended to study pose dependent as in Sect. 3.2 or a system in which VCM and sensor are not aligned perfectly. This will be analysed next.

3.4 Mode shapes non-ideal system

The ideal system in the previous section is modified. Firstly, the VCM and sensor are positioned such that they no longer operate aligned in the direction of the centre of mass. Next a stationary force is applied such that in the system's equilibrium configuration the leaf springs

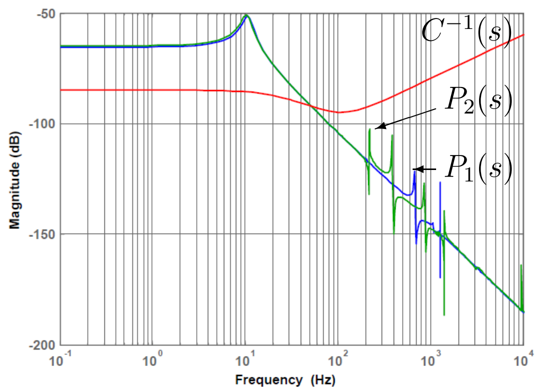


Figure 5: Bode magnitude plot of the inverse controller $C^{-1}(s)$ and mechanical system with ideal and non-ideal actuation, $P_1(s)$ and $P_2(s)$ respectively. The arrows indicate the first natural frequency above the bandwidth for each system.

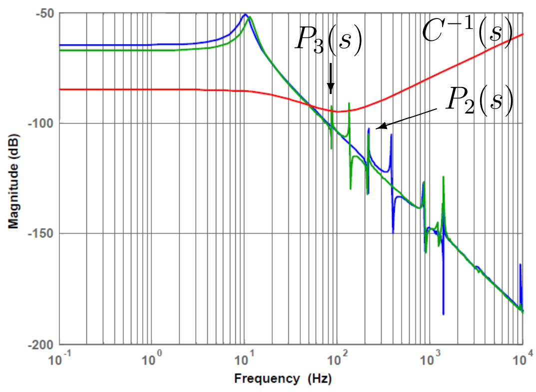


Figure 6: Bode magnitude plot of the inverse controller $C^{-1}(s)$ and mechanical system without and with manufacturing errors, $P_2(s)$ and $P_3(s)$ respectively. The arrows indicate the first natural frequency above the bandwidth for each system.

are deformed. Figure 5 shows that this system with transfer function $P_2(s)$ is still robustly stable, although it is also clear that the third natural frequency now starts to become relevant.

If the design considerations of Sect. 3.1 are ignored and the cut in the leaf spring is not present, the robust stability is no longer guaranteed. This is illustrated in Fig. 6 which presents the Bode magnitude plot of transfer function $P_3(s)$ of a system where one of the leaf springs is mounted with an angular offset of 3°. As was outlined before, this manufacturing error causes changes of the natural frequencies: The lowest natural eigenfrequency will increase and other natural frequencies decrease. More important, it appears that also the second natural frequency is clearly present in the Bode magnitude plot due to a change in the mode shape. The second and third natural frequencies peak above the frequency response of the inverse controller, indicating the risk of an unstable closed-loop system. In this particular example the analysis based on the SPACAR model of the mechanism reveals the undesired behaviour occurring due to the combination of non-linear effects, non-co-located control and misalignment of the leaf springs in an overconstrained system.

4 CONCLUSIONS

The rather simple example system of a one degree of freedom (1-DOF) support mechanism with elastic leaf springs (Fig. 1) demonstrates the applicability of the SPACAR software for design considerations and for analyses of the non-linear behaviour of the system. The formulation is based on a nonlinear finite element description for flexible multibody systems. Flexible joints like flexure hinges and leaf springs can be modelled adequately using only a few number of flexible beam elements as these elements account for geometric nonlinear effects such as geometric stiffening and interaction between deformation modes. In this way, a low dimensional system description can be obtained which is suitable for mechatronic design, i.e. the mechanical design as well as control system synthesis. It allows a designer to perform iterations quickly to optimise parameters. For more complex systems, the dimension of the system will increase but the advantages compared to other methods remain. The software has e.g. been applied successfully to model a hexapod-like manipulator with flexible joints with about 700 degrees of freedom, Fig. 7.

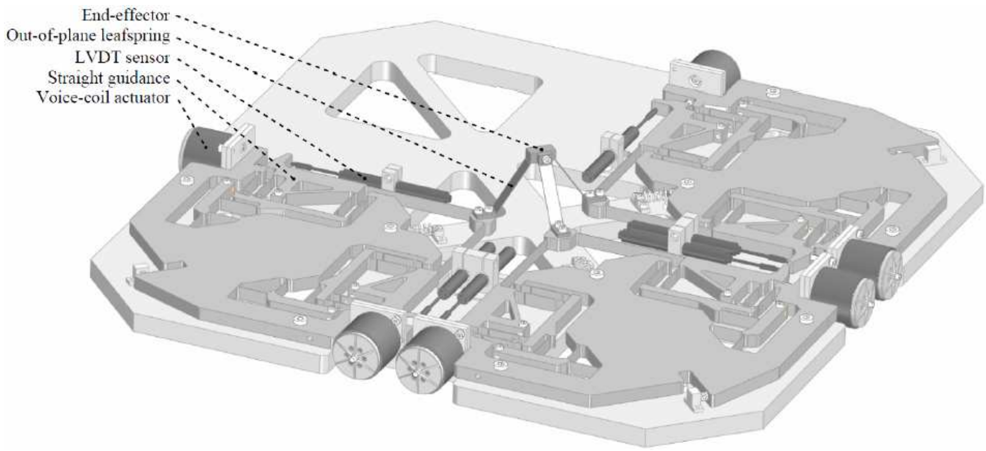


Figure 7: 6-DOF hexapod-like manipulator with flexible joints [7].

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