Intra-ply shear locking

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Abstract. Intra-ply shear locking results in unrealistic fibre stresses and spurious wrinkling in composite forming simulations. Three remedies are investigated: aligning the mesh, applying reduced integration and using multi-field elements. The bias extension simulation is used to test several triangular and quadrilateral elements on their capability to avoid locking. Their performance under large deformations is tested as well. The new multi-field element seems to be the best locking free element in random meshes.

Keywords: intra-ply shear locking, finite element, textiles, forming
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INTRODUCTION

Manufacturing processes can lead to unacceptable shape distortions in fibre reinforced products. These distortions depend on a wide variety of parameters from which geometry, material properties, lay-up, process temperatures and friction are just a few of the important factors. Numerical tools such as finite element (FE) simulations are a powerful tool to analyse these distortions and in the ideal case can lead to a first-time-right design. These optimizations require a robust, accurate and yet fast numerical procedure. This is not evident for anisotropic materials as shown in [1]. Besides this problem there is another important problem that has to be solved: intra-ply shear locking.

Intra-ply shear locking

Intra-ply shear locking is nothing more than the incapability of the element displacement field to represent the correct deformation mechanism of the fibre reinforced material. It is a numerical problem that has nothing to do with the physical jamming of the reinforcement. It leads to overestimation of fibre stresses, forces and stiffness and often leads to spurious wrinkles in 3D simulations. Some authors already suspected the existence of a locking phenomena, but the first article that addresses the problem correctly and illustrates the shortcomings of the current standard element formulation was presented by Yu et al. [2]. The locking phenomena can be illustrated by simulating the bias extension experiment. The bias extension experiment is frequently used to examine the shear response of biaxial reinforced materials. Figure 1a shows the undeformed and the deformed shape of the material. The two fibre directions are initially perpendicular to each other at $\pm 45°$. The specimen is gripped on the short edges and gradually extended. The stiffness of the fibres is dominant and the material deforms as a trellis frame, with each fibre crossing acting as a possible hinge point. Three deformation regions develop: an undeformed region (I), a central region with pure shear (II) and a region with intermediate shear (III). The paper of Potter [3] describes the bias extension experiment in detail. The finite element borders are aligned with the fibre directions in figure 1a and

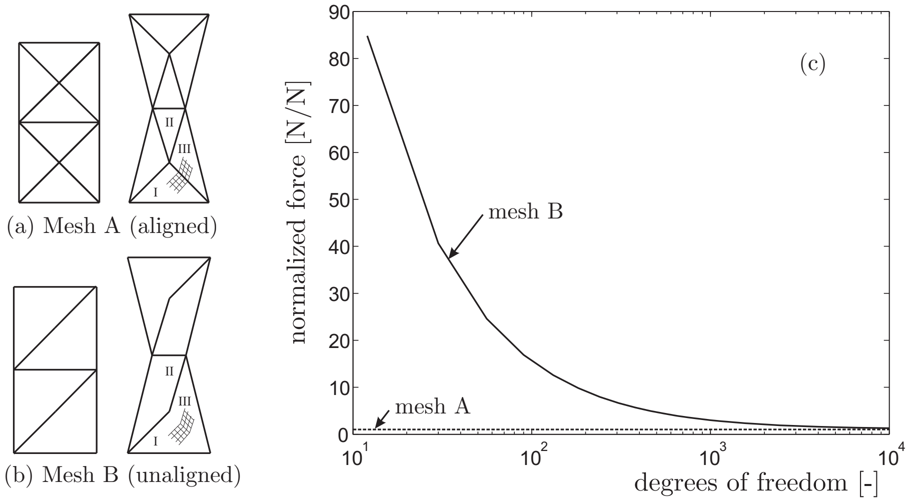

Figure 1. The three different deformation regions in a bias experiment: I, II and III. The aligned mesh (a) does not lock and the unaligned mesh (b) locks as illustrated in figure (c).

coincide with the lines along which the material develops ’hinge’ lines. This mesh can represent the trellis deformation correctly and locking is absent. The mesh shown in figure 1b shows severe locking, since a standard element cannot represent the discontinuous shear field. Fibres are stretched during the simulation, resulting in unrealistically high forces. Figure 1c shows the resulting tensile force of a bias extension simulation with both mesh types. Locking is reduced if the number of elements increases. This is the case in most types of locking. It is clear that the excessive amount of elements necessary to eliminate locking for this problem is unacceptable.

REMEDIES AGAINST INTRA-PLY SHEAR LOCKING

Three possible ways to avoid locking are discussed in this paper:

1. Aligned meshes
2. Selective reduced integration
3. Multi-field elements

Some of these options make it necessary to use higher order elements or quadrilaterals. Other options such as XFEM are not considered here.

Aligned meshes. Aligning the mesh with the fibre directions is probably one of the best remedies against intra-ply shear locking. Special attention is needed at curved edges of the blanks during meshing, where element edges cannot simply follow the curved boundary. This was already shown in the drape simulation in the article of Yu et al. [2]. This limits the use of automatic mesh generation. Alignment becomes impossible if the element contains more than two fibre directions. This can occur in multi-layer elements, where several layers are efficiently modelled within one element through the thickness as in [4]. Alignment also limits the use of Arbitrary Lagrangian Eulerian (ALE) methods, where the alignment of the fibres and the mesh will disappear during the simulation.

Selective reduced integration (SRI). Reduced integration or underintegration of elements is a well known way to eliminate several types of locking. Yu et al. [2] illustrated the use of reduced integration to eliminate intra-ply shear locking for woven fabrics. It was not considered a useful option due to the hourglassing introduced by the zero energy modes. A way to avoid this is to use Selective Reduced Integration (SRI). Reduced integration should only be applied to the fibre contribution of the material model. Using SRI in an element that is (accidentally) aligned with the fibre direction results in unnecessary deformation modes. The program becomes more robust if it is checked whether the element edge is aligned with the fibre or not. Full integration should be used if the element is aligned, otherwise SRI should be used. This process is called Adaptive Selective Reduced Integration (ASRI).

The locking problem cannot be solved for the linear triangle by reduced integration, since it is already fully integrated with only one point. Higher order elements are needed, e.g. quadratic triangles. The order of the displacement field of an element can be increased by adding degrees of freedom to existing nodes instead of adding nodes. An example is the Allmann88 triangle with vertex rotations [5, 6, 7]. Adding bubble modes or incompatible modes appeared to be ineffective.

Multi-field element (MF). Multi-field elements are also known as mixed elements. They contain degrees of freedom of different types, e.g. displacement and temperature. In this case the fibre strain is chosen as an additional degree of freedom beside the normal displacement degrees of freedom. A fibre f with direction $\mathbf{a}$ is located in a continuum at position $\mathbf{x}$. The strain rate $\dot{\varepsilon}_f$ of the fibre is the degree of freedom and must equal the velocity gradient in the direction of this fibre:

$$\dot{\varepsilon}_f = \mathbf{a} \cdot \mathbf{v}\overleftarrow{\nabla} \cdot \mathbf{a} \tag{1}$$

where $\mathbf{v}$ denotes the velocity. The weak FE formulation is found by weighing with a function $\mathbf{w}$ and integrating over the volume. With the use of the chain rule for differentiation and the divergence theorem of Gauss, the equation can be written as:

$$\int_V \mathbf{w}\dot{\varepsilon}_f \, dV + \int_V \mathbf{w}\overleftarrow{\nabla}(\mathbf{a} \cdot \mathbf{v})\mathbf{a} \, dV = \int_\Gamma \mathbf{w}\mathbf{n}(\mathbf{a} \cdot \mathbf{v})\mathbf{a} \, d\Gamma \tag{2}$$

The natural boundary condition for the strain field is zero velocity of the fibre with respect to the normal on the surface boundary. The nodes that contain the strain degrees of freedom can not be placed on the element edges. Placing the nodes on the edges would allow fibres to flow from one element into the adjacent element. The multi-field element combines strains and displacements within one element. Strain fields can have a different order of magnitude compared to displacements. A lot of significant digits can be lost due to this mismatch, making simulations less robust and less accurate. Another disadvantage is that the element stiffness matrix of the multi-field element can become non positive definite. This can be a problem for solvers used in FE calculations.

IMPLEMENTATION AND RESULTS

Ten different elements are selected based on the remedies against intra-ply shear locking that were discussed in the previous section. They are listed below and shown in figure 2.

1. Simplex (linear) triangle (LTR)
2. Quadratic triangle (QTR)
3. Quadratic triangle with selective reduced integration (QTR_SRI)
4. Quadratic triangle with adaptive selective reduced integration (QTR_ASRI)
5. Allman88 triangle (DRIL)
6. Allman88 triangle with selective reduced integration (DRIL_SRI)
7. Multi-field quadratic triangle with a constant strain field (QTR_MF)
8. Simplex (linear) quadrilateral (QUAD)
9. Simplex (linear) quadrilateral with selective reduced integration (QUAD_SRI)
10. Multi-field simplex quadrilateral with a constant strain field (QUAD_MF)

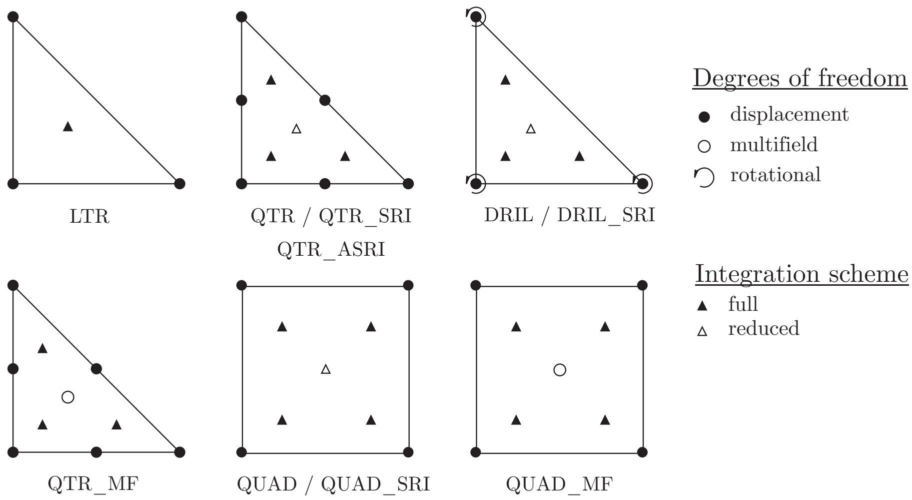

Figure 2. Ten different elements that are tested on their performance in a bias extension simulation.

Table 1. Parameters of the bias extension simulation.

width	[mm]	50	bulk stiffness	[MPa]	2.0
hight	[mm]	100	fibre stiffness	[MPa]	4000.0
thickness	[mm]	1	bulk volume fraction	[-]	0.50
mesh	[-]	2 x 4	fibre volume fraction (2x)	[-]	0.25

The elements were tested in a bias extension simulation as shown in figure 1 in a relatively coarse mesh. The number of elements ranged between 8 and 16, depending on the element type and mesh type. The geometry and the material parameters of the bias extension simulation can be found in table 1. Aligned and unaligned meshes of triangles were used, whereas the mesh of the quadrilaterals was necessarily unaligned. Figure 3 shows the resulting tensile forces of the simulations. The tensile forces are normalised with respect to the result of the simulation with simplex, aligned triangles. The fully integrated elements show accurate results in the aligned mesh and show severe locking in the unaligned meshes as expected. The quadratic triangle (QTR) gives quite accurate results when used in a random mesh with an overshoot of only 20%. Apparently, the curved boundaries of this element can conform to the discontinuous fields quite well in this case, were the elongation of the specimen is only 1%. The QTR shows severe locking if the elongation is increased. The results from the multi-field (MF) elements and the elements with reduced integration (SRI) are almost identical. Reduced integration reduces the number of evaluated fibre strains in an element to one, equal to the number

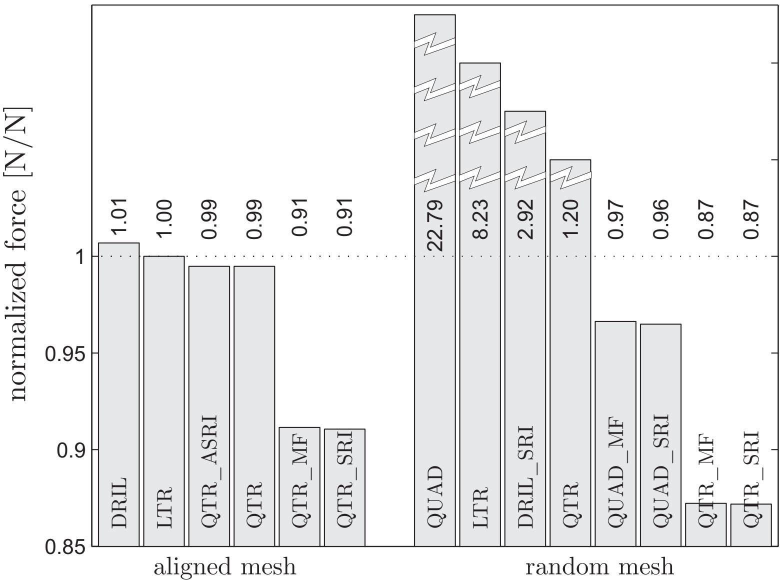

Figure 3. Tensile forces from simulation of the bias extension simulation. The forces are normalised with respect to the simulations with aligned simplex triangles.

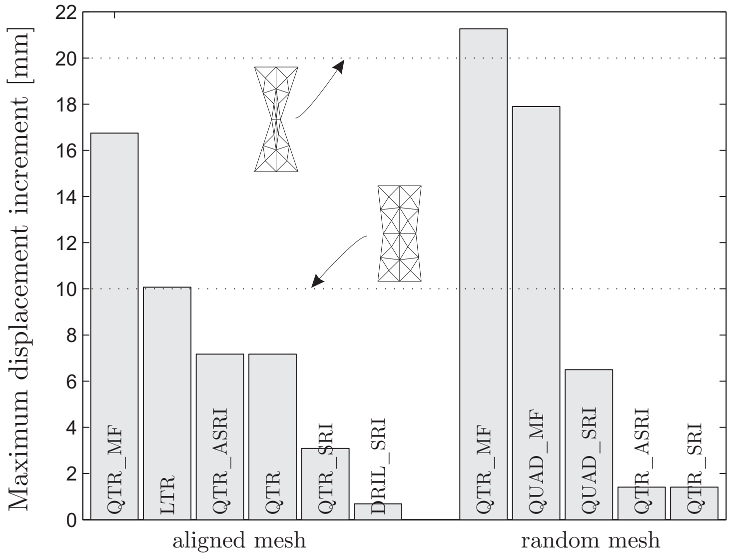

Figure 4. Maximum incremental displacement within one step. Testing the performance of the elements in large deformations.

of fibre strains in the multi-field element. It seems to be an effective way to eliminate locking. Elements become significantly less stiff, even less stiff than the aligned, fully integrated elements. This effect is known from other types of locking: reduced integration ’over’-eliminates the locking effect. The Allman88 element (DRIL) with drilling degrees of freedom and with SRI still locks in a random mesh. It seems that at least a quadratic field with six additional degrees of freedom is needed to avoid locking in triangular elements. Forming simulations of fibre reinforced composites must perform well with large deformations of highly anisotropic material. The locking free elements from figure

3 are tested on their performance in large deformations. The fibre stiffness is increased by a factor 10^3 and the maximum incremental displacement of one step is determined. The tangent matrix is calculated numerically for all element types and the maximum number of iterations is set to 8. Figure 4 shows the results of this analysis. The performance of elements deteriorates when reduced integration is used and improves for multi-field elements. The maximum step size for the aligned quadratic triangles is around 7% of the total specimens length. This value increases by almost 2.5 times for the multi-field element and decreases by almost 2.5 times for the element that uses SRI. For random meshes the difference is even larger. The quadrilaterals perform quite well and the difference between SRI and the multi-field element is not as large as for the quadratic triangle. The performance of the Allman88 triangle is poor.

CONCLUSIONS AND FUTURE WORK

Three remedies against intra-ply shear locking are investigated in this paper: aligning the mesh, applying selective reduced integration (SRI) and using multi-field elements. All fully integrated standard elements have to be aligned with the fibre directions to avoid locking. For a random mesh one can choose a higher order triangle (at least quadratic) with SRI or a simplex quadrilateral (or higher) with SRI. SRI indicates that only the fibre contributions to the material model are underintegrated. The new multi-field elements combine the standard displacement degrees of freedom with an additional field that interpolates the fibre strain. The elements are closely related to the SRI element and are free of locking for any mesh configuration. Aligning the mesh is the best option with respect to accuracy, but limits the use of automatic mesh generation and makes implementation of multi-layer elements impossible. Elements based on SRI and multi-field elements underestimate the stiffness, typically from 5% down to 15% in this particular study. The maximum allowable incremental step size during a simulation is much larger for the multi-field element than for the elements using SRI and even larger compared to the maximum step size of an aligned element. The multi-field element is the best element to use in large deformations of fibre reinforced materials in random meshes, based on the results from this analysis. The next step is the implementation of the multi-field element in a 3D simulation to verify the performance in realistic 3D simulations.

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