ANALYTICAL AND EXPERIMENTAL ANALYSIS OF A PARALLEL LEAF SPRING GUIDANCE

J. P. Meijaard*, D. M. Brouwer*,** & J. B. Jonker*
*Laboratory of Mechanical Automation and Mechatronics, Faculty of Engineering Technology,
University of Twente, Enschede, Netherlands
**DEMCON, Oldenzaal, Netherlands

Summary A parallel leaf spring guidance is defined as a benchmark problem for flexible multibody formalisms and codes. The mechanism is loaded by forces and an additional moment or misalignment. Buckling loads, changes in compliance and frequencies, and large-amplitude vibrations are calculated. A previously developed beam element for modelling the leaf springs is shown to be able to describe these phenomena with a limited number of elements. The results are validated by experiments.

INTRODUCTION

Precision manipulation of objects often requires elastic mechanisms. As elastic mechanisms introduce no play and only a low amount of friction, they are especially useful if a high repeatability is required. Furthermore, in micromechanical systems contacting bodies, such as roller bearings, with relative motion are better avoided. Examples of applications incorporating elastic mechanisms are manipulators for micro assembly, manipulation of substrates in microscopes and manipulation of optical elements in a beam path. Often these mechanisms are implemented in a statically indeterminate way. Misalignment due to assembly in combination with static indeterminacy causes internal stresses. Depending on the amount of misalignment the behaviour of a mechanism becomes more non-linear.

A parallel leaf spring guidance is considered as a system for testing flexible multibody modelling and analysis techniques. This mechanism is used in many machines as a standard design module. The purpose of this presentation is to provide a model system that can be used as a benchmark for flexible multibody modelling and analysis techniques to predict numerically the behaviour of an indeterminate system. On the one hand, this system is sufficiently simple so it can be completely specified by a small set of parameters and on the other hand, the system can be modelled at several levels of detail and shows complicated behaviour, such as non-linear stiffness and buckling.

SYSTEM DESCRIPTION

The system under consideration is shown in figure 1. It consists of a rigid guided body, a block, connected to the base by

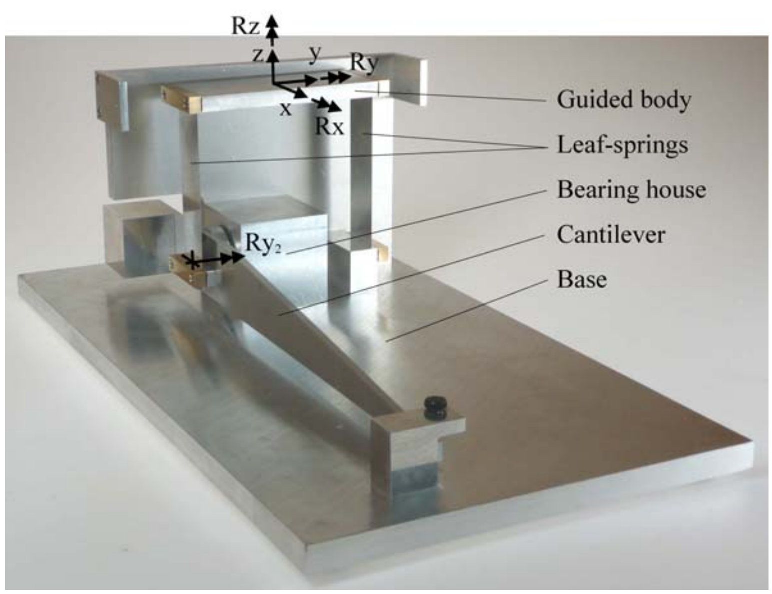

Figure 1. Leaf spring mechanism

two equal leaf springs. This mechanism has one large compliance in y-direction, whereas the stiffness in other motion directions is large. The system has the additional characteristic that one support point can rotate about an axis, Ry_2. This possibility of rotation removes the static indeterminacy in the direction of the large flexural rigidity of the leaf spring, so large internal stresses are avoided. If, however, a rotation about this axis is prescribed, which may represent an initial misalignment, large moments may develop, which can influence natural frequencies and may lead to buckling. Alternatively, a moment can be prescribed.

MODELLING AND ANALYSIS TECHNIQUES

The system is amenable to several levels of modelling detail. The easiest way is to use classical beams for the leaf springs, a rigid body for the block, a perfect hinge and rigid supports. The normal deformations of the leaf springs can be neglected, and also the bending in their planes if the system is loaded by a prescribed moment. A refinement is to include the effect of restrained warping and elastic clamping. Alternatively, the leaf springs can be modelled as thin plates. The factor $1/(1-\nu^2)$ between the flexural rigidity of beam theory and plate theory is a point to consider.

The system can be dealt with by mathematical analysis for the simplest conceivable model and by modelling the system by finite element techniques or other numerical methods. Here, the program SPACAR [1] is used with the flexible beam element described in [2].

A hardware model of the mechanism has been built, on which some standard measurements can be made. Frequencies for small amplitude as well as large amplitude vibrations, load–deflection characteristics and buckling loads can be measured.

RESULTS

Buckling and post-buckling behaviour

If the elastic pre-buckling displacements are neglected, the critical load $M_{\rm cr}$ is

$$M_{\rm cr} = 2\pi\sqrt{S_{\rm b}S_{\rm t}}/l \tag{1}$$

Here, $S_{\rm b}$ is the smaller flexural rigidity, $S_{\rm t}$ the torsional stiffness and l the length of either leaf spring. This is a threefold critical moment with three coinciding buckling modes. In two of these, the individual leaf springs buckle independently of each other, and in the third mode, the block has a displacement in the y-direction.

In the case that an additional vertical load P_z is present, which is due to gravity, the critical moments decrease and the lowest critical moment, which is associated with the mode with a displacement of the block, decouples from the other two modes, so the lowest critical moment is associated with a unique buckling mode. If the load is reversed in direction, for instance by holding the mechanism upside down, the double critical load becomes the lowest.

No spectacular destabilizing post-buckling behaviour due to interacting modes is observed, as can be found in other cases [3]. This is even less so if the rotation angle is prescribed instead of the moment. Imperfections have the result that the a true bifurcation disappears and a smooth load–deflection path is found.

The beam element in [2] is capable of capturing these phenomena and has a quadratic rate of convergence if the number of elements is increased for cases involving torsion.

Non-linear vibrations

The vibrations of the system are studied and the periods as a function of the amplitude are calculated. At large amplitudes far in the post-buckling region, some mode interaction can be observed.

Experiments

The experiments show qualitative agreement in frequencies, compliances and buckling loads with the numerical results. Quantitative differences are not large and can be explained from modelling assumptions made and imperfections of the experimental set-up.

CONCLUSIONS

The model system shows clearly that small misalignments may result in large deviations from the intended behaviour, such as changes in compliance and natural frequencies, and even buckling with coinciding or nearly coinciding critical loads.

The system is useful as a benchmark to test formalisms and codes for flexible multibody dynamics.

The beam element developed in [2] is capable of describing this system efficiently, although other elements may perform better in some respects.

References

[1] Jonker, J.B, Meijaard, J.P.: SPACAR – Computer Program for Dynamic Analysis of Flexible Spatial Mechanisms and Manipulators. In Schiehlen, W. (ed.): *Multibody Systems Handbook*, Springer, Berlin, 1990, pp. 123–143.

[2] Meijaard J.P.: Validation of Flexible Beam Elements in Dynamics Programs. *Nonlin. Dynam.* **9**:21–36, 1996.

[3] Koiter, W.T., Pignataro, M.: A General Theory for the Interaction Between Local and Overall Buckling of Stiffened Panels. In Ferrari, C. (ed.): *Problemi Attuali di Meccanica Teoretica e Applicata*, Torino, 1977, pp. 179–222.