On the microCHP scheduling problem

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***Abstract*—In this paper both continuous and discrete models for the microCHP (Combined Heat and Power) scheduling problem are derived. This problem consists of the decision making to plan runs for a specific type of distributed electricity generators, the microCHP. As a special result, one model variant of the problem, named n-DSHSP-restricted, is proven to be NP-complete in the strong sense. This shows the necessity of the development of heuristics for the scheduling of microCHPs, in case multiple generators are combined in a so-called fleet.**

Keywords: scheduling, complexity, microgeneration

I. Introduction

During the last years the way in which electricity is produced has changed. The availability of renewable energy sources, the development of distributed electricity generators and the demand for more energy efficient appliances are increasing [6]. A specific type of coming distributed electricity generators is microCHP (Combined Heat and Power on a domestic scale). A microCHP produces both heat and electricity for household usage; the electricity can also be delivered back to the electricity grid. This electricity production, while being connected to the grid, gives possibilities to increase stability in the grid, to replace a conventional power plant, and more. To effect these possibilities the individual microCHPs need to be controlled, as standalone devices, but also in cooperation with other microCHPs or generators. An important part in this control can be contained by online and offline scheduling problems. In this paper we present different kinds of scheduling models that are derived from the microCHP problem description.

The paper is organised as follows. In Section II the problem is explained in more detail. Sections III and IV translate this problem to two continuous models (single house and fleet scheduling) and three discrete models (single house and two fleet scheduling variants) respectively. One of these discrete problem variants, n-DSHSP-restricted, is proven to be NP-complete in the strong sense. The paper ends with conclusions and recommendations for future work.

II. Problem description

Since there are many ways to look at the problem of scheduling microCHP appliances (e.g. [2], [5]), we start with a general description of the available input of our problem. The setting consists of a number of houses, each equipped with a microCHP and a heat buffer. The advantage of a microCHP is that the standard engine generates heat and electricity simultaneously from natural gas, which improves the energy efficiency of the converted gas. However, an additional top burner can generate additional heat, in case of immediate heat demand that cannot be fullfilled by the normal configuration. In the presented model we do not use this top burner, since we want to use the full advantage of combined, highly efficient generation. So, the only two options for the microCHP in our problem are:

- to switch the machine on;
- to switch the machine off.

Certain technical constraints are attached to the use of the microCHP. Due to relatively long startup periods for the engine and the requirement not to start the engine too often, there is a minimum required period of time that the machine needs to run before it can be switched off. For a similar reason there is a minimum time between two consecutive runs of the microCHP.

The generated heat is used to supply the heat demand of the house. A heat buffer functions as an intermediate between heat production and consumption. This heat buffer has a given volume and the temperature in the heat buffer must stay between some limits, resulting in a certain heat capacity of the buffer. When the temperature drops below the given minimum value, the microCHP has to start in order to supply the heat buffer. If the temperature reaches the given maximum value, the microCHP has to be turned off.

So, the decisions to switch the microCHP in a house on and off are heat led. However, looking at objectives, the problem becomes electricity led. On a higher level, when a number of houses cooperate in a certain 'fleet', the total production of electricity of the microCHPs in the fleet can be seen as the production of a Virtual Power Plant (VPP) [4]. This production can be subjected to different objectives, e.g.:

- to match a certain predefined production pattern of the fleet, specifying the total electricity production at each moment in time of the day;
- to balance the inpredictable generation from e.g. a windmill park, such that the total combined generation again matches a predefined production pattern.

If the first objective is used to act with the VPP on the electricity market (e.g. the day ahead market, see [1]), this leads to an offline scheduling, in which a planning must be made for a complete day in advance. The second objective is an example for online scheduling, in which the total production of windmill park *and* VPP must be realtime controlled. In both cases, also for single houses, the objective stays electricity led, since electricity can be delivered back to the grid and is the product that can be sold.

III. Continuous models

The problem as given in Section II can be modelled in many ways. In this section we formulate two continuous models that are quite close to reality. Next, in Section IV we give alternative discrete variants which can be used in practice.

A. Single House Scheduling Problem

In this section we formalise the Single House Scheduling Problem (SHSP). The SHSP is the continuous time decision problem of switching on and off the microCHP in a house.

The planning horizon is defined as an interval $[0, T]$ (in hours). Let the (predicted) heat demand in a house be given by the non-negative continuous function $h(t)$ (in kW) on the interval $[0, T]$. For each time $\bar{t} \in [0, T]$ the total heat demand (in kWh) from the start of the planning horizon up to $\bar{t}$ equals:

$$\int_0^{\bar{t}} h(t)dt \tag{1}$$

The decisions to switch the microCHP on or off are taken at time moments in $[0, T]$, which we denote by ordered (increasing) sequences $\{t^{on}\}$ and $\{t^{off}\}$. Let l be the length of the sequence $\{t^{on}\}$ and m the length of the sequence $\{t^{off}\}$. For these sequences we have:

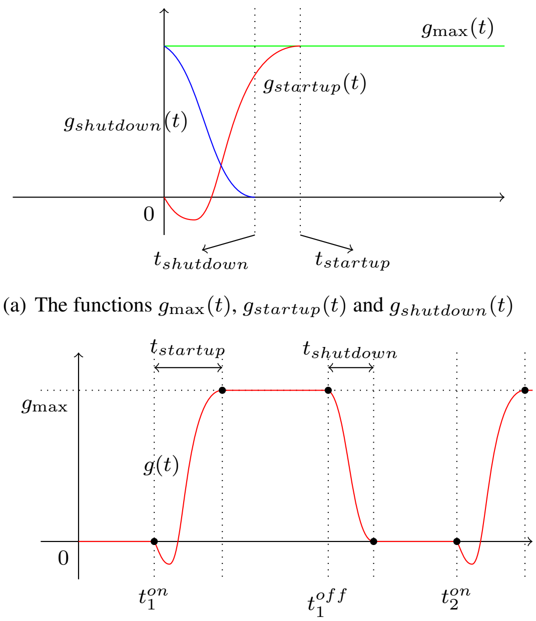
- $t_1^{on} \leq t_1^{off}$, since we assume that the microCHP is off at the start of the planning period;
- $m \in \{l-1, l\}$, since it is allowed that the microCHP runs at time moment T; thus, either $m = l$ or $m = l - 1$;
- $t_j^{on} \leq t_j^{off} \leq t_{j+1}^{on}$, $j = 1, \ldots, l-1$ and $t_m^{on} \leq t_m^{off}$, if $m = l$, indicating that decisions alter in time between switching on and switching off the microCHP;
- $t_j^{off} - t_j^{on} \geq MR$, causing the microCHP to keep running for a given minimum time MR, once the microCHP has been started;
- $t_{j+1}^{on} - t_j^{off} \geq MO$, causing the microCHP to stay off for a given minimum time MO, once the microCHP has been stopped;
- the values of MR and MO are chosen based on technical constraints, concerning efficiency loss and wearing, which is mostly due to the startup and shutdown periods of the microCHP.

The time moments t_j^{on} and t_j^{off} determine the intervals during which the microCHP is running. In case $m = l$, these intervals are $[t_j^{on}, t_j^{off}]$, $j = 1, \ldots, l$ and in case $m = l - 1$, these intervals are $[t_j^{on}, t_j^{off}]$, $j = 1, \ldots, l-1$, and $[t_l^{on}, T]$. Note, that in the latter case, the interval $[t_l^{on}, T]$ may be shorter than MR.

The generated heat and electricity are given by functions $g(t)$ and $e(t)$ (in kW). Based on the properties of the microCHP, the generated electricity is assumed to be equal to the generated heat, apart from a constant multiplication factor α:

$$e(t) = \alpha g(t). \tag{2}$$

Based on this, we restrict ourselves to the function $g(t)$ which represents the generated heat. This function is defined using



(a) The functions $g_{\max}(t)$, $g_{startup}(t)$ and $g_{shutdown}(t)$

(b) The function $g(t)$

Fig. 1. Heat generation

four types of intervals. We introduce startup and shutdown times $t_{startup}$ and $t_{shutdown}$ as the microCHP does not produce directly after its start at maximum level and does not immediately stop producing heat and electricity if it is turned off. In Figure 1(a) typical generation functions for the startup period and shutdown period are indicated. In between these intervals the microCHP produces at full power $g_{\max}$ (indicated by the function $g_{\max}(t)$ in Figure 1(a)). It is obvious to ask that $t_{startup} \leq MR$ and $t_{shutdown} \leq MO$. The heat generation function for a given interval in which the microCHP is running, is specified using four types of intervals:

1) startup intervals $[t_j^{on}, \min(t_j^{on} + t_{startup}, T)]$, $j = 1, \ldots, l$;
2) intervals $(\min(t_j^{on} + t_{startup}, T), t_j^{off})$, $j = 1, \ldots, l$ (where $t_l^{off} = T$ if $m = l - 1$) in which the microCHP is running at full power;
3) shutdown intervals $[t_j^{off}, \min(t_j^{off} + t_{shutdown}, T)]$, $j = 1, \ldots, m$;
4) intervals $[0, t_1^{on})$ (if $t_1^{on} > 0$), $(t_j^{off} + t_{shutdown}, t_{j+1}^{on})$, $j = 1, \ldots, l-1$ and $(t_m^{off} + t_{shutdown}, T]$ (if $t_m^{off} + t_{shutdown} < T$ and $l = m$), in which the production is zero.

The startup and shutdown functions of the heat generation and the electricity generation differ slightly in reality. During startup the electricity function becomes negative (the engine needs an electric 'push' to start), whereas the heat production slowly increases together with the increasing electricity production, but does not become negative. During the shutdown period the electricity and heat output decrease simultaneously, but again the engine needs a final electric 'push' to stop. However, these differences are negligible and the two functions are assumed to be directly comparable via the factor α.

The intervals of all types together form the planning horizon

$[0,T]$. Since we use different functions in the different types of intervals (see Figure 1(a)) we get a piecewise defined function for $g(t)$ (see Figure 1(b)):

$$g(t) = \begin{cases} g_{startup}(t - t_j^{on}) & \text{if } t_j^{on} \leq t \leq \min(t_j^{on} + t_{startup}, T), \\ & j = 1, \ldots, l \\ g_{\max}(t) & \text{if } \min(t_j^{on} + t_{startup}, T) < t < t_j^{off}, \\ & j = 1, \ldots, l \text{ (where } t_l^{off} = T \text{ if } m = l-1) \\ g_{shutdown}(t - t_j^{off}) & \text{if } t_j^{off} \leq t \leq \min(t_j^{off} + t_{shutdown}, T), \\ & j = 1, \ldots, m \\ 0 & \text{elsewhere,} \end{cases} \tag{3}$$

where $g_{\max}(t) = g_{\max}$, $g_{startup}(t)$ is a continuous function on $[0, t_{startup}]$ with $g_{startup}(0) = 0$ and $g_{startup}(t_{startup}) = g_{\max}$ and $g_{shutdown}(t)$ a continuous function on $[0, t_{shutdown}]$ with $g_{shutdown}(0) = g_{\max}$ and $g_{shutdown}(t_{shutdown}) = 0$.

Let the buffer capacity (in kWh) be denoted by BC and let the heat buffer level be given by the function $hl(t)$ (in kWh), representing the amount of extractable heat left in the buffer at time t. The 'fill rate' of the buffer is given by the heat generation function $g(t)$, the 'extraction rate' by the heat demand function $h(t)$, and finally there is some heat loss over time. To model this heat loss, we use a simplified version of Newton's Law of Cooling:

$$\frac{dT}{dt} = \frac{hA}{C}(T_{env} - T_{buffer}), \tag{4}$$

where h is the heat transfer coefficient, A the surface area of the heat transfer, C the heat capacity of water, T_{env} the environmental temperature and T_{buffer} the temperature of the buffer. This law implies exponential decay, if the water cools down to the environmental temperature. However, since we keep the buffer above a certain threshold that lies far above the environmental temperature, we assume that there is a constant heat loss rate in the buffer. Therefore we introduce the constant k as the heat loss rate. We assume that extra heat losses in the transport from and to the heat buffer are already incorporated in the functions $h(t)$ and $g(t)$. This results in the following derivative of the heat buffer level function $hl(t)$:

$$hl'(t) = g(t) - h(t) - k. \tag{5}$$

Thus, if we assume that the initial buffer level is $hl(0) = BL$, the heat buffer level function at time $\bar{t}$ is given by:

$$hl(\bar{t}) = BL + \int_0^{\bar{t}} g(t)dt - \int_0^{\bar{t}} h(t)dt - k\bar{t}. \tag{6}$$

At each moment in time $\bar{t}$ the heat buffer is not allowed to be empty and the buffer capacity may not be exceeded, i.e.:

$$0 \leq hl(\bar{t}) \leq BC, \tag{7}$$

has to be fullfilled for all time moments $\bar{t}$. Note that 0 and BC are used in this equation to reflect the extractable amount of heat from the buffer, so 0 coincides with the minimum allowed level of the buffer and does not mean that the buffer contains no heat, and BC equals the maximum amount of heat that is extractable and not the total amount of heat in the buffer.

The objective for the SHSP is to maximize the revenue from the generated electricity. Let the (possibly fluctuating) electricity prices be given by the function $p(t)$ on the interval $[0,T]$. Now, the objective may be:

$$\max \int_0^T p(t)e(t)dt \tag{8}$$

or, if we take a desired end level EL (i.e. the begin level of the heat buffer for the next day) into account:

$$\max \int_0^T p(t)e(t)dt - M|hl(T) - EL|. \tag{9}$$

Another way to take the end level of the heat buffer into account, without specifically using EL, may be by defining the objective:

$$\max \frac{\int_0^T p(t)e(t)dt}{\int_0^T e(t)dt}. \tag{10}$$

This last equation focuses more on the quality of the generation and less on the quantity, possibly solving the problem of finishing the day with a full buffer.

B. *Fleet Scheduling Problem*

The Fleet Scheduling Problem (FSP) is the continuous model in which a group of houses cooperates in a so-called fleet. Each house can be modelled by an SHSP. We use an additional subscript $n \in \{1, \ldots, N\}$ in the models, where N is the number of houses in the fleet, to indicate the individual differences of the houses. The only element that changes in the models of the individual houses for the FSP model, is the objective for the generation of the microCHPs. Whereas the focus in the SHSP only was on producing at the best possible times regarding revenue, according to equations (8), (9) and (10), now the generation of a single house has to consider the generation within the whole fleet too. This cooperation can be arranged in different ways:

- **Virtual Power Plant**
  One possible goal for the fleet of microCHPs is to cooperate in a Virtual Power Plant (VPP). In a VPP the fleet acts as one large electricity generator towards the outside world. This means that all individual microCHPs must be controlled in order to generate electricity like a normal power plant, i.e. the electricity must be generated on the right time, meaning that the sum of the complete production is controllable, not only over a large period (e.g. a complete day), but also on very small intervals (e.g. minutes). To model this, a predefined production function $P(t)$ (in kW) is defined as the production goal of the VPP. This function represents the agreed amounts and times in which electricity has to be delivered to the electricity grid. Under ideal circumstances, the FSP comes down to the problem of instantaneous matching the total electricity production of the VPP to $P(t)$, i.e.:

$$\sum_{n=1}^{N} e_n(t) = P(t). \tag{11}$$

  The FSP now consists of the assignment of runs to all microCHPs in such a way that constraint (11) is met (or approximated).

- **Balancing power**
A fleet can also be organized to be used as (backup) peak generator in order to balance fluctuating or hard-to-predict generation from renewable sources like windmills or photovoltaic cells. In this case, the system is divided in two categories: generators which are not controllable (the renewable sources) and which give an unpredictable output $\widetilde{O}(t)$, and a set of controllable microCHPs. We assume that the output $\widetilde{O}(t)$ matches (a large part of) a prespecified consumption function $C(t)$. The mismatch between the output and the given consumption must be corrected by the fleet. In this situation, decisions cannot be made in advance via offline scheduling techniques to plan the runs of the microCHPs, since there is no fixed generation pattern defined ($C(t) - \widetilde{O}(t)$ is a stochastic function). Instead of this, the fleet must be able to react realtime on fluctuations in the production of the unpredictable generators, such that:

$$\widetilde{O}(t) + \sum_{n=1}^{N} e_n(t) = C(t). \tag{12}$$

The objectives for the fleet can be to maximize the efficiency of the use of all microCHPs. In this case this means that the objective would be to minimize the number of different runs, hence maximizing the average run length. Since longer runs imply higher efficiency, the efficiency may be optimized by using the following objective function:

$$\min \sum_{n=1}^{N} l_n, \tag{13}$$

where l_n is the length of the sequence $\{t_n^{on}\}$ of house n.

If the production function $P(t)$ for the fleet is not totally specified (as in (11) or (12)), but leaves some freedom, a financial element can be added to the fleet objective. Let $P(t)^{\pm}$ be the space from which the production function of the fleet may be taken. Equation (11) then changes in:

$$\sum_{n=1}^{N} e_n(t) \in P(t)^{\pm} \tag{14}$$

The objective now may be:

$$\max \frac{A}{\sum_{n=1}^{N} l_n} + B \sum_{n=1}^{N} \int_0^T p(t) e_n(t) dt, \tag{15}$$

where A and B are proper weights, such that a balance between efficient use of the microCHPs and the revenue of the fleet is found. If the focus is completely on revenue, the objective may be:

$$\max \sum_{n=1}^{N} \int_0^T p(t) e_n(t) dt. \tag{16}$$

IV. Discrete models

The continuous models in Section III are the most direct transformation of the problem into a model. However, they are not easy to handle in practice, since the problem of determining values for the two sequences $\{t_n^{on}\}$ and $\{t_n^{off}\}$ for $n = 1, \ldots, N$ is hard to solve analytically. In order to simplify these models to variants which can be solved in practice, the models are transformed into discrete variants. This transformation simplifies the set of decisions.

A. Discrete Single House Scheduling Problem

The Discrete Single House Scheduling Problem transforms the SHSP into a discrete variant. Where the SHSP is defined on the interval $[0, T]$, this interval is now divided into a fixed number N_T of intervals $[t_i, t_{i+1}]$ of equal length $\frac{T}{N_T}$. The main difference between the continuous model and the discrete model is that the decision to have a microCHP running or switched off, is made for a complete interval $[t_i, t_{i+1}]$. Where the decision maker in the continuous models is completely free to instantaneously switch on or switch off a microCHP, this decision maker now can only decide to switch the microCHP on or off at the time moments $t_0 = 0, t_1 = \frac{T}{N_T}, \ldots, t_{N_T} = T$. To emphasize this idea we introduce decision variables x_j for the intervals:

$$x_j = \begin{cases} 1 & \text{if the microCHP is on during interval } j \\ 0 & \text{if the microCHP is off during interval } j, \end{cases} \tag{17}$$

where interval j is the interval $[t_{j-1}, t_j]$, $j = 1, \ldots, N_T$.

The heat demand is discretisized to input values H_j (in kWh) by integrating the original heat demand function $h(t)$ for each interval $j \in \{1, \ldots, N_T\}$:

$$H_j = \int_{(j-1)\frac{T}{N_T}}^{j\frac{T}{N_T}} h(t) dt. \tag{18}$$

The generation of heat and electricity now can be described by values g_j (in kWh) and e_j (in kWh) for each interval j. The production of electricity and heat still correspond to each other as in equation (2):

$$e_j = \alpha g_j. \tag{19}$$

The heat generation $G_{\max}$ during intervals in which the microCHP is running at full speed is given by:

$$G_{\max} = g_{\max} \frac{T}{N_T}. \tag{20}$$

Regarding startup and shutdown phases we assume a linear increase/decrease for the full intervals that have an overlap with the corresponding startup and shutdown times. So, we assume a linear increase during the $r = \lceil \frac{t_{startup}}{\frac{T}{N_T}} \rceil$ periods after the microCHP is switched on and a linear decrease during the $s = \lceil \frac{t_{shutdown}}{\frac{T}{N_T}} \rceil$ periods after the microCHP is switched off. Let $start_j$ be the binary variable for interval j, indicating whether the microCHP has been started during interval j or not:

$$start_j = \begin{cases} 1 & \text{if the microCHP is started in interval } j \\ 0 & \text{otherwise.} \end{cases} \tag{21}$$

Likewise $stop_j$ is defined as:

$$stop_j = \begin{cases} 1 & \text{if the microCHP is stopped in interval } j \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

To ensure that the variables $start_j$ are consistent with the x-variables (see (17)), the following constraints are added:

$$start_j \geq x_j - x_{j-1} \quad j \geq 2 \tag{23}$$

$$start_j \leq x_j \quad j \geq 2 \tag{24}$$

$$start_j \leq 1 - x_{j-1} \quad j \geq 2 \tag{25}$$

$$start_1 = x_1. \tag{26}$$

Similar equations for $stop_j$ have to be introduced. The heat generation g_j now can be calculated as follows:

$$g_j = G_{\max} x_j - G_{\max} \sum_{k=0}^{r-1} start_{j-k} \int_k^{k+1} (1 - \frac{v}{r}) dv$$
$$+ G_{\max} \sum_{k=0}^{s-1} stop_{j-k} \int_k^{k+1} (1 - \frac{w}{s}) dw. \tag{27}$$

Note, that the integrals in this expression do not depend on the decision variables and, thus, can be precalculated leading to a linear function for calculating g_j.

In the SHSP the decision sequences $\{t^{on}\}$ and $\{t^{off}\}$ had to fullfill the minimum runtime (MR) and minimum offtime (MO) constraints. For the DSHSP the minimum runtime constraint means that we have to run the microCHP for minimally $MR_D = \lceil \frac{N_T MR}{T} \rceil$ consecutive intervals. Likewise, the minimum offtime constraint implies that the microCHP has to be off for minimally $MO_D = \lceil \frac{N_T MO}{T} \rceil$ consecutive intervals. This leads to the following linear constraints for the discrete variant:

$$(MR_D - 1) x_{j-1} - \sum_{k=j-MR_D}^{j-2} x_k$$
$$\leq (MR_D - 1) x_j \qquad j > MR_D \tag{28}$$

$$x_{j-1} \leq x_j \quad 1 < j \leq MR_D \tag{29}$$

$$\sum_{k=j-MO_D+1}^{j-1} (x_{k-1} - x_k) \leq 1 - x_j \qquad j > MO_D. \tag{30}$$

For an explanation of these constraints (28)-(30) we refer to [2]. A necessary restriction is that $MO_D \leq MR_D$, which is usually the case for a microCHP.

The heat level hl_j in interval j (in kWh) can be calculated by:

$$hl_1 = BL \tag{31}$$

$$hl_j = hl_{j-1} + g_{j-1} - H_{j-1} - K \quad j > 1, \tag{32}$$

where $K = \frac{kT}{N_T}$ is the constant heat loss in an interval. This heat level indicates how much extractable thermal energy is available in the buffer at the begin of the interval j. Again, the buffer capacity has to be taken into account:

$$0 \leq hl_j \leq BC. \tag{33}$$

The SHSP objectives (8)-(10) can be transformed into:

$$\max \sum_j p_j e_j \tag{34}$$

$$\max \sum_j p_j e_j - M |(hl_{N_T} + g_{N_T} - H_{N_T} - K) - EL| \tag{35}$$

$$\max \frac{\sum_j p_j e_j}{\sum_j e_j}. \tag{36}$$

The presented DSHSP model can be easily formulated as an Integer Linear Programming (ILP) formulation. In [2] also a Dynamic Programming (DP) formulation is presented. The computational times to find an optimal solution for three types of instances for both formulations (ILP and DP) are given in Table I. For all instances and instance sizes (measured by the number of intervals) the DP approach is favourable to the ILP formulation. On the other hand, this DP grows exponentially when the instance size increases (see the final column in Table I). So, both formulations are presumably not suitable to be extended for the Discrete Fleet Scheduling Problem. In the following subsection, we will use this notice of 'complexity' to introduce a simplified version of the Discrete Fleet Scheduling Problem. Next, complexity results for this version are proven.

instance	interval		objective	computational time	
	# intervals	length (min)		ILP (s)	DP (s)
dtsh1[1]	24	60	eq. (34)	0.56	0.00
	48	30	eq. (34)	22.52	0.00
	120	12	eq. (34)	446.38	7.00
	240	6	eq. (34)	9613.28	326.00
	288	5	eq. (34)	1569.27	817.00
dtsh2[2]	24	60	eq. (34)	0.09	0.00
	48	30	eq. (34)	0.55	0.00
	120	12	eq. (34)	2868.67	4.00
	240	6	eq. (34)	8998.44[3]	183.00
	288	5	eq. (34)	27306.66	398.00
dtsh3[4]	24	60	eq. (34)	0.17	0.00
	48	30	eq. (34)	19.50	1.00
	120	12	eq. (34)	14.44	6.00
	240	6	eq. (34)	9127.72	327.00
	288	5	eq. (34)	5383.28	807.00

TABLE I
COMPUTATIONAL TIMES OF DSHSP

B. *Discrete Fleet Scheduling Problem*

We only describe the offline variant of the Discrete Fleet Scheduling Problem (DFSP), in which a VPP is considered. To model the DFSP we start with the the VPP variant where the production plan is fixed. A discrete model for a fleet is in principle a combination of individual models where constraints on the total production of the fleet are incorporated. However, already for 10 houses the computational times grow for the ILP formulation. On the other hand, combining the DP approach for different houses leads to an exploding state space, which is thus not suitable for solving the problem. This insight leads to the idea to use a precalculated set of possible production patterns (with enough variation among them) for each house. The fleet problem then is to select per house one of the precalculated patterns. These two models are listed below in more detail:

- **n-DSHSP**
  In the n-DSHSP model, the DSHSP models of N different houses are simply combined. To distinct between the

[1] $T = 24$h, $H_j = \frac{100.8}{N_T}$kWh, $\alpha = \frac{1}{8}$, $t_{startup} = \frac{1}{6}$h, $t_{shutdown} = \frac{1}{12}$h, $G_{\max} = \frac{192}{N_T}$kWh, $MR = 0.5$h, $MO = 0.5$h, $BL = 5$kWh, $BC = 10$kWh, p_j=APX(29-10-07)

[2] heat demand changed into $H_j = \frac{50.4}{N_T}$kWh

[3] terminated by solver, current solution/lower bound gap of 0.68%

[4] APX prices changed into p_j=APX(1-9-09)

houses, an index n is added for house n to each variable and parameter ($n = 1, \dots, N$). To achieve the given production plan only the following additional constraint has to be added:

$$\sum_{n=1}^{N} e_{n,j} = P_j \qquad j = 1, \dots, N_T, \tag{37}$$

where $P_j = \int_{(j-1)\frac{T}{N_T}}^{j\frac{T}{N_T}} P(t)dt$.

- **n-DSHSP-restricted**
  In the second model N DSHSP models with a restricted set of (feasible) local production patterns are combined. This restriction is motivated by the desire to reduce the solution space for the decision variables x_j. We only allow locally 'good' production patterns (allowing enough variation) and forget about other production patterns for this house. To introduce this concept we define a set of production patterns C_n for house n. Each pattern $p \in C_n$ is a {0,1} vector of dimension N_T fullfilling all constraints of DSHSP (i.e. p is a feasible solution for the DSHSP problem of house n). In this way, the constraints of the local houses are already incorporated in the sets $C_1, \dots, C_N$, and the only constraint that is left for n-DSHSP-restricted is to match the global predefined production plan $P = (P_1, \dots, P_{N_T})$. Let $pe(p)$ be the vector of generated electricity, corresponding to the production pattern p (note, that $pe(p)$ is independent of the actual house!). To match the production plan P, for each house $n = 1, \dots, N$ a production pattern $p_n \in C_n$ has to be chosen such that $\sum_{n=1}^{N} pe(p_n)_j = P_j$ for each $j \in \{1, \dots, N_T\}$. To summarize, we get the following:

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n-DSHSP-restricted

INSTANCE: Collection of sets $C_1, C_2, \dots, C_N$ of N_T-dimensional binary production patterns $p = (x_1, \dots, x_{N_T})$, a corresponding electricity generation function pe and a target production plan $P = (P_1, \dots, P_{N_T})$.
QUESTION: Is there a selection of production patterns $p_n \in C_n$, such that $\sum_{n=1}^{N} pe(p_n)_j = P_j$ for each $j \in \{1, \dots, N_T\}$?

Theorem 1 *n-DSHSP-restricted is NP-complete in the strong sense*

Proof The problem whether a feasible match exists between the production plan P and the sum of possible electricity production patterns of all houses is proven to be NP-complete in the strong sense by reducing 3-PARTITION to n-DSHSP-restricted. The problem 3-PARTITION, as described by [3], has the following form:

3-PARTITION

INSTANCE: Set A of $3m$ elements, a bound $B \in \mathbb{Z}^+$, and a size $s(a) \in \mathbb{Z}^+$ for each $a \in A$ such that $\frac{B}{4} < s(a) < \frac{B}{2}$ and $\sum_{a \in A} s(a) = mB$.
QUESTION: Can A be partitioned into m disjoint sets $A_1, A_2, \dots, A_m$ such that, for $1 \le i \le m$, $\sum_{a \in A_i} s(a) = B$

The specific instance of n-DSHSP-restricted that corresponds to a general instance of 3-PARTITION is as follows. First, the time horizon consists of $2mB$ time intervals. Next, for each $a \in A$, a cluster C_a is created with $m(B - s(a) + 1)$ production patterns. Each of these patterns has a sequence of $s(a)$ consecutive 1's at locations as can be seen in Figure 2. The dark gray areas correspond to sequences of 1's and light gray areas to sequences of 0's. Note, that in this way only production in the periods $[(2i + 1)B, 2(i + 1)B]$, $i = 0, \dots, m - 1$ is possible by the created houses. If MR is chosen as the smallest element of the 3-PARTITION instance and if the heat demand is such that at the end of the day the microCHP had to run for $s(a)$ time intervals in house a, the production patterns p are feasible for the microCHP model (note that MO is not important, since each pattern contains only one run). Finally, we define $pe(p)_j = E_{\max} p_j$ (meaning that startup and shutdown periods are ignored), and the target production plan by:

$$P_j = \begin{cases} E_{\max} & (2i+1)B < j \le 2(i+1)B \\ & \quad \text{for some } i \in \{0, \dots, m-1\} \\ 0 & \text{otherwise.} \end{cases} \tag{38}$$

For each house a now exactly one planning pattern from C_a must be chosen. Due to the definitions of P_j and pe, these patterns must be chosen such that two patterns never overlap and in all intervals within the m periods $[(2i+1)B, 2(i+1)B]$, $i = 0, \dots, m - 1$ of length B, exactly one pattern has to be active. This comes down to assigning to each interval $[(2i+1)B, 2(i+1)B]$ non overlapping patterns of total length exactly B. Since furthermore for each house exactly one pattern is used in this process, a feasible solution of the n-DSHSP-restricted instance exists if and only if 3-PARTITION has a solution. Thus, the constructed instance of n-DSHSP-restricted corresponds to a general instance of 3-PARTITION. ■

Note, that the used reduction is only pseudo-polynomial in the size of the 3-PARTITION instance. However, since 3-PARTITION is NP-complete in the strong sense, such a pseudo-polynomial reduction is sufficient to prove the mentioned result.

The construction in the proof is limited to only one run per day for each house and the minimum runtime depends on the smallest element a, which does not represent a very realistic example. However, a more realistic but also more complicated example can be constructed that broadens these limitations.

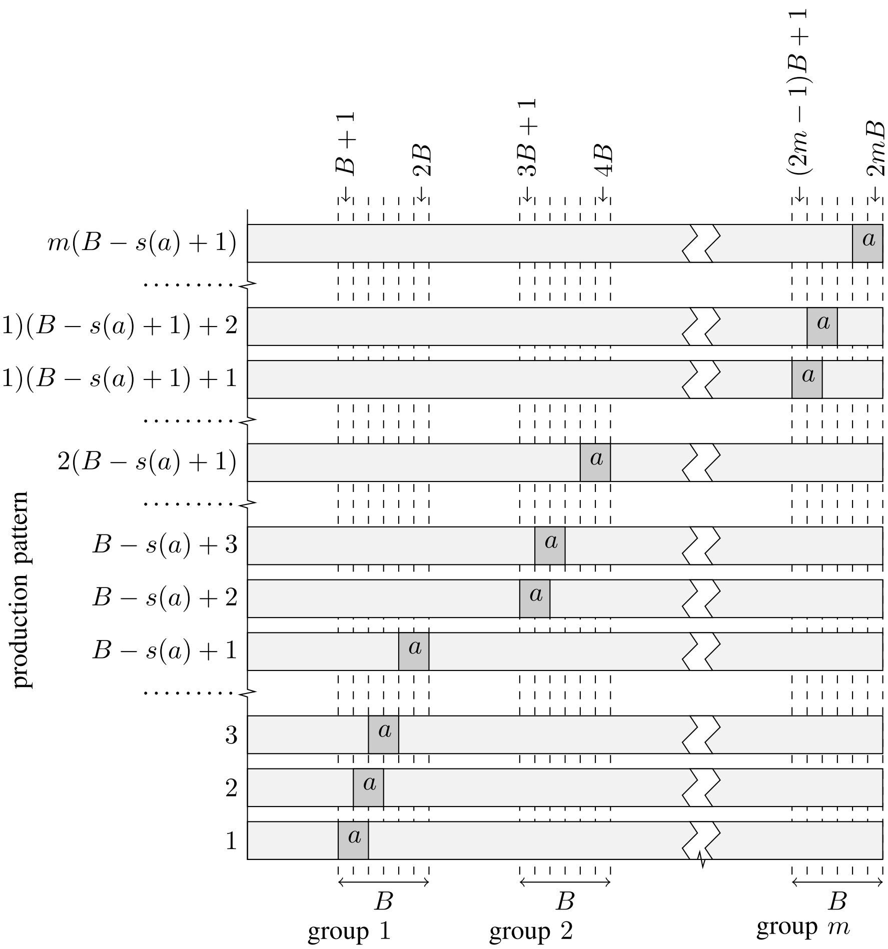

Fig. 2. The cluster C_a, consisting of $m(B-s(a)+1)$ production patterns for the house corresponding to the element a of length $s(a)$.

For this example we use each element a in $B-s(a)+1$ houses; each of them containing $m+1$ production patterns, and in total we use $\sum_{i=1}^{|A|} B-s(a_i)+1$ houses as in Figure 3. Each house has a basic pattern p^b, representing the runs of a normal day within a time horizon of $3(m+1)B$ time intervals. Next to the basic pattern, each house has m variations on this basic pattern, in which this basic pattern is copied and some adjacent production is done, as in Figure 3. We assume that heat demand and buffer level constraints are fullfilled, and that there is enough space left in the heat buffer to run for the additional $s(a)+1$ time intervals for each house, corresponding to element a. Each period $[3iB,(3i+1)B]$, $i=0,\dots,m$ is left idle in all patterns. Production is allowed in $[(3i+1)B,(3i+3)B)]$, $i=0,\dots,m$, where a run of length MR is positioned precisely in front of the runs of length $s(a)$ and the run of length 1. Obviously, these runs fullfill minimum runtime and offtime constraints if $MR=MO\leq B$. In the selection section of length $|A|$ exactly one 1 is added at the same time interval, for each cluster corresponding to the same $a\in A$. The target production plan is defined as: $P_j=\sum_{n=1}^{N} p_n^b+f_j$, where

$$f_j=\begin{cases}1 & (3i+2)B<t\leq(3i+3)B \qquad ,i=0,\dots,m\\ 0 & \text{otherwise.}\end{cases}$$

Due to this definition of P_j and the design of the selection section exactly one variated pattern belonging to a must be chosen from the $m(B-s(a)+1)$ variations based on the element a. Thus, only one of the corresponding $B-s(a)+1$ houses does not select its basic pattern; therefore all elements a are chosen exactly once, and they must fill the m periods of length B in the same way as in the proof.

Both n-DSHSP and n-DSHSP-restricted are feasibility problems: is it possible to coordinate the microCHPs such that the fleet produces a fixed production plan? As an objective we could minimize the number of total runs needed for this matching, like in FSP:

$$\min\sum_{j=1}^{N_T}\sum_{n=1}^{N} start_{n,j}. \tag{39}$$

If we consider the production function space $P(t)^{\pm}$ similar objectives as (15) and (16) can be defined:

$$\max\frac{A}{\sum_{j=1}^{N_T}\sum_{n=1}^{N} start_{n,j}}+B\sum_{j=1}^{N_T}\sum_{n=1}^{N} p_j e_{n,j} \tag{40}$$

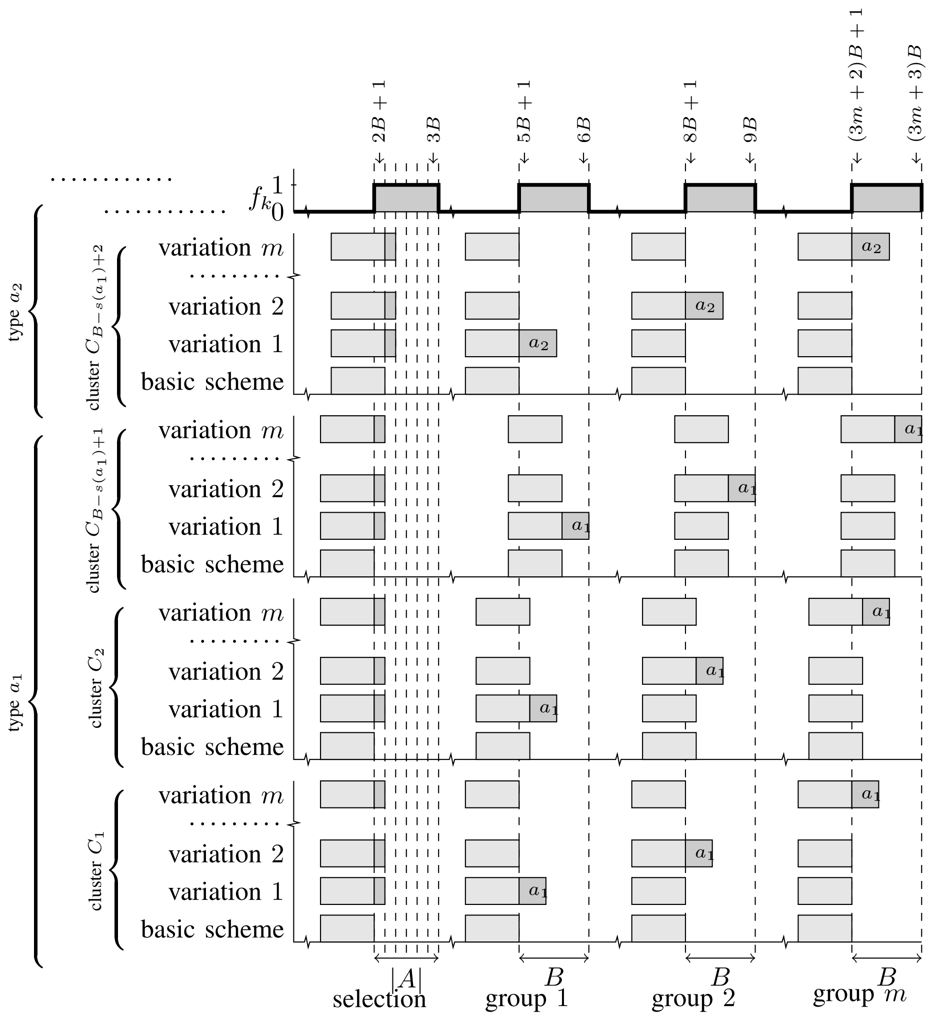

Fig. 3. Production patterns in a more realistic example

and

$$\max \sum_{j=1}^{N_T} \sum_{n=1}^{N} p_j e_{n,j}. \tag{41}$$

V. Conclusions

In this paper continuous and discrete models of the microCHP problem are presented. The discrete variant n-DSHSP-restricted is proven to be NP-complete in the strong sense. This means that a simplified version of the microCHP scheduling problem is already difficult to solve to optimality. To show that the instance that we used in the proof is close to reality, an extended example is given that links better to real world instances. For this example the same conclusion can be drawn regarding complexity.

VI. Recommendations

The complexity results in this paper give a stronger urge to focus on the development of approximation methods for DFSP. The development of fast and scalable heuristics is required, which give a good enough approximation of the optimum. One of these heuristics could use (fast) DP formulations for single houses and combine the individual output in a clever way. Column generation techniques, based on the n-DSHSP-restricted model, could be the basis for another heuristic.

VII. Acknowledgements

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