Decision Algorithm for the Stability of Planar Switching Linear Systems

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Abstract

This paper presents a decision algorithm for the analysis of the stability of a class of planar switched linear systems, modeled by hybrid automata. The dynamics in each location of the hybrid automaton is assumed to be linear and asymptotically stable; the guards on the transitions are hyper planes in the state space. We show that for every pair of an ingoing and an outgoing transition related to a location, the exact gain in the norm of the vector induced by the dynamics in that location can be computed. These exact gains are used in defining a gain automaton which forms the basis of an algorithmic criterion to determine if a planar hybrid automaton is stable or not.

1 Introduction

A hybrid automaton [1, 4] is an automaton with locations and transitions between the locations, together with continuous dynamics in the locations, usually described by differential equations, and constraints on both locations and transitions.

The analysis of the stability of a hybrid system is an important and interesting problem. Even in the case of switched linear systems with asymptotically stable dynamics in each location, it is possible that the switching regime gives raise to a global behavior of the system that is unstable (see e.g. [2]). For an overview of results on hybrid stability see [3, 7, 9]. In this paper we present a decision algorithm for the stability of planar switching linear systems. By restricting ourselves to the planar

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case, we show that the transition gains can be computed exactly by transforming the dynamics to its real Jordan form instead of using a quadratic Lyapunov function to provide an upper estimate of the gain.

We characterize a cycle of the automaton as being (strictly) contractive if the product of the associated gains is (strictly) less than 1, (strictly) expanding if it is larger than one. The absence of (non-strict) non-contractive cycles is a sufficient condition for the (asymptotic) stability of the hybrid automaton. The presence of expanding cycles is a sufficient condition for the instability of the hybrid automaton. In the planar case, the absence of non-contractive (or expanding) cycles is a *necessary and sufficient* condition for the stability of the hybrid automaton.

2 Planar LCH

A planar Linear Continuous Hyper plane [6] is a hybrid automaton [1, 4] such that the dynamics are linear in $\mathbb{R}^2$, the invariants of the locations are always true, the guards of the transitions are lines through the origin, and there are no resets associated with a transition.

2.1 Assumption
Throughout the paper we assume that the linear systems in the locations are of the form $\frac{d}{dt}x = A_\ell x$, with A_ℓ Hurwitz.

3 Stability

3.1 Definition (stability)
An LCH hybrid automaton is *stable* if and only if $\forall\epsilon > 0\ \exists\delta > 0 : \left|\left|x^0\right|\right| < \delta$: for all hybrid traces $x_1e_1x_2e_2\ldots$ with $x_1(0) = x^0$ and $\forall i \forall t \in [\tau_{i-1}, \tau_i] : ||x_i(t)|| < \epsilon$. An automaton that is not stable is called *unstable*.

3.2 Definition (asymptotic stability)
An LCH hybrid automaton is *asymptotically stable* if and only if it is stable and for any infinite hybrid trace $x_1e_1x_2e_2\ldots$ we have $\lim\limits_{i\to\infty} ||x_i|| = 0$.

3.1 Gains

Suppose a location ℓ is entered via a transition a with a state vector x_a and is left via a transition b with a state vector x_b. An indication as to how the location contributes to the stability or instability is the ratio of the norm of the outbound state and the inbound state. A ratio below one is in favor of stability whereas a ratio above one points at instability.

Since the actual ratio depends on the trace and the state trajectory (and in particular on the dwell time in a location) we consider the *maximal gain* that only depends on the pair of inbound and outbound transitions of a given location.

3.3 Definition (Maximal Gain)
The maximal gain corresponding to entering location ℓ via transition a and leaving it via transition b is denoted by $\gamma_{ab} \in \mathbb{R}^+$. It is defined as follows. For any solution $\mathbf{x}(t)$ of $\frac{d}{dt}\mathbf{x} = A_\ell \mathbf{x}$ with $\mathbf{v}_a^{\mathrm{T}}\mathbf{x}(0) = 0$ and $x(0) \neq 0$:

- $\gamma_{ab} = \perp$ if $\nexists t \geq 0$ s.t. $v_b^{\mathrm{T}} x(t) = 0$
- If t^* is the smallest $t > 0$ such that $v_b^{\mathrm{T}} x(t^*) = 0$, then $\gamma_{ab} = \frac{||x(t^*)||}{||x(0)||}$.

A maximal gain equal to $\perp$ means that location ℓ will never be left via b. A gain strictly greater than 0 means that the location can be left via b and that the corresponding gain in the norms of the vectors will be γ_{ab} in the worst case. The existence of the maximum is ensured because we consider stable locations and linear dynamics. It is easy to see that for planar systems the maximal gain is attained when the system leaves the location at the first possible occasion.

In the following, we assume that we know a lower and an upper bound of the maximal gain, i.e. $\alpha_{ab}, \beta_{ab} \in \mathbb{R}^+$ such that $0 \leq \beta_{ab} \leq \gamma_{ab} \leq \alpha_{ab}$.

3.2 Cycles

Of course, the stability properties of a hybrid automaton are completely determined by the stability of its cycles through the locations.

3.4 Definition
Let H be a hybrid automaton, then a *i(strictly) contractive cycle* of H is a sequence of transitions $C = e_1 e_2 \ldots e_m$ such that each e_i is a transition from ℓ_i to ℓ_{i+1}, with $\ell_1 = \ell_{m+1}$, and $\alpha_C = \alpha_{e_1 e_2} \cdot \alpha_{e_2 e_3} \cdot \ldots \cdot \alpha_{e_m e_1} \leq 1 (< 1)$. The scalar α_C is called the upper estimate of the cycle gain.

Theorem 3.5 provides a sufficient condition for the (asymptotic) stability of an LCH hybrid system based on the absence of (non-strict) non-contractive cycles.

3.5 Theorem
Let H be an LCH hybrid automaton with Hurwitz locations. If all cycles in H are (strictly) contractive then H is (asymptotically) stable.

3.6 Definition
Let H be a hybrid automaton, then an *(strictly) expanding cycle* of H is a sequence of transitions $C = e_1 e_2 \ldots e_m$ such that each e_i is a transition from ℓ_i to ℓ_{i+1}, with $\ell_1 = \ell_{m+1}$, and $\beta_C = \beta_{e_1 e_2} \cdot \beta_{e_2 e_3} \cdot \ldots \cdot \beta_{e_m e_1} \geq 1 (> 1)$. The scalar β_C is called the lower estimate of the cycle gain.

Theorem 3.7 provides a sufficient condition for the instability of an LCH hybrid system based on the detection of (strict) expanding cycles.

3.7 Theorem
Let H be an LCH hybrid automaton with Hurwitz locations. If H has a strict expanding cycle then H is unstable. If H has an expanding cycle then H is unstable.

4 Interval gain automata and cycle analysis

Theorem 3.5 provides us with a sufficient condition for stability, namely the absence of non-contractive cycles, and Theorem 3.7 with a sufficient condition for instability, namely the presence of expanding cycles. In order to check for non-contractive or expanding cycles we first transform a hybrid automaton into what we call *gain automaton*.

4.1 Definition
An *interval gain automaton* is a tuple $GA = (S, S^0, G)$ where

- S is the set of nodes,
- S^0 is the set of initial nodes,
- $G \subseteq S \times (\mathbb{R}^+ \times \mathbb{R}^+) \times S$ is the set of edges labeled with intervals of gains.

4.2 Definition
Let H be a planar LCH, then the gain automaton for H is defined by $GA(H) = (S_H, S_H^0, G_H)$ where

- The nodes of the gain automaton are the transitions of H, i.e. $S_H = E$.
- The initial nodes S_H^0 are the transitions from an initial location of H.
- For each pair of adjacent transitions a and b in H such that $\xrightarrow{a} l \xrightarrow{b}$ and $\alpha_{ab} \neq \perp$ there is an edge $a \xrightarrow{\beta_{ab},\alpha_{ab}} b$ in G_H.

It must be noted that there is an edge in the interval gain automaton only if the maximal gain corresponding to the pair of transitions in H is well defined, that is, not equal to $\perp$.

We present an algorithm on the gain automaton of a hybrid automaton for the detection of non-contractive and expanding cycles. This algorithm is inspired by the well-known algorithm for transforming an automaton into an equivalent regular expression (see e.g.[5, 8]). It works by successively deleting nodes of the gain automaton, while transforming the edges. The basic steps of the algorithm are:

- **Node elimination:** a node is eliminated, as illustrated in Figure 1(a). Each possible pair of an incoming and outgoing edge of this node leads to a new edge, labeled with the product of the interval gains defined as $(\beta_1, \alpha_1) \otimes (\beta_2, \alpha_2) = (\beta_1\beta_2, \alpha_1\alpha_2)$.
- **Double edge elimination:** As illustrated in Figure 1(c), if two edges have the same initial and final node they are transformed into a single edge, labeled with the union of the interval gains defined as $(\beta_1, \alpha_1) \oplus (\beta_2, \alpha_2) = (\min(\beta_1, \beta_2), \max(\alpha_1\alpha_2))$.

- **Loop edge analysis:** it is possible that deleting a node creates a loop edge, as illustrated in Figure 1(b). The algorithm analyzes the gain of a loop edge and then removes it. If the lower bound of the gain of such a loop edge is > 1 (i.e. an *expanding* loop edge) then the algorithm terminates and the system is unstable. If the upper bound of the gain of such a loop edge is > 1 (i.e. a *non-contractive* loop edge) then the algorithm marks the system as non stable.

The algorithm terminates when an expanding cycle is detected, and the system is unstable, or when all nodes have been removed, in which case the system is stable if no non-contractive cycle has been detected.

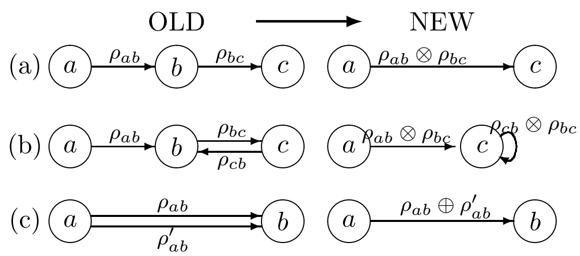

Figure 1. *Basic steps of the algorithm*

4.3 Theorem
Let H be an LCH hybrid automaton with Hurwitz locations. If algorithm detects a non-contractive (resp. expanding) loop edge in $GA(H)$ then H contains a non-contractive (resp. expanding) cycle.

The number of nodes in $GA(H)$ is quadratic in the number of nodes of H, and the complexity of the algorithm is linear in the number of nodes of GA, so the complexity of the algorithm is quadratic in the number of nodes of H. This means we have a computationally efficient way of checking the sufficiency condition for stability and instability.

5 Exact gain computation for planar systems

In this section we show that for planar LCH systems, the exact maximal gain for any pair of incoming and outgoing transitions can be obtained by computing the real Jordan form of the dynamics matrix. A case by case analysis of the different types of Jordan form shows that there is an analytic solution to the problem of finding, for any incoming state, the outgoing state with maximal gain, which corresponds in the planar case to leaving the location at the first possible occasion.

With the exact computation of maximal gains we obtain a necessary and sufficient condition for the stability of planar LCH from Theorems 3.5 and 3.7. The interval gain automaton of a planar LCH is such that the lower and upper bounds in every edge are equal to the corresponding maximal gain. In this case, the algorithm becomes a decision procedure for the stability (or instability) of the system.

Gain in real Jordan form

The real Jordan form for a 2×2 matrix is of one of the following forms:

$$\text{(a)} \begin{pmatrix} \lambda_0 & 0 \\ 0 & \lambda_1 \end{pmatrix} \qquad \text{(b)} \begin{pmatrix} \lambda_0 & 1 \\ 0 & \lambda_0 \end{pmatrix}$$

$$\text{(c)} \begin{pmatrix} \lambda_0 & 0 \\ 0 & \lambda_0 \end{pmatrix} \qquad \text{(d)} \begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix}$$

where λ_0, λ_1, α and β are real. The different types of Jordan forms correspond to the different possibilities of eigenvalues and eigenvectors. Case (a) corresponds to the case of 2 different real eigenvalues, case(b) to the case of 2 equal real eigenvalues, but one eigenvector, and case (d) is for 2 different complex eigenvalues. Case (c) is the case of 2 equal real eigenvalues, and two eigenvectors; it is not really interesting and trivial to deal with.

For every location ℓ with an incoming transition a and an outgoing transition b determined by vectors v_a and v_b respectively, we first compute the eigenvalues and eigenvectors of A_ℓ to determine its real Jordan form J_ℓ such that $A_\ell = MJ_\ell M^{-1}$, M being the matrix of the change of basis from J_ℓ to A_ℓ.

We show how to compute the maximal gain γ'_{ab} for a matrix in real Jordan form. For that, we assume that the guard of a is given by $a_1 y = a_2 x$ and the guard of b is $b_1 y = b_2 x$ in the basis of J_ℓ. To determine the maximal gain, we need to find if and when the solution of the system $\dot{\mathbf{x}} = J\mathbf{x}$, with $\mathbf{x}(0) = \left(\begin{smallmatrix} x_0 \\ y_0 \end{smallmatrix}\right) \neq \mathbf{0}$ an initial state in the incoming line (i.e. $a_1 y_0 = a_2 x_0$), intersects the switching line $b_1 y = b_2 x$.

Case (a): 2 different real eigenvalues

Let $J = \left(\begin{smallmatrix} \lambda_0 & 0 \\ 0 & \lambda_1 \end{smallmatrix}\right)$ be a stable matrix in diagonal form with $\lambda_0, \lambda_1 < 0$. The trajectory is given by

$$x(t) = x_0 e^{\lambda_0 t} \qquad y(t) = y_0 e^{\lambda_1 t}$$

We can determine exactly if and when the trajectory intersects the outgoing switching line $b_1 y = b_2 x$. It is easy to see that there is no intersection if any of a_1, a_2, b_1 or b_2 is equal to 0. Otherwise, let $a = a_2/a_1$ and $b = b_2/b_1$. Notice that the trajectory is caught in the location if $ab < 0$ as the trajectory would otherwise have to cross a coordinate axis containing an eigenvector. The intersection of the trajectory with the guard happens when $y_0 e^{\lambda_1 t} = b x_0 e^{\lambda_0 t}$, that is, when $a e^{\lambda_1 t} = b e^{\lambda_0 t}$. So the intersection happens at

$$t^* = \frac{\log(b/a)}{\lambda_1 - \lambda_0} \quad \text{iff } t^* > 0.$$

The exact maximal gain is then given by $\perp$ if $t^* < 0$, $a = 0$ or $b = 0$, otherwise $\gamma'_{ab} = \frac{||\mathbf{x}(t^*)||}{||\mathbf{x}(0)||}$ which, after simplification, yields

$$\gamma'_{ab} = (|\frac{a}{b}|)^{\frac{\lambda_0}{\lambda_0 - \lambda_1}} \sqrt{\frac{1+b^2}{1+a^2}} \qquad (1)$$

Case (b): 2 equal real eigenvalues, 1 eigenvector

Let $J = \left(\begin{smallmatrix} \lambda_0 & 1 \\ 0 & \lambda_0 \end{smallmatrix}\right)$ be a stable matrix with $\lambda_0 < 0$. The solution of this system is

$$x(t) = (1 + at)x_0 e^{\lambda_0 t} \qquad\qquad y(t) = ax_0 e^{\lambda_0 t}$$

If $a = 0$ or $b = 0$ there is no intersection of the trajectory with $y = bx$. Otherwise, the trajectory intersects the switching line when $x(t) = (\frac{1}{a} + t)y(t) = \frac{1}{b}y(t)$, that is for

$$t^* = \frac{1}{b} - \frac{1}{a} \quad \text{iff } t^* > 0.$$

The exact maximal gain is then $\perp$ if $t^* < 0$, $a = 0$ or $b = 0$, otherwise $\gamma'_{ab} = \frac{||\mathbf{x}(t^*)||}{||\mathbf{x}(0)||}$ yields, after simplification,

$$\gamma'_{ab} = \frac{a}{b} e^{\lambda_0 \frac{a-b}{ab}} \sqrt{\frac{1+b^2}{1+a^2}} \tag{2}$$

Case (d): 2 different complex eigenvalues

Let $J = \left(\begin{smallmatrix} \alpha & -\beta \\ \beta & \alpha \end{smallmatrix}\right)$ be a stable matrix with $\alpha \neq 0$. Using polar coordinates (r, θ) with $r = \sqrt{x^2 + y^2}$ and $\tan(\theta) = y/x$, it can be shown that the trajectories must satisfy $\dot{r} = \alpha r$ and $\dot{\theta} = \beta$ and the solution is

$$r(t) = r_0 e^{\alpha t} \qquad\qquad \theta(t) = \beta t + \theta_a$$

where $r_0 = \sqrt{x_0^2 + y_0^2}$ and $\theta_a = \text{atan2}(a, 1)$.

We denote by $\theta_b = \text{atan2}(b, 1)$ the angle of the switching line $y = bx$. The first possible switching occurs when $\theta(t) = \theta^*$ such that, if the trajectory is anti-clockwise (i.e. $\beta > 0$),

$$\theta^* = \begin{cases} \theta_b & \text{if } \theta_b - \pi < \theta_a < \theta_b \\ \theta_b + \pi & \text{if } \theta_b < \theta_a < \theta_b + \pi \\ \theta_b + 2\pi & \text{if } \theta_b + \pi < \theta_a < \theta_b + 2\pi \end{cases}.$$

and if the trajectory is clockwise (i.e. $\beta < 0$),

$$\theta^* = \begin{cases} \theta_b - \pi & \text{if } \theta_b - \pi < \theta_a < \theta_b \\ \theta_b & \text{if } \theta_b < \theta_a < \theta_b + \pi \\ \theta_b + \pi & \text{if } \theta_b + \pi < \theta_a < \theta_b + 2\pi \end{cases}.$$

This intersection will *always* happen for

$$t^* = \frac{\theta^* - \theta_a}{\beta}$$

and the exact maximal gain is then $\gamma'_{ab} = \frac{r(t^*)}{r_0} = e^{\alpha t^*}$ so

$$\gamma'_{ab} == e^{\frac{\alpha}{\beta}(\theta^* - \theta_a)}. \tag{3}$$

Gain from a change of basis

For planar systems we can associate a constant gain to a change of basis. Let $\mathbf{i}_a$ be a unit vector in the incoming line and $\mathbf{i}_b$ a unit vector in the outgoing line ($||\mathbf{i}_a|| = ||\mathbf{b}_b|| = 1$). Let M be a non-singular matrix representing a change of basis and $\mathbf{y} = M\mathbf{x} = \lambda M\mathbf{i}_a$ the image of an incoming vector $\mathbf{x}$ by M. Then,

$$\left(\frac{||\mathbf{y}||}{||\mathbf{x}||}\right)^2 = \frac{\lambda^2 \mathbf{i}_a^T.M^T M.\mathbf{i}_a}{\lambda^2 \mathbf{i}_a^T.\mathbf{i}_a} = \mathbf{i}_a^T.M^T M.\mathbf{i}_a$$

We define then the incoming gain γ_a^M and the outgoing gain γ_b^M for the change of basis M as:

$$\gamma_a^M = \left(\mathbf{i}_a^T.M^T M.\mathbf{i}_a\right)^{\frac{1}{2}} \qquad\qquad \gamma_b^M = \left(\mathbf{i}_b^T.M^T M.\mathbf{i}_b\right)^{-\frac{1}{2}}$$

Gain in original basis

The gain in the basis of the original matrix is then the product of the gain in the basis of the Jordan form times the gain for the change of basis:

$$\gamma_{ab} = \gamma_a^M \gamma'_{ab} \gamma_b^M.$$

6 Continuity and robustness

Although the calculation of the gains for the planar case, we distinguish several cases, depending on the location of the eigenvalues, the gains depend continuously on the matrices in the locations.

6.1 THEOREM
The gain as defined in Definition 3.3 depends analytically on matrix A and the switching lines.

A direct consequence of the above theorem is that stability of is robust with respect to small perturbations in the dynamics in the location as well in the switching lines.

7 Example

We illustrate the computation of the exact maximal gain with the simple planar LCH hybrid system of figure 2. The system has two locations ℓ_1 and ℓ_2, a transition a from ℓ_1 to ℓ_2, and a transition b from ℓ_2 back to ℓ_1. The dynamics are given by matrix A_1 and A_2 respectively with

$$A_1 = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \quad A_2 = \begin{pmatrix} -0.2 & 0.4 \\ -7.5 & -0.1 \end{pmatrix}$$

and the guards are orthogonal to $\mathbf{v}_a = \left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right)$ and $\mathbf{v}_b = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right)$.

We compute the exact gain of the cycle with the method of Section 5 and show that the system is (asymptotically) stable, and that stability cannot be determined using optimal QLF.

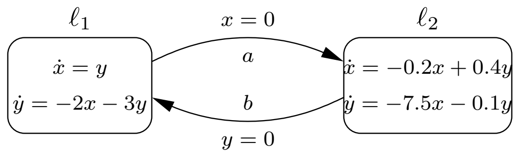

Figure 2. *A simple planar LHC.*

7.1 Gain in ℓ_1

We consider location ℓ_1 with incoming transition a with guard $x = 0$, and outgoing transition b with guard $y = 0$, and dynamics given by $\dot{\mathbf{x}} = A_1\mathbf{x}$. The eigenvalues of A_1 are $\lambda_0 = -1$ and $\lambda_1 = -2$. Matrix A_1 can be put in diagonal form J with the change of basis M corresponding to the eigenvectors, such that $A_1 = MJM^{-1}$ where

$$J = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \qquad M = \begin{pmatrix} 0.707 & -0.447 \\ -0.707 & 0.894 \end{pmatrix}.$$

The gains from the change of basis M^{-1} are computed for the unit vectors $\mathbf{i}_a = \binom{0}{1}$ and $\mathbf{i}_b = \binom{1}{0}$. We obtain $\gamma'_a = \sqrt{7} = 2.645$ and $\gamma'_b = 1/\sqrt{13} = 0.277$.

The gain for the diagonal matrix J is computed using equation 1. In this case we have

$$M^{-1}\binom{0}{1} = \binom{\sqrt{2}}{\sqrt{5}}$$

so $a = \sqrt{5/2}$, and

$$M^{-1}\binom{0}{1} = \binom{2\sqrt{2}}{\sqrt{5}}$$

so $b = \frac{1}{2}\sqrt{5/2}$. Therefore, the gain for J is $\gamma'_{ab} = \frac{1}{4}\sqrt{\frac{13}{7}}$.

The maximal gain in the original basis is

$$\gamma_{ab} = \gamma'_a \gamma'_{ab} \gamma'_b = \frac{1}{4}.$$

On the other hand, the upper bound on the maximal gain obtained with the optimal QLF as in [6] is $\rho_{ab} = 1/\sqrt{12.68} = 0.28$.

7.2 Gain in ℓ_2

We consider location ℓ_2 with incoming transition b with guard $y = 0$, and outgoing transition c with guard $x = 0$, and dynamics given by $\dot{\mathbf{x}} = A_2\mathbf{x}$. The conjugate complex eigenvalues of A_2 are $\lambda_0 = -0.15 + 1.731j$ and $\lambda_1 = -0.15 - 1.731j$. Matrix A_2 is similar to matrix J with a change of basis M such that $A_2 = MJM^{-1}$ where

$$J = \begin{pmatrix} -0.15 & -1.731 \\ 1.731 & -0.15 \end{pmatrix} \quad M = \begin{pmatrix} -0.35 & -1.731 \\ -7.5 & 0 \end{pmatrix}.$$

The gains from the change of basis M^{-1} are computed for the unit vectors $\mathbf{i}_b = \binom{1}{0}$ and $\mathbf{i}_a = \binom{0}{1}$. We obtain $\gamma'_b = 0.577$ and $\gamma'_a = 7.497$.

The gain for matrix J is computed using equation 3. In this case we have

$$M^{-1}\binom{1}{0} = \binom{0}{-0.577}$$

so $\theta_b = -\pi/2$, and

$$M^{-1}\binom{0}{-1} = \binom{0.133}{-0.004}$$

so $\theta_a = -0.029$. Since $\beta > 0$ and $\theta_a - \pi < \theta_b < \theta_a$, then $\theta^* = \theta_a$. Therefore, the gain for J is $\gamma'_{ba} = e^{\frac{\alpha}{\beta}(\theta_a - \theta_b)} = 0.875$.

The maximal gain for A is

$$\gamma_{ba} = \gamma'_b \gamma'_{ba} \gamma'_a = 3.78.$$

On the other hand, the upper bound on the maximal gain obtained with the optimal QLF as in [6] is $\rho_{ba} = \sqrt{15.77} = 3.97$.

7.3 Stability

We conclude that the system is (asymptotically) stable because the exact maximal gain of the cycle is $\gamma_{ab}\gamma_{ba} < 1$. On the other hand, using the upper bounds obtained with optimal QLF we obtain $\rho_{ab}\rho_{ba} > 1$ and therefore we cannot conclude on the stability of the system. It is not difficult to show that there does not exist a common QLF for the two locations.

8 Conclusions

We have derived a *necessary and sufficient* condition for the *stability* of a planar LCH hybrid automaton, namely the absence of expanding cycles (i.e. all cycles are contractive), together with an algorithm for efficiently checking this condition. We have made use of both systems theoretic concepts (in calculating the estimated gains) and computer science concepts (in checking the cycles in the gain automaton), thereby doing justice to both the continuous and the discrete aspects of hybrid systems. The choice for planar systems is, admitted, restrictive. However, by restricting to the planar case we are able to derive necessary and sufficient conditions rather than conservative sufficient conditions only.

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