



OPTIMIZATION OF THE RECONSTRUCTION AND ANTI-ALIASING FILTER IN A WIENER FILTER SYSTEM

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ABSTRACT

This paper discusses the influence of the reconstruction and anti-aliasing filters on the performance of a digital implementation of a Wiener filter for active noise control. The overall impact will be studied in combination with a multi-rate system approach. A reconstruction and anti-aliasing filter will be selected and its parameters will be varied to optimize the system level performance of the Wiener filter for a feedback controller based on an internal model control principle. It will be shown that the selection of the reconstruction and anti-aliasing filter is an important decision that can largely influence the overall system performance. This method can be used in combination with standard optimization algorithms to automatically find the optimal filters that will give the largest reduction for the overall system.

1 INTRODUCTION

In this paper the influence of the anti-aliasing and reconstruction filter will be studied. In most books like for instance [1] simple rules will be used. These rules are based on empirical data but do not seem to be directly applicable to active noise control. The optimal Wiener filter can be calculated if the transfer function between actuator and sensor is known. This transfer function will also include the influence from the anti-aliasing and reconstruction filters. A consequence from this is that the optimal Wiener filter needs to be recalculated when the anti-aliasing or reconstruction filter changes. The model formulated will be based on a system with a white Gaussian-noise input signal. This will result in a formulation in which only the impulse responses of the different parts are needed to quickly estimate the overall system performance. In this report a feedback control structure will be studied, for which group-delay is critical.

The method proposed is for a multi-rate system. The advantage of using a multi-rate system has already been shown in [2]. The impact of tuning the reconstruction and anti-aliasing filters will be shown. The assumption is that the analogue filters are already in place and only the digital interpolation and decimation filters are still tuneable.

Several criteria can be used to design the interpolation and decimation filters. In this report only the cut-off frequency, the transition width and the order of the filter will be tuneable.

Finally the usefulness will be shown by means of a number of simulations. It will be shown that the algorithm is simple and able to find a good compromise between the cut-off frequency, order and overall system performance.

2 THEORY

2.1 The Wiener filter

The Wiener filter will be derived as a FIR filter that minimizes the mean-square error (see also [1]). The system studied is a simple single-input single-output system. The block diagram for a Wiener filter system can be found in Figure 1.

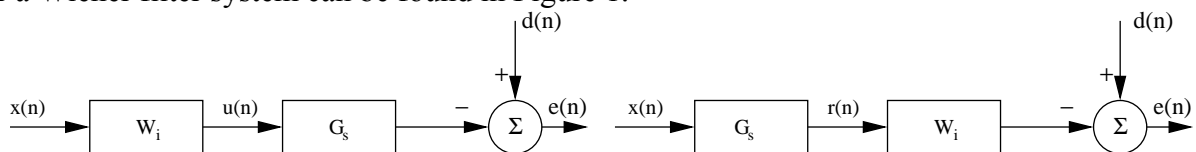


Figure 1 Wiener filter Model.

The systems studied in this paper are linear and time invariant. Therefore it is allowed to interchange the Wiener filter and the plant. The noise $x(n)$ is now filtered by the transfer function of the plant (left). The Wiener filter minimizes the error signal $e(n)$, which can be written as:

$$e(n) = d(n) + \mathbf{w}^T \mathbf{r}(n) = d(n) + \mathbf{r}^T(n) \mathbf{w}, \quad (1)$$

where

$$\mathbf{w} = [w_0 \quad w_1 \quad \cdots \quad w_{I-1}]^T, \quad (2)$$

$$\mathbf{r}(n) = [r(n) \quad r(n-1) \quad \cdots \quad r(n-I+1)]^T. \quad (3)$$

The idea is to find the values for the filter coefficients \mathbf{w} that minimize the quadratic cost criterion

$$J = E[e^2(n)] \tag{4}$$

in which E is the expectation operation. Taking the derivative of this and setting it equal to zero results in the following equation:

$$\frac{\partial J}{\partial \mathbf{w}} = 2[\mathbf{A}\mathbf{w} - \mathbf{b}] = 0. \tag{5}$$

The solution for $\mathbf{w} = \mathbf{w}_{opt}$ is obtained from :

$$\mathbf{w}_{opt} = \mathbf{A}^{-1}\mathbf{b}, \tag{6}$$

with

$$\mathbf{A} = E[\mathbf{r}(n)\mathbf{r}^T(n)], \tag{7}$$

$$\mathbf{b} = E[\mathbf{r}(n)d(n)]. \tag{8}$$

Equation (6) is valid when $r(n)$ is persistently exciting, resulting in a non singular \mathbf{A} . Finding the inverse of the \mathbf{A} matrix is not easy due to its size. However the \mathbf{A} matrix is in the Toeplitz-form and can be solved efficiently by exploiting the symmetry of it.

2.2 The controller

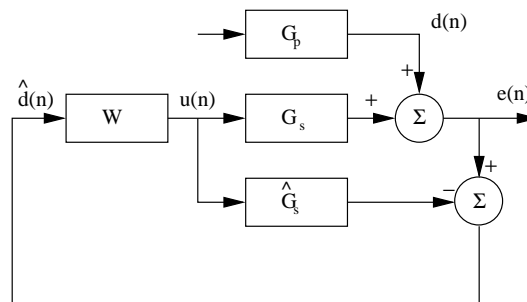


Figure 2 The feedback controller with IMC.

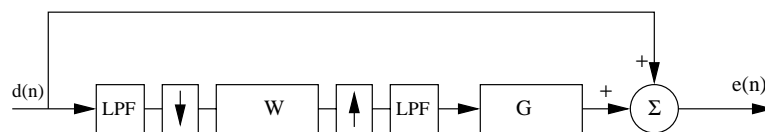


Figure 3 Multi-rate feedforward controller (feedback+IMC).

The derivation of the feedback controller starts with the normal feedforward controller. In a feedforward controller the following paths can be distinguished: the primary path, being the path from the source or reference to the error sensor and the secondary path, being the path from the actuator to the error sensor. In most feedback controllers there is also a path from the actuator to the source or reference sensor. In this article perfect cancellation from the actuator to the reference sensor is assumed by means of internal model control. The influences of this path are then subtracted from the input of the controller. In [1] it has been shown that the design methods for a feedforward controller can be used for a feedback controller under certain assumptions. The method proposed in this book is based on so-called internal model control for feedback controllers. The feedback controller with IMC can be found in Figure 2.

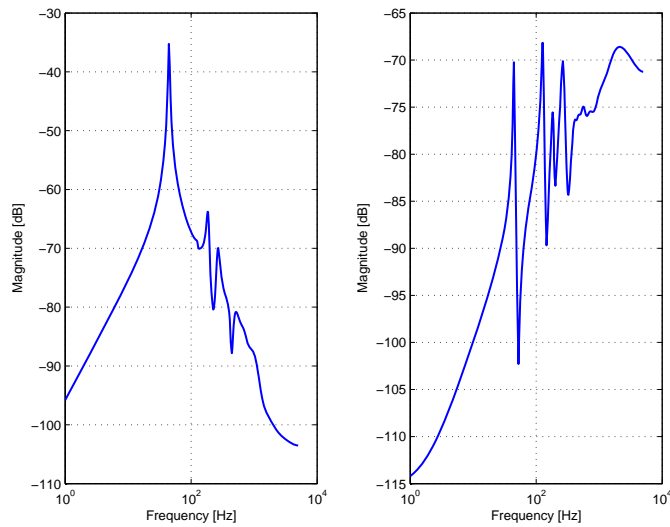


Figure 4 Plant, primary path (left) and secondary path (right).

If $G_s = \hat{G}_s$ then the model in Figure 2 is equivalent to a feedforward controller. The next extension is to add a multi-rate signal approach, in which the controller works on a lower sample rate than the actual system.

The multi-rate signal approach can be found in Figure 3. In this case the signals $x(n)$ and $d(n)$ are equal and the Wiener filter can be designed by observing both signals. The assumption is that both low-pass filters are digital filters. In this paper they will be simple FIR filters designed with the Matlab filter-design toolbox. These filters are based on the minmax filter design method. The filters are minimum-phase filters. The pass-band is set to 1, the stop-band to zero and the transition width and order are still tuneable.

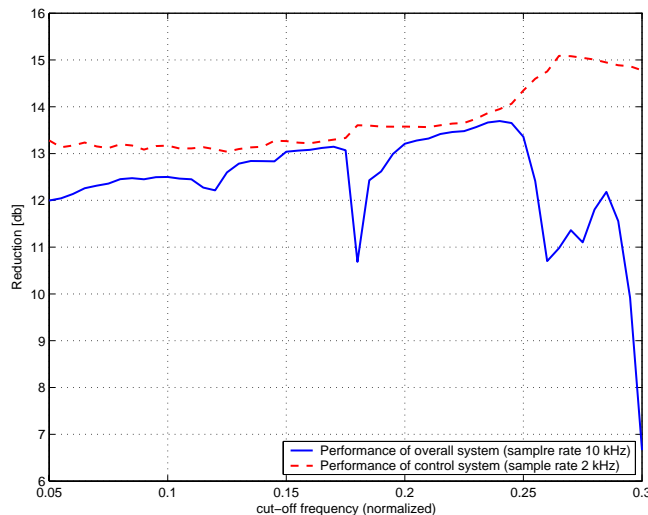


Figure 5 Performance of the overall system as function of the cut-off frequency.

3 SIMULATION RESULTS

The question is whether modifying the cut-off frequency of the interpolator and decimator filter is a good optimization criterion. The plant model used for this simulation is based on a

so called smart-panel. This panel consists of a sandwich panel, a piezoelectric patch actuator, and an error sensor measuring the local velocity of the panel. The model used is a state-space representation of the models described in [3]. The primary and secondary path of the plant can be found in Figure 4. The setup consists of a discrete model of the primary and secondary path of the smart-panel running at 10 kHz. The sample rate is brought down to 2 kHz by performing a decimation and interpolation operation with a factor of 5. The performance is evaluated at 2 kHz and 10 kHz. The reduction is plotted as function of the cut-off frequency and can be found in Figure 5. In this plot, the transition width is set to a normalized frequency of 0.1 in which normalization carried out with respect to the sampling frequency. Two scenarios have been plotted. The first scenario only the reduction of the system in the low-sample rate domain (2 kHz) is plotted, using a controller-internal estimate of the error signal. In the second scenario the same system is evaluated in the high sample rate domain (10 kHz), i.e. the overall system. From this figure it can be concluded that the cut-off frequency influences the overall performance. There exists an optimal value for which the reduction is maximal. However there is also a value for the cut-off frequency for which the system performance degrades substantially.

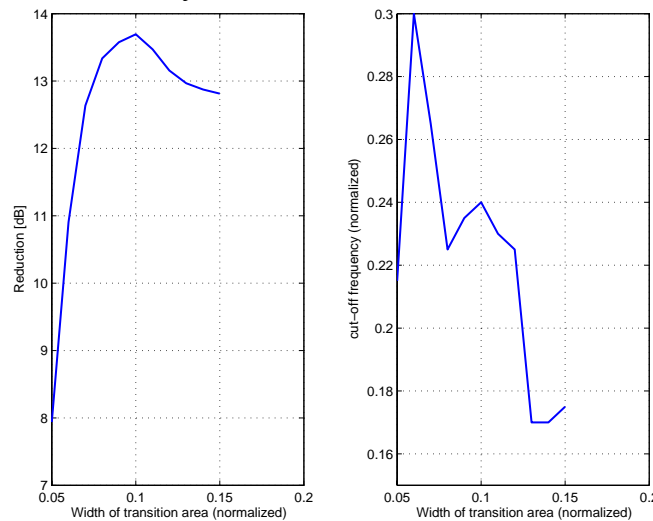


Figure 6 Influence of transition width on overall performance (left) and the cut-off frequency belonging to a certain transition width (right).

The next question that arises is if the transition width of the FIR filter will influence the overall performance of the system. In an optimization loop the optimal cut-off frequency will be found for each transition width of the decimation and interpolation filters. The results are plotted in two figures. In the first figure the reduction is plotted as a function of the transition width. In the second plot the transition width is plotted as function of the cut-off frequency. Both plots are necessary to estimate the optimal cut-off and transition width figures. The results can be found in Figure 6.

In the last simulation the impact of the order of the filter was studied. The idea is to change the order of the filter and to calculate the optimal cut-off frequency. The transition width is set to 0.1 (normalized). The result from this experiment can be found in Figure 7. It seems that if the order is too high then it will decrease the overall performance again. On the other hand, if the filter is too short it will also degrade the performance again.

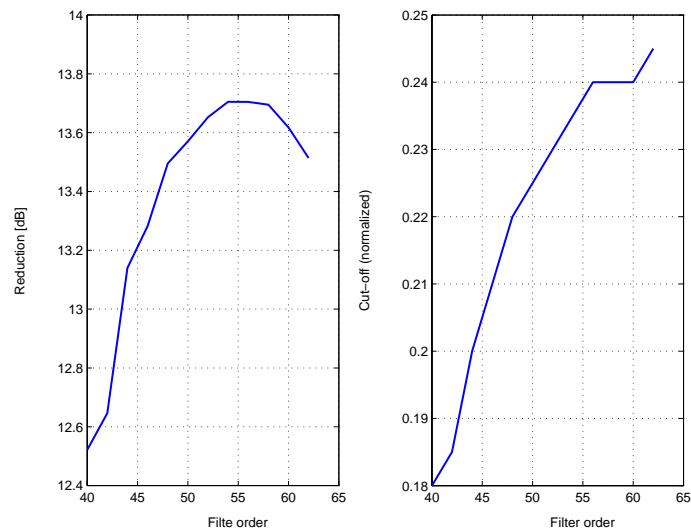


Figure 7 Impact of the filter order on the reduction (left) and on the cut-off frequency (right).

4 CONCLUSIONS

In this paper a method has been proposed in which the cut-off frequency, the transition width and the order of a decimation and interpolation filter can be optimized. The proposed method implements a multi-dimensional optimization procedure by tuning only one parameter at a time, thereby simplifying the optimization process considerably. If the cut-off frequency is too high then reconstruction and anti-aliasing errors will degrade the performance. However if the cut-off frequency is too low then the group-delay will degrade the performance.

REFERENCE

- [1] Stephen Elliott, *Signal Processing for Active Control*, Academic Press, 2001.
- [2] Mingsian R. Bai, Yuanpei Lin, and Jienwen Lai, "Reduction of electronic delay in active noise control systems a multi rate signal processing approach," *Journal of the Acoustical Society of America*, vol. **111**, pp. 916–924, February 2002.
- [3] C.R. Fuller, S.J. Elliott and P.A. Nelson, *Active Control of Vibration*, Academic press limited, 24-28 Oval Road, London, 1996.