

# RESTORATION OF MISSING LINES IN GRIP PATTERNS FOR BIOMETRICS AUTHENTICATION ON A SMART GUN

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## ABSTRACT

The Secure Grip project<sup>1</sup> aims to develop a grip-pattern recognition system, as part of a smart gun. Its target users are the police officers. The current authentication algorithm is based on a likelihood-ratio classifier. The grip pattern is acquired by sensors on the grip of the gun. Since in practice various factors can result in missing lines in a grip pattern, restoration of these missing lines will be useful and practical. We present a restoration algorithm based on null-space error minimization. The simulation results of the restoration and authentication experiments show that this restoration algorithm effectively restores grip patterns, and is, therefore, capable of improving the system's authentication performance when missing lines are present.

## 1. INTRODUCTION

The Secure Grip project aims to develop a prototype recognition system as part of a smart gun, where the grip-pattern recognition ensures that it can only be fired by its rightful user. This application is intended for use by the police officers, since carrying a gun in public brings considerable risks [1]. The first prototype of this smart gun system (see Figure 1) was described in [2] and [3], in terms of its design, implementation and evaluation. Authentication simulations were made based on a likelihood-ratio classifier, by using the grip-pattern data collected from a group of police officers (see Figure 2). The simulation results indicated that the grip pattern contains sufficient information to authenticate the police officers [3].

During data collection we observed that sometimes the grip-pattern images had a couple of lines missing, due to some damage in the cable of the demonstrator (see Figure 3). This will degrade the authentication performance of the system. In addition, the registration result of data before authentication can be affected, which will further add to degradation of the authentication performance. In practice there can be various factors resulting in missing lines in a grip-pattern image, while using a smart gun,

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Figure 1: *Prototype of a smart gun.*

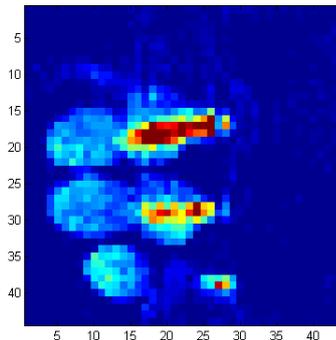


Figure 2: *An example of the grip-pattern image.*

such as damages in the cable or in the sensor, or lose contacts between electric components. Thus, restoration of these missing lines will be useful and practical. In our work, we apply a restoration algorithm based on null-space error minimization. We assume that the missing lines are only present in grip patterns of the test set, while in those of the training set complete data are available. This paper presents the restoration algorithm and analyzes its effect on the authentication performance of the smart gun system. Section 2 reviews the authentication algorithm. The restoration algorithm is described in Section 3. Subsequently, Section 4 presents and analyzes the simulation results of the restoration and authentication experiments. Finally, the conclusions are given in Section 5.

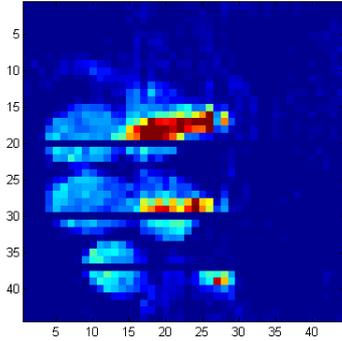


Figure 3: *Grip-pattern image with missing lines.*

## 2. AUTHENTICATION ALGORITHM

The authentication algorithm is based on a likelihood-ratio classifier for Gaussian probability densities. The likelihood-ratio classifier has been proved to be optimal, in terms of the average overall error rates if the data has a known probability density function [4], [5]. We assume that both the overall data, and that of each individual subject, are Gaussian distributed. The pixel values of each grip-pattern image are aligned into a column vector, and are used as the features in the algorithm. In our system, the feature space has the dimension of 1936, since each grip pattern was recorded as a 44 by 44 image.

In practice, the exact probability density function of neither the overall data, nor the data of each individual subject is known, and therefore needs to be estimated from the training data. In total, four parameters need to be estimated. They are the mean vector of the overall data, the mean vector and the covariance matrix of each individual subject's data, and the transformation matrix. In our case, the number of training samples from each subject should be much more than 1936. This is to prevent the estimated covariance matrices from being singular, and to prevent the classifier suffering from overtraining. Otherwise, the algorithm would suffer from the so-called small-sample-size problem. However, we cannot make this large number of measurements, since it would be rather impractical for the user enrollment.

To solve the small-sample-size problem, we assume that each subject has the same within-class covariance matrix. Thus, this common covariance matrix can be estimated more accurately, by using data from all the subjects. As a further step to relieve the effect of the small-sample-size problem, we reduce the dimension of the feature space. Firstly, we apply a PCA (Principal Component Analysis) to project the overall data to a subspace with the biggest variances. In this new space, the directions contributing to the authentication, are not more than the number of sub-

jects minus one. Also, these directions have the smallest variances of each individual subject's data [2]. Accordingly, a further dimension reduction is achieved, by applying a PCA to each individual subject's data, and discarding all the directions, except only the 40 (41 subjects in total) with the smallest variances.

## 3. RESTORATION ALGORITHM

We assume that the covariance matrix of the feature vectors has a null space of a dimensionality that is at least the number of unknown features. Let the column vector  $x$  denote a feature vector with  $N$  elements. Furthermore, let

$$x = \begin{pmatrix} u \\ v \end{pmatrix}, \quad (1)$$

with  $u$  containing the  $m$  unknown features and  $v$  containing the  $N - m$  known ones. We assume that  $x$  has a zero mean. Let  $\hat{u}$  denote the estimate for  $u$ :

$$\hat{u} = \mathbf{H}^T v. \quad (2)$$

Let the feature space be an  $M$ -dimensional subspace of  $\mathbb{R}^N$  described by

$$\begin{aligned} O &= \{x | x = \mathbf{F}\phi, \mathbf{F} \in \mathbb{R}^N \times \mathbb{R}^M, \\ &M \leq N - m, \phi \in \mathbb{R}^M, \mathbf{F}^T \mathbf{F} = \mathbf{I}\}. \end{aligned} \quad (3)$$

The columns of  $\mathbf{F}$  compose an  $M$ -dimensional orthonormal basis of the feature subspace. Note that  $M$  is assumed to be much smaller than  $N$ . We look for  $\hat{u}$ , such that the norm of the signal component in the null space, the null-space error, after restoration

$$E_0 = \|(\mathbf{I} - \mathbf{F}\mathbf{F}^T) \begin{pmatrix} \hat{u} \\ v \end{pmatrix}\|^2 \quad (4)$$

is minimized [6]. The matrix  $(\mathbf{I} - \mathbf{F}\mathbf{F}^T)$  projects onto the null-space of  $\mathbf{F}$ , therefore

$$(\mathbf{I} - \mathbf{F}\mathbf{F}^T)^2 = (\mathbf{I} - \mathbf{F}\mathbf{F}^T). \quad (5)$$

We partition  $\mathbf{F}$  in two sub-matrices:

$$\mathbf{F} = \begin{pmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{pmatrix}, \quad (6)$$

with  $\mathbf{F}_1$  an  $m \times m$  matrix and  $\mathbf{F}_2$  an  $(N - m) \times m$  matrix. By using (5) and (6), the null-space error (4) can be rewritten as

$$\begin{aligned} E_0 &= (\hat{u}^T v^T)(\mathbf{I} - \mathbf{F}\mathbf{F}^T) \begin{pmatrix} \hat{u} \\ v \end{pmatrix} \\ &= \|\hat{u}\|^2 + \|v\|^2 - \|\mathbf{F}_1^T \hat{u} + \mathbf{F}_2^T v\|^2. \end{aligned} \quad (7)$$

Setting the derivative with respect to  $\hat{u}$  equal to zero yields the following equation

$$(\mathbf{I} - \mathbf{F}_1 \mathbf{F}_1^T) \hat{u} = \mathbf{F}_1 \mathbf{F}_2^T v, \quad (8)$$

from which  $\hat{u}$  can be solved.

We briefly look at the sensitivity of this restoration algorithm to model errors, which occur when the observed  $v$  deviates from the assumptions. We assume that

$$x = x' + g \quad (9)$$

$$= \begin{pmatrix} u' \\ v' \end{pmatrix} + \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}, \quad (10)$$

with  $g$  an error component in the null space of  $\mathbf{F}$ . This means that  $\mathbf{F}_1^T g_1 + \mathbf{F}_2^T g_2 = 0$ . The error component in the restoration is now given by

$$\begin{aligned} \epsilon &= (\mathbf{I} - \mathbf{F}_1 \mathbf{F}_1^T)^{-1} \mathbf{F}_1 \mathbf{F}_2^T g_2 \\ &= -(\mathbf{I} - \mathbf{F}_1 \mathbf{F}_1^T)^{-1} \mathbf{F}_1 \mathbf{F}_1^T g_1 \\ &= g_1 - (\mathbf{I} - \mathbf{F}_1 \mathbf{F}_1^T)^{-1} g_1. \end{aligned} \quad (11)$$

The first term reflects the presence of model errors in the part that is to be restored. The second term reflects the error due to the compensation for model errors in the given data. The compensation error grows strongly when the eigenvalues of  $\mathbf{F}_1 \mathbf{F}_1^T$  approach 1.

Note that this restoration algorithm works only in the case that the missing lines are present in grip patterns of the test set, while in those of the training set complete data are available. If there are also missing lines in data of the training set, some other type of restoration algorithm should be applied. Linear interpolation, for example, could be a choice.

## 4. EXPERIMENT, RESULTS AND DISCUSSION

In this section simulation results of the restoration and authentication experiments are presented and analyzed, in terms of the average restoration error per pixel of the missing lines and the overall Equal-Error Rate (EER), respectively.

### 4.1. Experiment set-up and results

We collected data from a group of 41 police officers, with 25 grip-pattern images recorded from each. We assumed that the missing lines were only present in data of the test set, while in that of the training set, no lines were missing. We made simulations of restoration in different cases, where the number of missing lines increased from one till ten in each sample of the test set. In all cases 75% of the data was randomly chosen for training, and the rest 25%

Number of missing lines	Average restoration error per pixel
1	105.01
2	119.27
3	118.04
4	117.84
5	114.11
6	117.89
7	118.73
8	118.62
9	117.27
10	114.92
$\sigma_W^2$	239.62

Table 1: Restoration error per pixel.

for testing. The missing lines in each sample were randomly chosen as well. To evaluate the restoration quality we calculated the average restoration error per pixel. The simulation results are shown in Table 1. As a reference, the average within-class variance per pixel  $\sigma_W^2$  of the training set is also presented.

To evaluate the effect of the restoration of missing lines on the authentication performance, we did two experiments. In the first experiment, the test set consisted of samples where a certain number of lines were missing in each sample. That is, pixel values in those lines were all set to zero. While in the second experiment, the test set was composed of samples, where the missing lines were restored with our restoration algorithm. The authentication performance was assessed by the overall Equal-Error Rate (EER). This was calculated by taking into account all the likelihood-ratios of both the genuine subjects and the imposters. We averaged the overall EERs obtained from 20 runs of simulation as the final result. In each single run, we randomly chose 75% of the data for training, and used the rest 25% for testing. This whole procedure was repeated, with the number of missing lines in each sample of the test set increasing from one till ten. The simulation results are shown in Table 2. For easy comparison, the simulation result when the test set is without missing lines is also presented. *MLs* and *RLs* represent missing lines and restored lines, respectively, and  $\sigma_{\text{EER}}$  represents the standard deviation of the EERs in all 20 runs. The authentication results are further shown in Figure 4, as the number of missing lines increases.

### 4.2. Discussion

One can see from Table 1 that the average restoration error per pixel is fairly constant, when different numbers of missing lines in the data are present. That is, the restoration error per pixel seems to be robust against the increase

Complete test set	EER (%) = 0.55		
Number of <i>MLs</i>	EER (%) with <i>MLs</i>	EER (%) with <i>RLs</i>	$\sigma_{\text{EER}}$ (%)
1	0.65	0.59	0.19
2	0.70	0.67	0.27
3	0.65	0.62	0.20
4	0.81	0.66	0.21
5	1.03	0.65	0.21
6	1.06	0.70	0.28
7	1.20	0.79	0.33
8	1.20	0.81	0.25
9	1.52	0.85	0.26
10	1.50	0.93	0.31

Table 2: *EERs obtained in authentication experiment.*

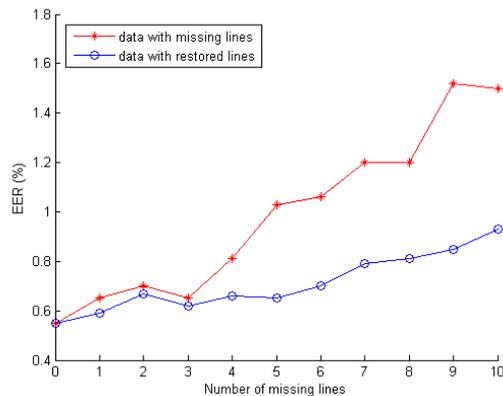


Figure 4: *EERs obtained by using data with missing and restored lines as function of the number of missing lines.*

in the number of missing lines. Also, the average restoration error per pixel is much smaller, compared to the average within-class variance per pixel of the training set. This indicates that the restoration algorithm restores data effectively.

In addition, Table 2 shows that up to three missing lines, the authentication results are not affected very much in comparison with that by using a complete data set. This suggests that the current authentication algorithm is robust against the presence of a small number of missing lines. However, as the number of missing lines in the test set becomes higher, the authentication performance degrades more. Thus, restoration of missing lines in the grip patterns is useful, especially when the number of missing lines is relatively high.

Finally, the presented restoration algorithm effectively restores the missing lines, and therefore improves the system's authentication performance. On the other hand, as shown in Figure 4, the authentication result by using the

restored data becomes worse as the number of missing lines in each sample increases. This is because that the more missing lines are present, the higher restoration error results in a grip-pattern image. However, on the whole this restoration algorithm works well to improve the authentication performance of the system when missing lines are present.

## 5. CONCLUSIONS

In the smart gun system the current authentication algorithm seems to be robust against the presence of a small number of missing lines. However, restoration of missing lines in the grip patterns is necessary and practical, when the number of missing lines becomes higher, say more than five in the case of a grip pattern of  $44 \times 44$  pixels.

The presented restoration algorithm has been proved to be able to effectively restore the missing lines in grip-pattern data. In addition, the average restoration error per pixel seems to be robust against the increase in the number of missing lines. On the other hand, the authentication result by using the restored data becomes worse as the number of missing lines increases, due to the accordingly increased restoration error. Yet, on the whole this restoration algorithm works well to improve the authentication performance of the system when missing lines are present.

## 6. REFERENCES

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